



# ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

First Term Examination - 2018

Class :11 A1



Sub :Physics

F.M.:70

DURATION:3 Hrs15Mins

DATE:03.08.2018

### SECTION-I

(1x14=14)

Answer the following questions (Multiple Choice Questions)

(1) The frequency of vibration  $f$  of a mass  $m$  suspended from a spring of spring constant  $k$  is given by the relation  $f = am^x k^y$ , where  $a$  is a dimensionless constant and the unit of  $k$  is  $N/m$ . The values of  $x$  and  $y$  are:

(a)  $x = 1/2, y = 1/2$

(b)  $x = -1/2, y = -1/2$

(c)  $x = 1/2, y = -1/2$

(d)  $x = -1/2, y = 1/2$

(2) Percentage error in the measurement of mass and velocity are 2% and 3% respectively. The error in the estimate of kinetic energy obtained by measuring mass and velocity will be :

a. 8%

b. 2%

c. 12%

d. 10%

(3) If a unit vector is represented by  $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$ , then the value of  $c$  is:

a. 1

b.  $\sqrt{0.11}$

c.  $\sqrt{0.01}$

d.  $\sqrt{0.39}$

(4) The velocity of a particle is  $v = v_0 + gt + ft^2$ . If its position is  $x=0$  at  $t=0$ , then its displacement after time  $t=1s$  is :

a.  $v_0 + g/2 + f$

b.  $v_0 + 2g + 3f$

c.  $v_0 + g/2 + f/3$

d.  $v_0 + g + f$

(5) If  $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$  and  $V_2$  is finite then

a.  $\vec{V}_1$  is parallel to  $\vec{V}_2$

b.  $\vec{V}_1$  is perpendicular to  $\vec{V}_2$

c.  $V_1 > V_2$

d.  $V_1 = V_2$

(6) A particle located at  $x=0$  at time  $t=0$ , starts moving along the positive  $x$  direction with a velocity  $v$  that varies as  $v = \alpha\sqrt{x}$ . The displacement of the particle varies with time as

a.  $t^{1/2}$

b.  $t^3$

c.  $t^2$

d.  $t$

(7) Speeds of two identical cars are  $u$  and  $4u$  at a specific instant. The ratio of the respective distance in which the two cars are stopped from that instant is :

a) 1:1

b. 1:4

c. 1:8

d. 1:16

(8) The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in measurement of force and length are respectively 4% and 2%, the maximum error in the measurement of pressure is:

a. 1%

b. 2%

c. 6%

d. 8%

(9) A boat is moving with a velocity  $4\hat{i} + 9\hat{j}$  with respect to ground. The water in river is flowing with a velocity  $-3\hat{i} + 4\hat{j}$ . The relative velocity of boat with respect to water is:

- a.  $8\hat{i}$  b.  $-6\hat{i} - 8\hat{j}$   
 c.  $7\hat{i} + 5\hat{j}$  d. 5

(10) The percentage error in the determination of  $g = 4\pi^2 l/t^2$ , when  $l$  and  $t$  are measured with 2% and 3% errors is:

- a. 8% b. 6%  
 c. 4% d. 9%

(11) A boat is sent across a river with a velocity of 8km/hr. If the resultant velocity of the boat is 10km/hr, then the velocity of river is:

- a. 3km/hr b. 6km/hr  
 c. 8km/hr d. 128km/hr

(12) A particle under the action of a constant force moves from rest upto 20s. If distance covered in first 10s is  $S_1$  and that covered in next 10s is  $S_2$ , then :

- a.  $S_1 = S_2$  b.  $S_2 = 3 S_1$   
 c.  $S_2 = 2 S_1$  d.  $S_2 = 4 S_1$

(13) The resultant of two forces is 20N. When one of the force is  $10\sqrt{3}$  N and the angle between two forces is  $30^\circ$ , then what is the value of second force?

- a. 10N b. 20N  
 c.  $20\sqrt{3}$  N d.  $10\sqrt{3}$  N

(14) The respective number of significant figures for the numbers 21.021, 0.0005 and  $5.1 \times 10^{-3}$  are:

- a. 4,4,2 b. 5,1,2  
 c. 5,1,5. d. 5,2,3

**SECTION -II**

**GROUP -A**

Answer the following questions in one or two sentences (Alternatives are to be noted): (1x4=4)

(1) The position  $x$  of a particle at time  $t$  is given by  $x = \frac{V}{a} (1 - e^{-at})$ , where  $V$  is a constant and  $a > 0$ , Find out the dimensions of  $V$  and  $a$ .

**OR**

A force  $F$  is given by  $F = at + bt^2$ , where  $t$  is time. Find the dimensions of  $a$  and  $b$ .

- (2) What is the angle made by vector  $\vec{A} = 2\hat{i} + 2\hat{j}$  with  $x$  axis ?  
 (3) The displacement time graph for two particles A and B are straight lines inclined at an angles of  $30^\circ$  and  $45^\circ$  with time axis. What is the ratio of their velocities?  
 (4) The position coordinate of a moving particle is given by  $x = 6 + 18t + 9t^2$  ( $x$  in metres and  $t$  in seconds). What is the velocity at  $t = 2s$ ?

**OR**

Consider a vector  $\vec{F} = 4\hat{i} - 3\hat{j}$ . Which unit vector is perpendicular to  $\vec{F}$ ?

**GROUP -B**

Answer the following questions in short (Alternatives are to be noted) : (2x5=10)

(5) What is the magnitude and direction of  $(\hat{i} + \hat{j})$ ?

OR

At what angle must the two forces  $(A+B)$  and  $(A-B)$  act so that the resultant may be  $\sqrt{A^2+B^2}$ .

- (6) The initial and final temperatures of water as recorded by an observer are  $(40.6 \pm 0.2)^\circ\text{C}$  and  $(78.3 \pm 0.3)^\circ\text{C}$ . Calculate the rise in temperature with proper error limit.
- (7) To a diver going east in a car with velocity of  $40\text{km/hr}$ , a bus appears to move towards north with a velocity of  $40\sqrt{3}\text{km/hr}$ . What is the actual velocity and direction of motion of the bus?

OR

Rain is falling vertically with a speed of  $30\text{m/s}$ . A woman rides a bicycle with a speed of  $10\text{m/s}$  in the north to south direction. What is the relative velocity of rain with respect to woman? In which direction the woman should hold her umbrella to protect herself from rain.

- (8) Kepler discovered that the orbital periods  $T$  of the planets about the sun are related to their distances  $r$  from the sun. From Newton's laws, the following relationship may be derived,  $T^2 = r^3[4\pi^2/GM]$ . Here  $M$  is the mass of the sun. Using dimensional analysis find out the units of  $G$  in terms of the base units of S.I.
- (9) A car moving with a velocity of  $30\text{m/s}$  is stopped by the application of brakes which impart a retardation of  $6\text{m/s}^2$  to the car. (a) How long does it take for the car to come to a stop? (b) How far does the car travel during the time brakes are applied?

### GROUP-C

Answer the following questions in short (Alternatives are to be noted):  $(3 \times 9 = 27)$

(10) A liquid having a small depth but a large volume is forced by the applied pressure to escape through an orifice with a velocity  $v$ . Obtain an expression for the velocity of liquid in terms of applied pressure and density using the method of dimensional analysis.

(11) Find the angle between two vectors  $\hat{i} - 2\hat{j} - 5\hat{k}$  and  $2\hat{i} + \hat{j} - 4\hat{k}$

(12) A car accelerates from rest at a constant rate  $\alpha$  for some time, after which it decelerates at a constant rate  $\beta$  to come to rest. If the total time elapsed is  $t$  second then calculate the maximum velocity attained by the car.

(13) If unit vectors  $\hat{g}$  and  $\hat{u}$  are inclined at an angle  $\phi$  then prove that  $|\hat{g} - \hat{u}| = 2\sin\phi/2$

OR

Two forces whose magnitude are in the ratio of  $3:5$  give a resultant of  $28\text{N}$ . If the angle of their inclination is  $60^\circ$ , find the magnitude of each force.

(14) Two resistors of resistances  $R_1 = (100 \pm 3)\Omega$  and  $R_2 = (200 \pm 4)\Omega$  are connected in series and in parallel combination. Find the equivalent resistances in both the cases considering the errors.

(15) What is the distance travelled by a point during the time  $t$ , if it moves in X-Y plane, according to the relation  $x = a \sin \omega t$  and  $y = a(1 - \cos \omega t)$ ?

(16) The distance  $x$  of a particle moving in 1D under the action of a constant force is related to time by the equation  $t = \sqrt{x} + 3$  where  $x$  is in meter and  $t$  in seconds. Find the displacement of the particle when its velocity is 0.

(17) A body travels uniformly through a distance of  $(13.8 \pm 0.2)\text{m}$  in a time  $(4.0 \pm 0.3)\text{s}$ . Calculate its velocity with error limits rounding off to 2 significant figures. What is the % error in velocity?

(18) An aeroplane takes off at an angle of  $30^\circ$  to the horizontal. If the component of its velocity along the horizontal is  $250\text{km/hr}$  what is the actual velocity? Also find the vertical component of velocity.

### Group-D

Answer the following questions (Alternatives are to be noted):

$(5 \times 3 = 15)$

(19) The position vector of a particle is  $\vec{r} = 3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}$  metre, where  $t$  is in seconds. (a) Find the velocity and the acceleration vector of the particle. (b) What is the magnitude and the velocity of the particle at  $t = 2\text{s}$ ?  $(1.5 + 1.5 + 1 + 1)$

OR

(i) Write down the expression of position vector of a particle in 3D space.

(ii) A bullet fired into a target loses  $1/x$  of its velocity after traversing a distance  $s$  meter into the target. Show that it covers a further distance of  $s(x-1)^2 / (2x-1)$ .

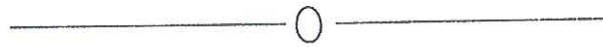
(1+4)

(20) The motion of a particle along a straight line is given by the equation  $x = 8 + 12t - t^3$ , where  $x$  is in m, and  $t$  is in s. Calculate the retardation of the particle when its velocity becomes zero.

(21) (i) Determine the vector which when added to the resultant of  $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$  and  $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$  gives unit vector along  $y$  direction.

(ii) A body constrained to move along  $Z$  axis of a coordinate system is subjected to a constant force  $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$  N. Calculate the work done by the force in displacing the body through 4m along  $z$  axis.

(3+2)





ST. LAWRENCE HIGH SCHOOL  
FIRST TERM – 2018(ANSWER KEY)

Submitted by  
Nandini Chatterjee  
8/8/18

Sub:Physics

Class:XI-A2

F.M :70

Date: 3.08.2017

Duration:3.15hrs

SECTION-I

(1x14=14)

Answer the following questions (Multiple Choice Questions)

- (1) The frequency of vibration  $f$  of a mass  $m$  suspended from a spring of spring constant  $k$  is given by the relation  $f = am^xk^y$ , where  $a$  is a dimensionless constant and the unit of  $k$  is  $N/m$ . The values of  $x$  and  $y$  are:

Ans: (d)  $x = -1/2, y = 1/2$

- (2) Percentage error in the measurement of mass and velocity are 2% and 3% respectively. The error in the estimate of kinetic energy obtained by measuring mass and velocity will be :

Ans:(a)8%

- (3) If a unit vector is represented by  $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$ , then the value of  $c$  is:

Ans: (b)  $\sqrt{0.11}$

- (4) The velocity of a particle is  $v = v_0 + gt + ft^2$ . If its position is  $x=0$  at  $t=0$ , then its displacement after time  $t=1s$  is :

Ans:(c)  $v_0 + g/2 + f/3$

- (5) If  $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$  and  $V_2$  is finite then

Ans:(b).  $V_1$  is perpendicular to  $\vec{V}_2$

- (6) A particle located at  $x=0$  at time  $t=0$ , starts moving along the positive  $x$  direction with a velocity  $v$  that varies as  $v = \alpha\sqrt{x}$ . The displacement of the particle varies with time as

Ans: (c)  $t^2$

- (7) Speeds of two identical cars are  $u$  and  $4u$  at a specific instant. The ratio of the respective distance in which the two cars are stopped from that instant is :

Ans:(d)1:16

- (8) The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in measurement of force and length are respectively 4% and 2%, the maximum error in the measurement of pressure is:

Ans:(d). 8%

- (9) A boat is moving with a velocity  $4\hat{i} + 9\hat{j}$  with respect to ground. The water in river is flowing with a velocity  $-3\hat{i} + 4\hat{j}$ . The relative velocity of boat with respect to water is:

Ans: (c)  $7\hat{i} + 5\hat{j}$

- (10) The percentage error in the determination of  $g = 4\pi^2 l/t^2$ , when  $l$  and  $t$  are measured with 2% and 3% errors is:

Ans: (a) 8%

- (11) A boat is sent across a river with a velocity of 8km/hr. If the resultant velocity of the boat is 10km/hr, then the velocity of river is:

Ans (b) 6km/hr

- (12) A particle under the action of a constant force moves from rest upto 20s. If distance covered in first 10s is  $S_1$  and that covered in next 10s is  $S_2$ , then :

Ans: (d)  $S_2 = 4S_1$

(13) The resultant of two forces is 20N. When one of the force is  $10\sqrt{3}$  N and the angle between two forces is  $30^\circ$ , then what is the value of second force?

Ans: 20N

(14) The respective number of significant figures for the numbers 21.021, 0.0005 and  $5.1 \times 10^{-3}$

Ans: (b) 5, 1, 2

## SECTION -II

### GROUP -A

Answer the following questions in one or two sentences (Alternatives are to be noted): (1x4=4)

(1) The position  $x$  of a particle at time  $t$  is given by  $x = \frac{v}{a} (1 - e^{-at})$ , where  $v$  is a constant and  $a > 0$ , Find out the dimensions of  $v$  and  $a$ .

Ans: Dimension of  $v$  is  $[LT^{-1}]$  and  $a$  is  $[T^{-1}]$

OR

A force  $F$  is given by  $F = at + bt^2$ , where  $t$  is time. Find the dimensions of  $a$  and  $b$ .

Ans:  $[MLT^{-2}] = a[T]$  and  $[MLT^{-2}] = b[T^2]$

Therefore,  $a = [MLT^{-3}]$  and  $b = [MLT^{-4}]$

(2) What is the angle made by vector  $\vec{A} = 2\hat{i} + 2\hat{j}$  with  $x$  axis?

Ans: Here,  $A_x = 2, A_y = 2$

Let  $\theta$  be the angle which The  $\vec{A}$  makes with  $x$  axis then,  
 $\tan \theta = A_y/A_x = 2/2 = 1$  or,  $\theta = 45^\circ$

Alternative method,  $\cos \theta = (2\hat{i} + 2\hat{j}) \cdot \hat{i} / \sqrt{8} = 2/2\sqrt{2} = 1/\sqrt{2}$   
or,  $\theta = 45^\circ$

(3) The displacement time graph for two particles A and B are straight lines inclined at an angles of  $30^\circ$  and  $45^\circ$  with time axis. What is the ratio of their velocities?

Ans:  $V_1/V_2 = \tan 30^\circ / \tan 45^\circ = 1/\sqrt{3}$

(4) The position coordinate of a moving particle is given by  $x = 6 + 18t + 9t^2$  ( $x$  in metres and  $t$  in seconds). What is the velocity at  $t = 2s$ ?

Ans:  $v = dx/dt = 18 + 18t$

hence velocity at  $t = 2s$  is  $v = 18 + 18 \times 2 = 18 + 36 = 54m/s$

OR

Consider a vector  $\vec{F} = 4\hat{i} - 3\hat{j}$ . Which unit vector is perpendicular to  $\vec{F}$ ?

Ans: Let the vector be  $x\hat{i} + y\hat{j}$ , for two vectors to be perpendicular their dot product must be zero,

therefore,  $(4\hat{i} - 3\hat{j}) \cdot (x\hat{i} + y\hat{j}) = 0$

or,  $4x = 3y$

or,  $x/y = 3/4$

hence the unit vector is  $3\hat{i} + 4\hat{j}$

**GROUP -B**

Answer the following questions in short (Alternatives are to be noted) : (2x5=10)

- (5) What is the magnitude and direction of  $(\hat{i} + \hat{j})$ ?

Ans: Magnitude :  $|\hat{i} + \hat{j}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2} = 1.414$  units  
 Direction :  $\tan \theta = 1/1 = 1$  or,  $\theta = 45^\circ$

**OR**

At what angle must the two forces  $(A+B)$  and  $(A-B)$  act so that the resultant may be  $\sqrt{A^2+B^2}$ .

Ans:  $A^2+B^2 = (A+B)^2 + (A-B)^2 + 2(A+B)(A-B)\cos\theta$

or,  $A^2+B^2 = A^2+B^2 + 2AB + A^2+B^2 - 2AB + 2A^2 - B^2 \cos\theta$

or,  $\theta = \cos^{-1}[-(A^2+B^2)/2(A^2-B^2)]$

- (6) The initial and final temperatures of water as recorded by an observer are  $(40.6 \pm 0.2)^\circ\text{C}$  and  $(78.3 \pm 0.3)^\circ\text{C}$ . Calculate the rise in temperature with proper error limit.

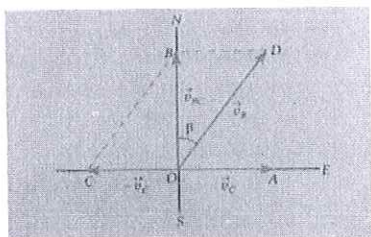
Ans:  $\theta_1 = (40.6 \pm 0.2)^\circ\text{C}$ ,  $\theta_2 = (78.3 \pm 0.3)^\circ\text{C}$

Rise in temperature  $\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7^\circ\text{C}$

$\Delta\theta = \pm(0.2 + 0.3) = \pm 0.5^\circ\text{C}$

Hence rise in temperature =  $(37.7 \pm 0.5)^\circ\text{C}$

- (7) To a diver going east in a car with velocity of 40km/hr, a bus appears to move towards north with a velocity of  $40\sqrt{3}$  km/hr. What is the actual velocity and direction of motion of the bus?



Ans: The true velocity of the car is  $OA = 40\text{km/hr}$  due east

relative velocity of bus wrt car  $OB = 40\sqrt{3}$  km/hr due north

Let the true velocity of bus be along OD and angle  $BOD = \beta$ , then  $V_b = OD = \sqrt{40^2 + (40\sqrt{3})^2} = 80\text{km/hr}$ ,

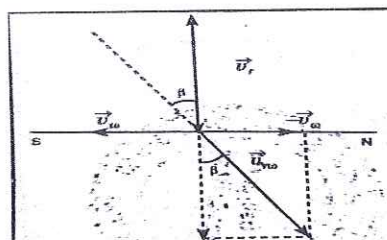
$\tan\beta = BD/OB = 40/40\sqrt{3} = 1/\sqrt{3}$ , or  $\beta = 30^\circ$  east of north

**OR**

Rain is falling vertically with a speed of 30m/s. A woman rides a bicycle with a speed of 10m/s in the north to south direction. What is the relative velocity of rain with respect to woman? In which direction the woman should hold her umbrella to protect herself from rain.

Ans : In order to protect herself from rain, she must hold her umbrella in the direction of relative velocity of rain w.r.t her

$\vec{V}_{rw} = \vec{V}_r - \vec{V}_w = 30 - 10 = 20\text{m/s}$



$\tan \beta = 10/30 = 1/3 = 0.333$ , or  $\beta = 18^\circ 26'$

- (8) Kepler discovered that the orbital periods  $T$  of the planets about the sun are related to their distances  $r$  from the sun. From Newton's laws, the following relationship may be derived .  $T^2 = r^3[4\pi^2/GM]$ . Here  $M$  is the mass of the sun. Using dimensional analysis find out the units of  $G$  in terms of the base units of S.I.

Ans: From the given equation :  $G = [4\pi^2 / T^2 M] r^3$

Using dimensional analysis,

$$G = 1X [L]^2 / [T^1]^2 X [M] = [M^{-1}L^2T^{-2}]$$

Hence the units of G on SI are  $kg^{-1}m^2s^{-2}$

- (9) A car moving with a velocity of 30m/s is stopped by the application of brakes which impart a retardation of  $6m/s^2$  to the car. (a) How long does it take for the car to come to a stop? (b) How far does the car travel during the time brakes are applied?

Ans: Initial velocity  $u = 30m/s$ , final velocity  $v = 0$  acceleration  $a = -6 m/s^2$

- (i) Applying  $v = u + at$ ,  $0 = 30 - 6t$ ,  $t = 5s$   
 (ii) Applying  $v^2 - u^2 = 2as$ ,  $0 - 30 \times 30 = 2x(-6)s$ ,  $s = 900/12 = 75m$

### GROUP-C

Answer the following questions in short (Alternatives are to be noted): (3x9=27)

- (10) A liquid having a small depth but a large volume is forced by the applied pressure to escape through an orifice with a velocity  $v$ . Obtain an expression for the velocity of liquid in terms of applied pressure and density using the method of dimensional analysis.

Ans: Let velocity  $v$  depends upon pressure 'p' and density 'd' as follows:

$v$  proportional  $p^a d^b$ , or,  $v = k p^a d^b$

Taking dimensions on both sides:

$$[LT^{-1}] = [ML^{-1}T^{-2}]^a X [M^1L^{-3}]^b$$

Using principle of homogeneity:

$$a = 1/2 \quad b = -1/2$$

$$\text{Thus, } v = k \frac{\sqrt{p}}{\sqrt{d}}$$

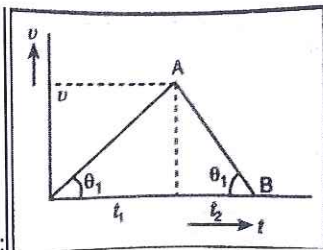
- (11) Find the angle between two vectors  $\hat{i} - 2\hat{j} - 5\hat{k}$  and  $2\hat{i} + \hat{j} - 4\hat{k}$

Ans:  $\cos \theta = (\hat{i} - 2\hat{j} - 5\hat{k}) \cdot (2\hat{i} + \hat{j} - 4\hat{k}) / (\sqrt{30} \cdot \sqrt{21})$

$$\theta = \cos^{-1} = 2 - 2 + 20 / 25.09 = 20 / 25.09$$

$$\text{or, } \theta = 37.14^\circ$$

- (12) A car accelerates from rest at a constant rate  $\alpha$  for some time, after which it decelerates at a constant rate  $\beta$  to come to rest. If the total time elapsed is  $t$  second then calculate the maximum velocity attained by the car.]



Ans:

$$\alpha = \tan \theta_1 = v/t_1$$

$$\text{therefore } 1/\alpha = t_1/v$$

$$\beta = \tan \theta_2 = v/t_2$$

$$\text{therefore } 1/\beta = t_2/v$$

$$\text{Adding, } 1/\alpha + 1/\beta = \beta + \alpha/\alpha\beta = t_1 + t_2/v = t/v$$



therefore  $v = t(\alpha\beta / \beta + \alpha)$

(13) If unit vectors  $\hat{g}$  and  $\hat{u}$  are inclined at an angle  $\phi$  then prove that  $|\hat{g} - \hat{u}| = 2\sin\phi/2$

$$\begin{aligned}\text{Ans } |\hat{g} - \hat{u}|^2 &= (\hat{g} - \hat{u}) \cdot (\hat{g} - \hat{u}) \\ &= \hat{g} \cdot \hat{g} - \hat{g} \cdot \hat{u} - \hat{u} \cdot \hat{g} + \hat{u} \cdot \hat{u} \\ &= 2 - 2\cos\phi = 2(1 - \cos\phi) = 2 \cdot 2\sin^2\phi/2 = 4\sin^2\phi/2\end{aligned}$$

$$\text{or, } |\hat{g} - \hat{u}| = 2\sin\phi/2$$

**OR**

Two forces whose magnitude are in the ratio of 3:5 give a resultant of 28N. If the angle of their inclination is  $60^\circ$ , find the magnitude of each force.

Ans: Let A and B be the two forces, then  $A = 3x$ ;  $B = 5x$ ;  $R = 28\text{N}$ ,  $\theta = 60^\circ$

Thus  $A/B = 3/5$

$$\text{Now, } 28 = \sqrt{9x^2 + 25x^2 + 30x^2 \cos 60^\circ} = 7x$$

$$\text{or, } x = 28/7 = 4$$

Forces are  $A = 3 \times 4 = 12\text{N}$  and  $B = 4 \times 4 = 20\text{N}$

(14) Two resistors of resistances  $R_1 = (100 \pm 3)\Omega$  and  $R_2 = (200 \pm 4)\Omega$  are connected in series and in parallel combination. Find the equivalent resistances in both the cases considering the errors.

Ans: In series :  $R = R_1 + R_2 = 100 + 200 = 300\Omega$

$$\Delta R = 3 + 4 = 7\Omega$$

Therefore Equivalent resistance in series is  $(300 \pm 7)\Omega$

In parallel  $1/R = 1/R_1 + 1/R_2 = 1/100 + 1/200 = 200/3$ , or,  $R = 3/200 = 66.66\Omega$

$$\Delta R = 3 + 4 = 7\Omega$$

therefore equivalent resistance in parallel is  $(66.66 \pm 7)\Omega$

(15) What is the distance travelled by a point during the time  $t$ , if it moves in X-Y plane, according to the relation  $x = a \sin \omega t$  and  $y = a(1 - \cos \omega t)$ ?

Ans: Velocity of the point along x axis

$$\vec{V}_x = dx/dt = a\omega \cos \omega t$$

$$\vec{V}_y = dy/dt = a\omega \sin \omega t$$

$$\text{As } \vec{V} = V_x \hat{i} + V_y \hat{j}, \text{ and } V = \sqrt{V_x^2 + V_y^2}, V = \sqrt{(a\omega \cos \omega t)^2 + (a\omega \sin \omega t)^2} = a\omega$$

distance travelled in time  $t$  is  $Vt = a\omega t$

(16) The distance  $x$  of a particle moving in 1D under the action of a constant force is related to time by the equation  $t = \sqrt{x} + 3$  where  $x$  is in meter and  $t$  in seconds. Find the displacement of the particle when its velocity is 0.

Ans: Given  $t = \sqrt{x} + 3$

$\sqrt{x} = t - 3$ , squaring both sides,

$$x = t^2 - 6t + 9$$

$$v = dx/dt = 2t - 6$$

when  $v = 0$ , then  $2t - 6 = 0$  or,  $t = 3\text{s}$

at  $t = 3\text{ s}$ , displacement is,  $x = 3^2 - 6 \cdot 3 + 9 = 0$

hence displacement of the particle is 0 when its velocity is 0.

(17) A body travels uniformly through a distance of  $(13.8 \pm 0.2)\text{ m}$  in a time  $(4.0 \pm 0.3)\text{ s}$ . Calculate its velocity with error limits rounding off to 2 significant figures. What is the % error in velocity?

Ans:  $s = (13.8 \pm 0.2)\text{ m}$ ,  $t = (4.0 \pm 0.3)\text{ s}$

therefore velocity  $v = s/t = 13.8/4 = 3.45 = 3.4\text{ m/s}$  (round off to two significant fig)

$$\Delta v/v = \pm (\Delta s/s + \Delta t/t) = \pm (0.2/13.8 + 0.3/4) = \pm 4.94/13.8 \times 4 = \pm 0.0895$$

$$\Delta v = \pm 0.0895 \times v = \pm 0.0895 \times 3.45 = \pm 0.3087 = \pm 0.31 \text{ (upto 2 significant fig)}$$

hence  $v = (3.4 \pm 0.31)\text{ m/s}$

% error in velocity =  $\pm 0.0895 \times 100 = \pm 8.95\% = \pm 9\%$

(18) An aeroplane takes off at an angle of  $30^\circ$  to the horizontal. If the component of its velocity along the horizontal is  $250\text{ km/hr}$  what is the actual velocity? Also find the vertical component of velocity.

Ans: Here  $\theta = 30^\circ$ , horizontal component  $A_x = 250\text{ km/hr}$ , Actual velocity  $A = ?$  Vertical component  $A_y = ?$

we know  $A_x = A \cos\theta$ ,  $A = A_x/\cos\theta = 250/\cos 30^\circ = 250/2/\sqrt{3} = 288.7\text{ km/hr}$

Also  $A_y = A \sin\theta = 288.7 \sin 30^\circ = 144.35\text{ km/hr}$

#### Group-D

Answer the following questions (Alternatives are to be noted):

(5x3=15)

(19) The position vector of a particle is  $\vec{r} = 3.0t\hat{i} - 20t^2\hat{j} + 4.0\hat{k}$  metre, where  $t$  is in seconds. (a) Find the velocity and the acceleration vector of the particle. (b) What is the magnitude of velocity of the particle at  $t = 2\text{ s}$ ? (1.5+1.5+1+1)

Ans: (a) Velocity  $\vec{v} = d\vec{r}/dt = (3.0\hat{i} - 4.0t\hat{j})\text{ m/s}$

Acceleration  $\vec{a} = -4\hat{j}\text{ m/s}^2$

(b) At  $t = 2\text{ s}$ ,  $\vec{v} = 3\hat{i} - 4 \times 2\hat{j} = (3\hat{i} - 8\hat{j})\text{ m/s}$

magnitude of  $\vec{v} = \sqrt{9 + 64} = 8.54\text{ m/s}$

Direction  $\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(-8/3) = -69.5^\circ$  ie below x axis.

**OR**

(i) Write down the expression of position vector of a particle in 3D space

Ans: Position vector of a particle in 3D space  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

(ii) A bullet fired into a target loses  $1/x$  of its velocity after traversing a distance  $s$  meter into the target. Show that it covers a further distance of  $s(x-1)^2/(2x-1)$ . (1+4)

Ans: Let  $u$  be the initial velocity of the bullet before entering the target

final velocity after covering a distance  $s = u - u/x = u(1 - 1/x)$

using  $v^2 - u^2 = 2as$

$u^2(1 - 1/x)^2 - u^2 = 2as$

Let  $s'$  be the further distance travelled by the bullet before it stops  $v = 0$

Total distance covered in target  $= s + s'$

using,  $0 - u^2 = 2a(s + s')$

$$a = -u^2/2(s+s')$$

$$\text{Therefore, } u^2(1-1/x)^2 - u^2 = 2[-u^2/2(s+s')]s$$

$$\text{solving, } s' = s(x-1)^2/(2x-1)$$

(20) The motion of a particle along a straight line is given by the equation  $x = 8 + 12t - t^3$ , where  $x$  is in m, and  $t$  is in s. Calculate the retardation of the particle when its velocity becomes zero.

$$\text{Ans: Given } x = 8 + 12t - t^3$$

$$\text{velocity } v = dx/dt = 12 - 3t^2, \text{ retardation 'a' } = dv/dt = -6t$$

$$\text{when velocity is 0 ie } 12 - 3t^2 = 0, t = 2\text{s}$$

$$\text{hence retardation 'a' } = -2 \times 2 = -12\text{m/s}^2$$

(21) (i) Determine the vector which when added to the resultant of  $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$  and  $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$  gives unit vector along y direction.

$$\text{Ans: } \vec{R} = \vec{A} + \vec{B} = (5\hat{i} - \hat{j} + 4\hat{k})$$

the unit vector along y direction is  $\hat{j}$

$$\text{hence the required vector is: } \hat{j} - (5\hat{i} - \hat{j} + 4\hat{k}) = -5\hat{i} + 2\hat{j} - 4\hat{k}$$

(ii) A body constrained to move along Z axis of a coordinate system is subjected to a constant force  $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$  N. Calculate the work done by the force in displacing the body through 4m along z axis. (3+2)

$$\text{Ans: Displacement } \vec{S} \text{ along z axis, } \hat{k},$$

$$\text{hence work done } W = \vec{F} \cdot \vec{S} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot 4\hat{k} = 12\text{J}$$

