ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

1st Term Examination – 2018

Class : 11 B , C

Sub : Mathematics **F.M.: 80**

DURATION:3 Hrs15Mins **DATE:30.07.2018**

Group- A

(I) Choose the correct option:- (8 X 1 = 8)

1) If B be the power set of A, state which of the following is true?
   a) A - B b) B - A c) $A \in B$ d) A = B

2) State which of the following is the value of Cot (- 870 °)?
   a) √ 3 b) 1/√3 c) -1/√3 d) - √3

3) If Cos A + Cos B = 2 , then which of the following is the value of Cos (A + B)?
   a) 1 b) 0 c) -1 d) 2

4) Cot 2θ + tan θ =
   a) $\sin^2 2\theta$ b) $\cot^2 2\theta$ c) $\text{Cosec}^2 2\theta$ d) $\tan^2 2\theta$

5) 2 Sin 40° Sin 10° =
   a) Cos 30° + Cos 50° b) Cos 30° - Cos 50° c) Cos 50° - Cos 30° d) none of these

6) The maximum value of $4x - x^2 - 2$ is
   a) 0 b) 1 c) 2 d) 3

7) The straight line joining the points (-3 , - 4) and (2 , 5) is
   a) 5x - 9y = 21 b) x - 2y + 8 = 0 c) 9x - 5y + 7 = 0 d) 4x - 3y + 7 = 0

8) The perpendicular of the straight line 3x + 4y +15 = 0 from the origin is
   a) 3 units b) 4 units c) 5 units d) 15 units

Group - B

(II) Answer the following questions:- (4 X 6 = 24)

1) A, B, C, D are the four angles taken in order of a cyclic quadrilateral. Prove that
   Cot A + Cot B + Cot C + Cot D = 0

2) If A, B and C are the angles of a triangle, show that

$$\frac{\text{Cos A Cos C} + \text{Cos}(\text{A}+\text{B})\text{Cos}(\text{B}+\text{C})}{\text{Cos A Sin C} - \text{Sin}(\text{A}+\text{B})\text{Cos}(\text{B}+\text{C})} = \text{Cot C}$$

3) If $0 < x < \pi/2$ and $\text{Cos } x + \text{Sin } x = \sqrt{2}$, then find the value of Sin 3x.

4) Find the maximum and minimum values of 5 Cos θ + 12 Sin θ + 12

5) Find the value of ½ Sec 80° - 2 Cos 20°

6) Show that 4 Sin A Sin B Sin C = Sin(A+ B - C) + Sin (B + C – A) + Sin (C+A - B) - Sin (A+B+C)

Group – C

(III) Answer the following questions:- (4 X 6 = 24)

1)For any two sets A and B, prove that $(A \cup B)^C = A^C \cap B^C$

2) If S = { a , b , c , d , e , f } be the universal set and A , B , C are three subsets of S where A = { a , c , d , f }, B ∩ C = { a , b , f }, find (A U B) ∩ (A U C) and B' U C'.

3) Applying the laws of algebra of sets, prove that (A U B) ∩ A = A

4) In an examination 45% of the candidates have passed in English, 40% have passed in Bengali, while 30% have passed in both the subjects. Find the total number of candidates if 90 of them have failed in both the subjects.

5) If one root of the equation $x^2 + rx - s = 0$, is square of the other, prove that $r^3 + s^2 + 3sr - s = 0$

6) If a , b , c are rational and a + b + c = 0, show that the roots of the equation $ax^2 + bx + c = 0$ are rational.

Group – D

(IV) Answer the following questions:- (4 X 6 = 24)

1) Find the equation of the straight line parallel to y axis and passing through the point (-2 , 3)

2) If 3a + 2b + c = 0 for all positions of the moving line ax + by + c = 0, show that the line always passes through a fixed point. Find the co ordinates of the fixed point.

3) If the angle between the lines y = x - 6 and y = mx + 6 be 60°, find m.

4) A straight line if perpendicular to the straight line 3x - 4y = 6 and passes through (2 , 1). Find the equation of the straight line.

5) Find the perpendicular distance of the straight line 4x - y = 5, from the point (2 , -1)

6) Find the equation of the straight line equidistant from the point (-2, 3) and the line 8y = 9x -12.

Clay
1.8.18.

ST. LAWRENCE HIGH SCHOOL

First Term Examination

Sub: Mathematics **Class: XI B , C** **FM: 80**

Duration: 3 hrs 15mins **Date: 30.07.18**

Answers

Group- A

(I) 1)C 2)A 3)A 4) C 5) B 6) C 7) C 8)A

Group - B

(II) 1) Opp. Angles of a cyclic quadrilateral are supplementary.

A + C = 180 and B + D = 180

LHS = Cot A + Cot B + Cot(180-A) + Cot(180-B)

= Cot A + Cot B – Cot A – Cot B

2) Consider A + B + C = 180

Hence A + B = 180 – C

And B + C = 180 – A

3) cos x + sin x = √2

Or, sin π/4 Cosx + Cos π/4 Sinx = 1

Or, x + π/4 = π/2

Or, x = π/4

Hence Sin 3x = Sin 3 π/4 = 1/√2

4) Refer to Example 15 of page no 125

5) $\frac{1}{2Cos80}$ - 2 Cos20 $= \frac{1-2\,(Cos\,100+\cos 60)}{2\,Cos80} = \frac{1+2\cos 80}{2\,Cos\,80} = 1$

6) L.H.S = 2 Sin A [Cos (B - C) - Cos { B + C)]

= 2 Sin A Cos (B - C) - 2 Sin A Cos (B + C)

= Sin (A + B - C) + Sin (A - B + C) - [Sin (A + B - C) - Sin (B + C - A)]

Group – C

(III) 1) Refer to De Morgan's Law

2) (A U B) ∩ (A U C) = A U (B ∩ C) [by distributive law]

= { x : x ∈ A v x ∈ (B ∩ C) [by definition of union]

= { a , b , c , d , f }

Again B' U C' = (B ∩ C)' [by De Morgan's law]

= { c , d ,e }

3) (A U B) ∩ A = (A U B) ∩ (A U Ø) [by identity law]

= A U (B ∩ Ø) [by distributive law]

= A U Ø [by identity law]

= A [by identity law]

4) Let A and B be the sets of candidates who passed in English and Bengali resp. We have n(A) = 45% , n(B) = 40% , and n(A∩B) = 30%

By using formula n(A U B) = n(A) + n(B) – n(A ∩ B)
We get, n (A U B) = 55%
Therefore 55% candidates passed in at least one of the two subjects
Hence (100 – 55) % = 45% candidates failed in both the subjects.
Hence 90 candidates failed in both subjects when total no of candidates = $\frac{100X90}{45}$ = 200
5) One root of the equation is the square of the other. Let us assume that the roots of the equation are α and α^2
Hence α (1 + α) = - r --------(1) and α^3 = - s
Now cubing both side of eq (1) we get
-s [1 - s - 3r] = - r^3
Or $r^3 + s^2 + 3$ sr - s = 0
6) b = - (c + a)
The discriminant of the equation is
{ - (c + a) }^{2} - 4ac = (c +a)^2 – 4ac = (c -a)^2
Since a , b , c are rational and the discriminant of equation is perfect square, hence the roots of the equation are rational.

Group – D

(IV)1) Let us assume that the equation of the reqd. straight line is x = a. As it passes through pt (-2, 3) we have, a = -2.
Hence the eq. of the straight line is x + 2 = 0
2) From the given condition we have c = -3a - 2b. Now eq. of the moving straight line is ax + by -3a -2b = 0 , or a(x - 3) + b (y - 2) = 0 ----------(1).
The straight line (1) passes through the point of intersection of the straight lines x – 3 = 0 and y – 2 = 0, whose point of intersection is (3,2)
Hence the straight line (1) passes through a fixed point and its co ordinates are (3, 2)
3) Slopes of the given line are 1 and m resp. By the problem we have
tan60= $\pm\frac{1-m}{1+m.1}$, taking positive and solving we get m = -2 + √3
And by taking - sign and solving we get m = - 2 - √3 which are the reqd. values of m.
4) The eq. of the straight line perpendicular to the straight line 3x – 4y = 6 is 4x + 3y = c. If this line passes though pt (2 , 1) then we have 4x2 + 3x1 = c, or c = 11
Therefore the reqd eq of the straight line is 4x + 3y = 11
5) If p be the perpendicular distance of the straight line from the pt (2, -1) then we have
P $\pm\frac{4.2-(-1)-5}{\sqrt{4^2+(-1)^2}}$ = 4/√17.
The reqd. perpendicular distance of the straight line 5x - 12y + 7 = 0 from the pt (3,4) is 26/13 = 2 unit by following the above procedure.
6) 8y – 9x + 12 = 0 ----------(1).
Let us assume the eq. of the straight line is 8y - 9x + c = 0----------(2)
By question the perpendicular distance of (-2,3) form the line (1) = 2 X perpendicular distance of (-2,3) from the line (2)
Or, 24 + 18 + 12 = 2 (24 + 18 + c) , or c = -15
Hence the reqd eq is 8y - 9x - 15 = 0