ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

1st Term Examination - 2018

Class : 11 A1 , D

Sub : Mathematics — F.M.: 80

DURATION:3 Hrs15Mins — DATE:30.07.2018

[Relevant rough work must be done in the margin of the page containing the answers]

GROUP : A

Select the correct alternatives : **1x8=8**

1. If $A=\{a,b,c\}; B=\{a,b\}; C=\{a,b,d\}$ and $D=\{c,d\}$ then

   a) $B\supset A$ b) $D\subset A$ c) $B\subset C$ d) $D\subset C$

   or

   Find the value of $(1 + i)\left(1 - \frac{1}{i}\right)$

2. Which one of the following statement is correct?

   a) $\{1\}\in\{1,2,3\}$ b) $1\in\{1,2,3\}$ c) $1\subset\{1,2,3\}$ d) $\{1\}\subset\{1,2,3\}$

   or

   Find $\sin 45^{0}\cos 65^{0}+\sin 135^{0}\cos 115^{0}$

3. The value of $\tan 1575^{0}$ is

   a) 1 b) -1 c) $-\sqrt{3}$ d) $\sqrt{3}$

4. The smallest value of $5\cos\theta+12$ is

   a) 5 b) 12 c) 7 d) 17

5. The maximum value of $\sin x+\cos x$ is

   a) $\sqrt{2}$ b) 2 c) $\frac{1}{\sqrt{2}}$ d) 1

6. If n is a positive integer; then $i^{n}+i^{n+1}+i^{n+2}+i^{n+3}$ is equal to

   a) 1 b) i c) i^{n} d) 0

7. The straight line $ax+by+c=0$ will be parallel to x-axis if

   a) $a=0, c=0$ b) $a=0, c\neq 0$ c) $b=0, c=0$ d) $b=0, c\neq 0$

8. The gradient of the line parallel to y-axis is
a) −1 b) 0 c) 1 d) undefined

GROUP : B

Answer any six questions : **4x6=24**

9. If ϖ be an imaginary cube root of unity and $\sqrt[3]{x}=\varpi.\sqrt[3]{a}+\varpi^2.\sqrt[3]{b}$; show that $(x-a-b)^3=27abx$.

10. Find the cube roots of i.

11. Using principle of mathematical induction prove that
$1^3+3^3+5^3+....+(2n-1)^3=n^2(2n^2-1)$

12. Prove that for any three sets A, B and C; $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

13. The sum of 4 integers in A.P. is 24 and their product is 945. Find the integers.

14. Solve graphically : (Graph paper is not required)
$3x+4y\le 12,\ x+2\ge 0;\ y\le 3$

15. For what value of x, the relation $|x|+|x-1|\ge 5$ will be satisfied?

16. Express $\dfrac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ in the polar form.

GROUP : C

Answer any six questions : **4x6=24**

17. Evaluate : $\sin 10^0 \sin 50^0+\sin 50^0 \sin 250^0+\sin 250^0 \sin 10^0$

18. If $\sin\theta=n\sin(\theta+2\alpha)$; prove that $\tan(\theta+\alpha)=\dfrac{1+n}{1-n}\tan\alpha$

19. Evaluate : $\cos\dfrac{2\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{8\pi}{15}\cos\dfrac{16\pi}{15}$

20. Express $u=\sin^6 x+\cos^6 x$ in the form of $A+B\cos 4x$ and hence obtain the maximum and minimum value of u.

21. If $\theta+\phi=\dfrac{\pi}{4}$; show that $(1+\tan\theta)(1+\tan\phi)=2$. Hence prove that
$\tan\dfrac{\pi}{8}=\sqrt{2}-1$

22. Show that $\sin 16^0 + \cos 16^0 = \frac{1}{\sqrt{2}}\left(\sqrt{3}\cos 1^0 + \sin 1^0\right)$

23. If $2\tan\theta = \cot\phi$; then prove that $\cos(\theta - \phi) = 3\cos(\theta + \phi)$

GROUP : D

Answer any six questions :

4x6=24

24. Find the ratio is which the straight line $4x + y = 0$ divides the line segment joining the joins (6,4) and $(-1,-7)$

25. Find the equation of the straight line which passes through the point of intersection of the two straight lines $2x + 3y = 3$ and $x - 2y = 1$ and at a unit distance from the origin.

26. If the angle between the straight lines $2x - 2y + 5 = 0$ and $y = mx + 4$ is 60^0; find the value of m.

27. Find the equation of the straight line passing through the point (3,5) and perpendicular to the straight line $14x - 3y + 1 = 0$

28. A ray of light coming along the straight line $x - 2y + 5 = 0$ reflects at the mirror which is situated along the straight line $3x - 2y + 7 = 0$. Find the equation of the reflected ray.

29. Find the locus of the mid point of the portion of the line $x\cos\alpha + y\sin\alpha = p$ along the axes of co-ordinates.

30. The co-ordinates of the two vertices of a triangle are $(-2,3)$ *and* $(5,-1)$ respectively. If the ortho centre of the triangle be of the origin; find the co-ordinates of the third vertex of the triangle.

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3.8.16

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CLASS-XI A1&D

SOLUTION

GROUP : A

1. (c) 2. (d) 3. (b) 4. (c) 5. (a) 6. (d) 7. (b) 8. (d)

GROUP : B

9. $\therefore \sqrt[3]{x} = \omega\sqrt[3]{a} + \omega^2 \sqrt[3]{a} \Rightarrow x = \omega^3 a + \omega^6 b + 3\omega^3 \sqrt[3]{x}\sqrt[3]{ab}$

$\Rightarrow x = a + b + 3\sqrt[3]{x}\sqrt[3]{ab}$

$\Rightarrow (x-a-b)^3 = 27abx$

10. Let, $x = \sqrt[3]{i} \Rightarrow x^3 = i = -i^3 \Rightarrow (x+i)(x^2 - ix - 1) = 0$

$\therefore x = -i;\ xh2 - ix = 0 \Rightarrow x = \frac{i \pm \sqrt{i^2+4}}{2} = \frac{i \pm \sqrt{3}}{1}$

11. $P(n): 1^3 + 3^3 + 5^3 + ... + (2n-1)^3 = n^2(2n^2 - 10)$

$P(1): LHS = (2.1-1)^3 = 1$

RHS = $1^2(2.1^2 - 1) = 1$

$P(m+1): 1^3 + 3^3 + 5^3 + ... + (2m-1)^3 + (2m+1)^3$

$= m^2(2m^2 - 1) + (2m+1)^3$

$= 2m^4 - m^2 + 8m^3 + 12m^2 + 6m + 1$

$= 2m^4 + 8m^3 + 11m^2 + 6m + 1$

$= 2m^4 + 2m^3 + 6m^3 + 6m^2 + 5m^2 + 5m + m + 1$

$= 2m^3(m+1) + 6m^2(m+1) + 5m(m+1) + (m+1)$

$= (m+1)(2m^3 + 6m^2 + 5m + 1)$

$= (m+1)[2m^3 + 2m^2 + 4m^2 + 4m + m + 1]$

$= (m+1)[2m^2(m+1) + 4m(m+1) + 1(m+1)]$

$= (m+1)2(2m^2 + 4m + 1)$

$= (m+1)^2[2(m\_1)^2 - 1]$

$\therefore P(n)$ holds true for n = m+1.

12. Consult Text Book.

13. Let the four numbers be $(a-3d), (a-d), (a+d), (a+3d)$

By the problem, $a - 3d + + a - d + a + d + a + 3d = 24 \Rightarrow a = 6$

Again, $(6-3d)(6-d)(6+d)(6+3d) = 945$

$\Rightarrow (4-d^2)(36-d^2) = 105$

$\Rightarrow d^4 - 40d^2 + 39 = 0$

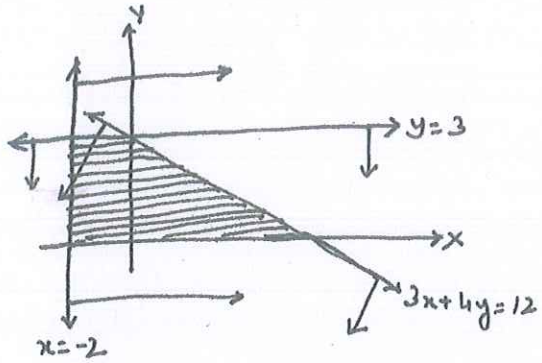
$\Rightarrow d^2 = 1;\ d^2 = 39$

$\therefore d = \pm 1;\ d = \pm\sqrt{39}$

$\therefore$ The numbers are integers, $d \neq \pm\sqrt{39};\ d = \pm 1$

The required numbers are 3,5,7,9 or 9,75,3 (Ans.)

14. $3x+4y+12 \Rightarrow \frac{x}{4}+\frac{y}{3}=1$



15. $|x|+|x-1| \geq 5$

$\pm x \pm (x-1) \geq 5$

$\therefore x+(x-1) \geq 5 \;\; or \; -x(x-1) \geq 5$

$2x \geq 6 \; -2x \geq 4$

$x \geq 3 \;\; x \leq -2$

$\therefore (-\infty < x \leq -2) \cup (3 \leq x < \infty)$ (Ans)

16. Given Exprn. $= \frac{1}{2}\left[\cos 40^0 - \cos 60^0 + \cos 200^0 - \cos 300^0 + \cos 240^0 - \cos 260^0\right]$

$= \frac{1}{2}\left[\cos 40^0 + \cos 200^0 - \cos 260^0 - \frac{1}{2} - \frac{1}{2} - \frac{1}{2}\right]$

$= \frac{1}{2}\left[2\cos 120^0 \cos 80^0 - \cos(180^0 + 80^0) - \frac{3}{2}\right]$

$= \frac{1}{2}\left[-\cos 80^0 + \cos 80^0 - \frac{3}{2}\right] = -\frac{3}{4}$ (Ans)

17. $\sin\theta = n\sin(\theta+2\alpha) \Rightarrow \frac{\sin\theta}{\sin(\theta+2\alpha)} = n \Rightarrow \frac{1+n}{1-n} = \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta}$

$= \frac{2\sin(\theta+\alpha)\cos\alpha}{2\cos(\theta+\alpha)\sin\alpha}$

$\tan(\theta+\alpha) = \frac{1+n}{1-n}\tan\alpha = an(\theta+\alpha)\cot\alpha$

18. Let, $\frac{2\pi}{15} = \theta$ Given Expression $= \cos\theta + \cos 2\theta \cos 4\theta \cos 8\theta$

$= \frac{1}{2\sin\theta}(\sin 2\theta \cos 2\theta \cos 4\theta \cos 8\theta)$

$= \frac{1}{4\sin\theta}\sin 4\theta \cos 4\theta \cos 8\theta$

$= \frac{1}{8\sin\theta}\sin 8\theta \cos 8\theta$

$= \frac{1}{16\sin\theta}\sin 16\theta$

$= \frac{1}{16\sin\theta}\sin(2\pi+\theta) = \frac{1}{16}$

19. $u=\sin^6 x+\cos^6 x$

$= 1-3\sin^2 x\cos^2 x$

$= 1-\frac{3}{4}\sin^2 2x$

$= 1-\frac{3}{8}(1-\cos 4a)$

$= \frac{5}{8}+\frac{3}{8}\cos 4a$

$\therefore -1\le \cos 4x \le 1$

$\therefore u_{\min}=\frac{5}{8}-\frac{3}{8}=\frac{1}{4}$

$\therefore u_{\min}=\frac{5}{8}+\frac{3}{8}=1$

20. $\theta+\phi=\frac{\pi}{4}\Rightarrow \tan(\theta+\phi)=1\Rightarrow \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}=1$

$\Rightarrow \tan\theta+\tan\phi+\tan\theta\tan\phi=1$

$\Rightarrow (1+\tan\theta)(1+\tan\phi)=2$

Let $\theta=\phi=\frac{\pi}{8}$

$\therefore \left(1+\tan\frac{\pi}{8}\right)^2=2$

$\therefore 1+\tan\frac{\pi}{8}=\pm\sqrt{2}$

$\therefore \tan\frac{\pi}{8}=\pm\sqrt{2}-1$

But, $\frac{\pi}{8}$ is an acute angle, $\tan\frac{\pi}{8}>0$

$\therefore \tan\frac{\pi}{8}=\sqrt{2}-1$

21. $\sin 16^0+\cos 16^0=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 16^0+\frac{1}{\sqrt{2}}\cos 16^0\right)$

$= \sqrt{2}(\sin 45^0\sin 16^0+\cos 45^0\cos 16^0)$

$= \sqrt{2}\cos 29^0$

$= \sqrt{2}\cos(30^0-1^0)$

$= \sqrt{2}(\cos 30^0\cos 1^0+\sin 30^0\sin 1^0)$

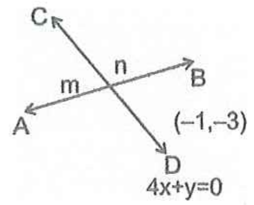
$= \sqrt{2}\left(\frac{\sqrt{3}}{2}\cos 1^0+\frac{1}{2}\sin 1^0\right)=\frac{1}{\sqrt{2}}(\sqrt{3}\cos 1^0+\sin 1^0)=RHS$

22. $2\tan\theta=\cot\phi\Rightarrow \tan\theta\tan\phi=\frac{1}{2}$

$\Rightarrow \frac{\sin\theta\sin\phi}{\cos\theta\cos\phi}=\frac{1}{2}$

$\Rightarrow \frac{\cos\theta\cos\phi+\sin\theta\sin\phi}{\cos\theta\cos\phi-\sin\theta\sin\phi}=\frac{2+1}{2-1}$

$\Rightarrow \cos(\theta-\phi)=3\cos(\theta+\phi)$

23. 

Let $A(6,4)$; $B(-1,-7)$ be the line segment.

$\therefore 0:\left(\frac{-m+6n}{m+n},\frac{-7m+4n}{m+n}\right)$

The above co-ordinate satisfy the line $4x+y=0$

$\therefore \frac{-4m+24n}{m+n}+\frac{-7m+4n}{m+n}=0$

$\Rightarrow -4m+24n-7m+4n=0$

$\Rightarrow 11m=28n \Rightarrow \frac{m}{n}=\frac{28}{11}$

$\therefore$ Reqd. ratio is 28:11 (Ans)

24. Equation of the straight line that passes thro' the point of intersection of intersection of $2x-3y=3$ *and* $x-2y=1$ is

$2x-3y-3+\lambda(x-2y-1)=0$

$\Rightarrow (z+\lambda)x-(2+2\lambda)y=\lambda+3$

$\therefore$ The above line is at a unit distance from the origin.

$$\frac{\lambda+3}{\sqrt{(2+\lambda)^2+(3+2\lambda)^2}}=1$$

$\therefore \lambda^2+9+6\lambda=4+4\lambda+\lambda^2+9+4\lambda^2+12\lambda$

$\therefore 4\lambda^2-10\lambda+4=0$

$\Rightarrow 2\lambda^2-5\lambda+2=0$

$\Rightarrow \lambda=2;\frac{1}{2}$

If $\lambda=2$; Reqd. equation; $4x-7y=5$

$\lambda=\frac{1}{2}$; Reqd. Equation; $\frac{5}{2}x-4y=\frac{7}{2}$

$5x-8y=7$ (Ans)

25. Slopes of the straight lines are 1 and m respectively.

$\therefore \tan 60^0=\pm\frac{1-m}{1+m}\Rightarrow\frac{1-m}{1+m}=\pm\sqrt{3}$

On solving; $m=\sqrt{3}-2$ *or* $-(2+\sqrt{3})$ (Ans)

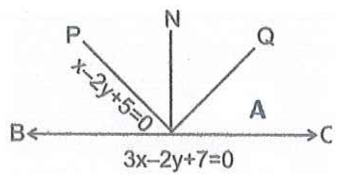
26. Equation of the line perpendicular to $14x-3y+1=0$ *is* $3x+14y+k=0$

The above line passes thro' (3,5);

$9+70+k=0\Rightarrow k=-79$

Required equation is $3x+14y=79=0$ (Ans)

27.



$\because \angle PAN = \angle QAN$

$\therefore \angle PAB = \angle QAC$

Equation of AQ is

$(x-2y+5)+\lambda(3x-2y+7)=0$

$\Rightarrow (1+3\lambda)x-(2+2\lambda)y+(5+7\lambda)=0$

$\therefore m_{PA}=\frac{1}{2};\ m_{BC}=\frac{2}{3}\ m_{AQ}=\frac{1+3k}{2+2k}$

$\tan\angle PAB = \tan\angle QAC$

$$\Rightarrow \frac{\frac{3}{2}-\frac{1}{2}}{1+\frac{3}{2}\cdot\frac{1}{2}}=\frac{\frac{1+3k}{2+2k}-\frac{3}{2}}{1+\frac{1+3k}{2+2k}\cdot\frac{3}{2}} \Rightarrow k=-\frac{14}{13}$$

$\therefore$ Required equation of the reflected ray $\Rightarrow 29x-2y+33=0$ (Ans)

28. $x\cos\alpha + y\sin\alpha = p \Rightarrow \frac{x}{\frac{p}{\cos\alpha}}+\frac{y}{\frac{p}{\sin\alpha}}=1$

The above line cuts x-axis at $\left(\frac{p}{\cos\alpha},0\right)$ & y-axis $\left(0,\frac{p}{\sin\alpha}\right)$

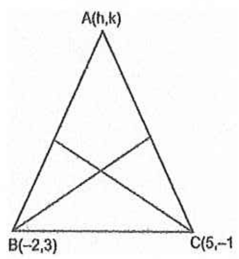
Co-ordinate of the mid point : $\left(\frac{p}{2\cos\alpha},\frac{p}{2\sin\alpha}\right)$

Let $p(h,k)$ be the locus of the moving point.

$\therefore h=\frac{p}{2\cos\alpha};k=\frac{p}{2\sin\alpha}$

$\therefore 1=\frac{p^2}{4h^2}+\frac{p^2}{4k^2}\Rightarrow \frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$ (Ans)

29.



$m_{AO}=\frac{k}{h};\ m_{BC}=-\frac{4}{7};\ m_{BO}=-\frac{3}{2}$

$m_{AC}=\frac{k+1}{h-5}$

$\therefore AO \perp^r BC \Rightarrow 4k=7h$

$\therefore BO \perp^r AC \Rightarrow 2h=3k+13$

$\therefore h=-4;\ k=-7$

Co-ordinates of 3rd vertex : $(-4,-7)$