FOR GOD AND COUNTRY
St. Lawrence High School, Kolkata

ST. LAWRENCE HIGH SCHOOL
PRE-ANNUAL EXAMINATION - 2018
CLASS : XI
MATHEMATICS

TIME : 3 HRS 15 MIN **FULL MARKS : 80**

[Relevant rough work must be done in the margin of the page containing the answers]

PART-A

1. Select the correct Alternatives : **10x1=10**

a) If ${}^{n}P_r = 720$ *and* ${}^{r}C_r = 120$ then the value of r is
i) 2 ii) 3 iii) 4 . iv) none of these

b) The equation of the line passing through (2,3) and perpendicular to the line $3x+4y-5=0$ is i) $4x-3y+1=0$ ii) $4x-3y+5=0$ iii) $4x+4y-5=0$ iv) $4x-3y-7=0$

c) The value of $\tan\frac{\Pi}{8}$ is i) $\sqrt{2}-1$ ii) $-\sqrt{2}+1$ iii) $\sqrt{2}+1$ iv) none of these

d) If $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$ then the value of x is i) 10 ii) 100 iii) 1000 iv) 1

e) The value of $\sqrt{3}\operatorname{cosec}20^0-\sec 20^0$ is
i) 2 ii) 4 iii) 6 iv) $\frac{1}{2}$

f) The value of $\lim_{x\to 1}(1-x)\tan\frac{\Pi x}{2}$ is
i) $\frac{2}{\Pi}$ ii) $\frac{\Pi}{2}$ iii) does not exist iv) 1

g) $\frac{d}{dx}\sin(x^0)$ = i) $\frac{\Pi}{180}\cos x^0$ ii) $\cos x^0$ iii) $-\frac{\Pi}{180}\cos x^0$ iv) none

h) $\frac{d}{dx}(\tan\sqrt{x})$= i) $\frac{\sec^2\sqrt{x}}{2\sqrt{x}}$ ii) $\frac{2\sec\sqrt{x}}{\sqrt{x}}$ iii) $\frac{\tan\sqrt{x}}{\sqrt{x}}$ iv) none of these

i) The length of the L.R. of the ellipse $16x^2+y^2=16$ is
i) $\frac{3}{4}$ ii) $\frac{1}{4}$ iii) $\frac{1}{2}$ iv) $\frac{1}{8}$

j) The eccentricity of the hyperbola $9y^2-4x^2=36$
i) $\frac{\sqrt{13}}{2}$ ii) $\frac{2}{\sqrt{13}}$ iii) $\frac{\sqrt{13}}{4}$ iv) $\frac{4}{\sqrt{13}}$

GROUP – B

2. a) Answer any two questions. 2X2

i. If $n(x) = 4$ and $n(y) = 8$. Find the maximum and minimum numbers of elements of $x \cup y$.

ii. $A = \{ (a,b): a+3b = 12, a, b \in n \}$, Find the range of A.

iii. Show that $\cos^6 x + \sin^6 x = \frac{1}{8}(5 + 3\cos 4x)$

iv Find the value of $[\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}]$ where $(n \in z^+)$,

b) Answer any two questions. 2X2

i. If $(1 + i)(2 + I)(3 + i) \ldots\ldots (n + i) = a + ib$, show that $2.5.10\ldots\ldots(n^2+1) = a^2 + b^2$.

ii. If n is an even number, find the sum of the series upto n terms $1^2 - 2^2 + 3^2 - \ldots\ldots\ldots - \frac{n(n+1)}{2}$

iii. How many 5 digits telephone number be formed from the digits from 0 to 9 when every number start with 23 (no digit being repeated in any number).

iv. Find the value of $3^{2/3} 3^{2/9} 3^{2/27} \ldots\ldots\ldots\ldots$

c) Answer any one questions. 2X1

i. If the gradient of the line joining the points (2a, -2) and (1, -a) is (-2). Find the value of a.

ii. Find the ratio in which the plane $2x + 2y - 2z = 1$ divides the line segment joining the points A(2, 1, 5) and B(3, 4, 3)

d) Answer any one questions. 2X1

i. Evaluate $\lim_{x \to 0} \frac{|\sin x|}{x}$

Ii. Find the condition of existence of $\lim_{x \to 0} f(x)$

e) Answer any one questions. 2X1

i. Find the variance of first 4 natural numbers

ii. Four letters are placed in four addressed envelopes. Find the probability that no letter goes to the correct envelope.

Group - C

3a) Answer any **Two** questions:- **(4 X 2 = 8)**

i) For any three sets A, B and C prove that $A - (B \cap C) = (A - B) \cup (A - C)$

ii) Prove that $16 \cos \pi/15 \cos 2\pi/15 \cos 4\pi/15 \cos 8\pi/15 = -1$

iii) In any triangle ABC if $8R^2 = a^2 + b^2 + c^2$, Prove that the triangle is rt. angled.

b) Answer any **Two** questions:- **(4 X 2 = 8)**

i) Prove by induction that the sum of the cubes of three successive positive integers is divisible by 9.

ii) If the vertices of an equilateral triangle be represented by the complex numbers z_1, z_2, z_3 on the Argand diagram, then prove that $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

iii) How many words can be made using all the letters in the word MONDAY? How many of them begin with M and do not end with Y.

iv) Find the coefficient of x^{10} in the expansion of $(1-2x+3x^2)(1-x)^{15}$

v) The sum of three numbers in A.P is 18. If 2, 4, and 11 be added to them respectively the resulting numbers arein G.P. Determine the numbers.

c) Answer any **Two** questions:- **(4 X 2 = 8)**

i) A straight line has slope 5/12 and it passes through the point P (3, - 7/2). Find the co ordinates of a point Q on this line which is at a distance of 13/2 unit from P.

ii) The equation of the hypotenuse of a right angled isosceles triangle is $3x + 4y = 4$

and the co ordinates of the opposite vertex are (2 , 2). Find the equation of its other sides.

iii) The co ordinates of the two ends of latus rectum of a parabola are (3 , 4) and (3 , 0). Find the equation of the parabola.

d) Answer any **One** question:- (4 X 1 = 4)

i) Evaluate $\lim_{x \to 0} \frac{Cosec\,x - Cot x}{x}$

ii) Find from the first principle the derivative of Cos(logx)

e) Answer any **One** question (4 X 1 = 4)

i) Show that $\sqrt{2}$ is not rational (Use the method of contradiction)

ii) By giving a counter example show that the following statement is false:
The equation $4x^2 - 25 = 0$ does not have a root lying between (-3) and (-2).

f) Answer any One question (4 X 1 = 4)

i) Two unbiased dice are rolled together. Find the odds in favour of getting two digits, the sum of which is 7.

ii) Find the S.D from the following data

Daily wages (in Rs)	20 – 24	25 – 29	30 – 34	35 – 39
No. of employees	16	28	14	12

Group – D

4. a) Answer any one question : **5 x 1 = 5**

i) If $\frac{\tan 3A}{\tan A} = k$; show that $\frac{\sin 3A}{\sin A} = \frac{2k}{k-1}$; show also that k can not lie between $\frac{1}{3}$ and 3.

ii) Find the domain and range of the function $f(x) = \frac{x}{x^2 - 5x + 4}$.

iii) If $\tan\frac{\theta}{2} = \operatorname{cosec}\theta - \sin\theta$; show that $\cos^2\frac{\theta}{2} = \cos 36^{\circ}$

b) Answer any two questions : **5 x 2 = 10**

i) Solve : $|x-1| + |x-2| + |x-3| \geq 6 \forall x \in R$

ii) If G.M. of two unequal positive numbers a and b is $\frac{a^{p+1} + b^{p+1}}{a^p + b^p}$ then find the value of p.

iii) From 10 candidates, how many selections of 5 can be made so as to
a) Include both the youngest and the oldest
b) Exclude the youngest if it includes the oldest?

iv) Solve : $|z| + z = 2 + i$ (z being a complex number)

c) Answer any one question : **5 x 1 = 5**

i) Prove that SP + S'P = 20 for the ellipse $\frac{x^2}{100} + \frac{y^2}{100} = 1$, S and S' are the two foci of the ellipse and P is any point on the ellipse.

ii) The co – ordinates of the vertices of a hyperbola are (9, 2) and (1, 2) and the distance between its two foci 10 units. Find the equation of the hyperbola and also the length of its latusrectum :

iii) Prove that the least length of the focal chord of the parabola $y^2 = 4ax$ is its latus rectum.

Sanjay Bhattacharya 23/2/18

ST. LAWRENCE HIGH SCHOOL

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PRE-ANNUAL EXAMINATION- 2018

Mathematics Solution

Class: XI

F. M. 80 **Date of Examination: 27.01.2018**

Part A

1. a(ii), b(i), c(i), d(ii), e(ii), f(i), g(i),, h(i), i(iii), j(i).
2. a.i. 12 maximum when disjoined.

   ii. {1,2,3}

   iii. LHS= 1- $3\cos^2x.\sin^2x$ = 1-$\frac{3}{8}$(1- cos4x) = $\frac{1}{8}$(5 + 3cosx)= RHS

   iv. $\tan(\frac{n\pi}{2}+(-1)^n\ \frac{\pi}{4})$ = $\tan\frac{\pi}{4}$ = 1.

   b.i. (1+i)(2+i).......(n+i) = a+ib

   (1-i)(2-i)..........(n-i) = a-ib. So 2.5.10.......(n^2 +1) = a^2 + b^2.

   ii. (

   iii. 8_{p_3} =336.

   iv. Sum of the degrees of 3 is 3(using infinite GP). So 3^3 = 27.

   c.i. $\frac{-a+2}{1-2a}$ = -2 ,ie, a=$\frac{4}{5}$

   ii. A(2, 1, 5) and B(3, 4, 3) for 2x + 2y- 2z =1. The required ratio is 7:5.

   d.i. Limit does not exist.

   ii. LHL = RHL f(x=a)

   e.i. Var (x) = (n^2 -1)/12. Here n=4. Ans = 5/4.

   ii. 4 letters can be put in 4 envelope in 3X3=9 ways where no letter goes to the correct

   envelope and total number of ways is 4!=24. Hence probability is 3/8.

Group – C

3(a) i) Let x be an arbitrary element of the Set A - (B∩ C)

Then x ∈ A – (B∩ C) ⇒ x ∈ A ∧ x ∉ (B ∩ C)

Or, (x ∈ A ∧ x ∉ B) V (x ∈ A ∧ x ∉ C

Or, x ∈ (A – B) U (A – C)

Therefore A – (B∩ C) ⊆ (A - B) U (A – C)

Again let y be an arbitrary element of the set (A – B) U (A – C)

Then y ∈ (A – B) V y ∈ (A –C)

Or y ∈ A ∧ (y ∈ B V y ∉ C) ⇒ y ∈ A ∧ y ∉ (B ∩ C)

Or y ∈ A – (B∩ C)

Or A – (B∩ C) = (A – B) U (A – C)

(ii) Let $\pi/15 = \theta$.
Therefore $15\theta = \pi$, and then proceed as example 17.

iii) We have $8R^2 = 4R^2(\sin^2A + \sin^2B + \sin^2C)$
Or, $1 - \cos 2A + 1 - \cos 2B + 2 - 2\cos^2C = 4$
$2\cos(A + B).\cos(A - B) + 2\cos^2C = 0$
Or, $-\cos C\,[\cos(A - B) - \cos C] = 0$
Or, $\cos C.2\cos A\cos B=0$
So either $\cos A=0$ i.e $A = 90°$, or $\cos B = 0$ i.e $b = 90°$, or $\cos C= 0$ i.e $C = 90°$
Therefore the triangle is right angled

3) (b) i) Let P(n) be the statement given by
$P(n) = n^3 + (n+1)^3 + (n + 2)^3$ is divisible by 9.
Then P(1) = 36 which is divisible by 9.
Let us assume that P(m) is true. Then $m^3 + (m+1)^3 + (m+2)^3$ is divisible by 9.
$=[m^3 + (m+1)^3 + (m+2)^3] + 9(m^2 + 3m + 3)$ which is always divisible by 9.
Thus we see P(1) and P(m+1) is always true whenever P(m) is true.

(ii) Assume that the vertices of the equilateral triangle are A,B and C and these points are represented by z_1, z_2 and z_3 respectively in the Argand diagram. Further assume that
$z_2 - z_3 = p$, $z_3 - z_1 = q$, and $z_1 - z_2 = r$
Therefore $p + q + r = 0$ ....(1)
Again by questions BC = CA = AB
Or $|p|^2 = |q|^2 = |r|^2$, or $p\bar{p} = q\bar{q} = r\bar{r} = k\ (\neq 0)$
Hence from (1) we get $k/p + k/q + k/r = 0$, or $qr + rp + pq = 0$
Or, $(z_3 - z_1)(z_1 - z_2) + (z_1 - z_2)(z_2 - z_3) + (z_2 - z_3)(z_3 - z_1) = 0$ proved

iii) There are 6 different letters . So they can be arranged among themselves in 6! = 720 ways. To find the number of words beginning with M, we put the letter M in the first place. So the rest 5 letters can be arranged among themselves in 5! = 120 ways. Again number of words beginning with M and ending with Y we put M in first and Y in last. The remaining 4 letters can be arranged in 4! = 24 ways. Therefore the number of words beginning with M and not ending with Y = 120-24= 96.

(iv) The given expression
$(1-2x + 3x^2)\,[1+ {}^{15}C_1.(-x)+ {}^{15}C_2(-x)^2+\ldots\ldots + {}^{15}C_{10}.(-x)^{10}+\ldots..+ (-x)^{15}$.
Therefore the reqdcoeff of x^{10} is ${}^{15}C_{10}+(-2).(-{}^{15}C_9) + 3.\,{}^{15}C_8$
= 3003 + 10010+19305 = 32318

(v)Let the reqdnos in A.P be a-b , a and a +b. By condition a – b + a + a + b = 18, or a= 6
Again the nos are in G.P.
Hence $\frac{a+4}{a-b+2} = \frac{a+b+11}{a+4}$
Or, putting a = 6, we get $b^2 + 9b - 36 = 0$
Or, b = -12 or 3.
Hence the reqdnos are 6+12, 6 and 6-12, or 18 , 6 and -6 when b =-12
Or, 6-3 , 6 and 6 +3, or 3 ,6 ,9 when b = 3

3(c) i) We have tanθ = 5/12. Hence sinθ = ∓ 5/13 and cos θ = ∓ 12/13

Hence the equation of the straight line through P (3 , -7/2) is

$\frac{x-3}{Cos\,\theta} = \frac{y+7/2}{Sin\,\theta}$ ....(1)

By the problem Point Q lies on the line (1) where $\overline{PQ}$ = 13/2 unit.

Let (h,k) be the co ordinates of Q. Hence (h,k) will satisfy equation (1).

Solving we get h = 3 + $\frac{13}{2}$ $Cos\theta$ = 9 or -3.

And k = -7/2 + $\frac{13}{2}$ $Sin\theta$ = -1 , or -6.

Therefore the reqdco ordinates of Q are (9 , -1) or (-3 , -6)

(ii) Eq of the hypotenuse is 3x + 4y = 4, Hence slope = - ¾

If m be the slope of one of the equal sides of the triangle then we have

Tan45° = ∓ $\frac{4m+3}{4-3m}$ ....(1) ,

Taking positive sign we get m = 1/7.

And by taking negative sign we get m = -7.

Therefore eq of its other sides are

x - 7y + 12 = 0 (taking m = 1/7) or

7x + y = 16 (taking m = - 7)

iii) Here the axis of the parabola is parallel to x axis. Hence the equation of the reqd parabola is

$(y - \beta)^2 = 4a(x - \alpha)$. Co ordinates of the ends of latus rectum of the parabola are $(\alpha + a, \beta + 2a)$ and $(\alpha + a, \beta - 2a)$. By question

$\alpha + a = 3, \beta + 2a = 4, \beta - 2a = 0$

Solving we get ß =2 and ∝ = 2. Therefore the reqd equation is $y^2 - 4y + 4 = 4x - 8$

Or, $y^2 - 4(x + y) + 12 = 0$

3(d) i)

$$\lim_{x \to 0} \frac{1}{x}\left[\frac{1}{Sinx} - \frac{Cosx}{Sinx}\right] = \lim_{x \to 0} \frac{2\,Sin^2 x/2}{x^2} = \lim_{u \to 0} \left(\frac{Sinu}{u}\right)^2 = 1/2$$

(ii) Let f(x) = cos (logx) and u = log x. Then we have u + k = log(x + h). By using the derivatibe of f(x) w.r.t x at the point x, we get $f'(x) = \lim_{h \to 0} \frac{f(x+h)=f(x)}{h} = \lim_{k \to 0} \frac{\cos(u+k) - \cos u}{k}$

$$= \lim_{k \to 0} Sin(u + k/2) . \lim_{k \to 0} \left[\frac{-sink/2}{k/2}\right] . \lim_{h \to 0} \frac{1}{h} \log \frac{x+h}{x}$$

$$= - sinu \lim_{h \to 0} \frac{\log(1+z)}{z} . 1/x \quad = - sin\,u .1 . 1/x = - 1/x\ Sin(\log x)$$

(e) i) Let p be the given mathematical statement and if possible let us assume that p is not true . i.e $\sqrt{}$ 2 is rational. Therefore $\sqrt{2}$ = x/y, where x and y are positive integers prime to each other and y >1 .

Hence 2 = $\frac{x^2}{y^2}$, or x^2 / y = 2y ....(1)

By assumption x and y are positive integers prime to each other. Hence x^2 and y are also positive integers prime to each other.

Therefore form (1) we get a positive rational number which is not an integer = a positive integer which is clearly impossible. Hence our assumption is not true.

(ii) We have $4x^2 - 25 = 0$. Or x $\mp$ 5/2

Clearly the equation $4x^2 - 25 = 0$, has a root (-5/2) and this root lies between (-3) and (-2). Therefore the counter example of the given statement is the root x = - 5/2 and hence the given statement is not true.

(f) i) The first die may have 6 different outcomes each of which can be associated with 6 different outcomes of the second die. Let A denote the event that the sum of the digits in the two dice is 7. Clearly event A contains equally likely event points (1,6), (2,5), (3,4),(4,3),95,2) and (6,1). Therefore we get P(A) = 6/36 = 1/6. Therefore odds in favour of event A are 1: (6-1), or 1: 5

(ii) Rs 5.26

4. a.i. RHS = $\frac{2k}{k-1} = \frac{\frac{2tan3A}{tanA}}{\frac{tan3A}{tanA}-1} = \frac{2\ SinA\ CosA}{Sin\ 2A} = \frac{sin3A}{sinA}$

$\frac{tan3A}{tanA} = k$ => (3k-1) tan^2A = k-3, ie, tanA= $\pm\sqrt{\frac{k-3}{3k-1}}$, ie, k>3. K<1/3

ii. Domain R{1, 4}

For Range of y $(5y+1)^2 - 16y^2 \geq 0$· or $(9y+1)(y+1) \geq 0$

R: (-∞ < y< -1/9) U (-1 < y < ∞)

iii. LHS => $2(1-\cos^2\frac{\theta}{2}) = (2\cos^2\frac{\theta}{2}-1)^2$ or, $\cos^2\frac{\theta}{2} = \frac{2\pm2\sqrt{5}}{8}$ So $\cos^2\frac{\theta}{2} = \frac{\sqrt{5}+1}{4}$ = cos 36^0

b.i. If x ≥ *from the* given equation –(x-1) – (x – 2) – (x – 3) ≥6

So the solution is (- ∞ ,0] U [4 , ∞)

ii. By the problem $a^{p+1} + b^{p+1} = a^{p+1/2}\sqrt{b} + \sqrt{a}.\, b^{p+1/2}$

or, $a^{p+1/2} = b^{p+1/2}$ or, p = ½

iii. a) 8_{C_3} = 56 b) 8_{C_4} = 105

iv. IzI +z = 2 + I => x + $\sqrt{x^2+y^2}$ =2 : y= 1 => x = ¾ => z = $\frac{3}{4}$ +i.

c.i. e=8/10Coordinates of foci = $(\pm 8, 0)$

SP + SP' = $\sqrt{(10\cos\theta-8)^2+36\sin^2\theta} + \sqrt{(10\cos\theta+8)^2+36\sin^2\theta}$ =20.

ii. Coordinate of the centere: (5, 2)

Let the equation of hyperbola be $\frac{(x-5)^2}{16} - \frac{(y-2)^2}{9} = 1$

2a = 8 and 2ae = 10 => $b^2 = 9$ and $a^2 = 16$.

iii. Length of the focal chord = $a(t+\frac{1}{t})^2$ unit = a $[(t-\frac{1}{t})^2 + 4]$ = $a(t-\frac{1}{t})^2$ 4a ≥ 4a

So the least length of the focal chord of the parabola is 4a which is the latus rectum.