ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

Pre – Test Examination – 2018

Sub :Mathematics **Class: 12** **FM: 80**

Duration:3hrs 15 Mins. **Date: 06.08.2018**

Part – A

Group- A

(I) Answer the following questions :- (2 X 10 = 20)

1) Show that the relation " is congruent to " on the set A of all triangles in a plane is an equivalence relation.
2) If Z be the set of integers, prove that the function $f : Z \to Z$, defines by $f(x) = |x|$, for all $x \in Z$, is a many one function.
3) Find the principal value of $\cot^{-1}(-1)$
4) Prove that $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3) = 15$.
5) Evaluate $\begin{vmatrix} x-1 & 1 \\ x^3 & x^2+x+1 \end{vmatrix}$
6) Solve the following system of homogeneous equations $x + 2y - z = 0$, $x - 2y + 2z = 0$, $x + z = 0$
7) Examine the continuity of $f(x) = 2x^2 + 1$ at $x = 1$.
8) Find the derivative w.r.t x, $\sin^{-1} x + \sin^{-1}\sqrt{1-x^2}$
9) If $y = \tan^{-1}(x/a)$, find y_2.
10) Determine the value of p and q for which the vectors $p\hat{i}+2\hat{j}+6\hat{k}$ and $3\hat{i} - 3\hat{j} + q\hat{k}$ are collinear.

Group - B

(II) Answer any 10 questions :- (4 X 10 = 40)

1) A relation R is defined on the set of natural numbers N as follows:- $R = \{(x, y): x, y \in N \text{ and } 2x + y = 41\}$. Show that R is neither reflexive nor symmetric and transitive.
2) Let R be the set of real numbers and $f : R \to R$ be defined by $f(x) = 2x^2 - 5x + 6$. Find $f^{-1}(5)$
3) Prove that $\sin^{-1}\cos\sin^{-1} x + \cos^{-1}\sin\cos^{-1} x = \pi/2$.
4) Find the value of $\cos\sin^{-1}(3/5)$.
5) Find the values of x , y , z and t when the following matrices are equal

$$\begin{pmatrix} x+y & y-z \\ 5-t & 7+x \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} t-x & z-t \\ z-y & x+z+t \end{pmatrix}$$

6) Solve by Cramer's rule:- $3x + y + z = 10$, $x + y - z = 0$, $5x - 9y = 1$

7) If $A = \begin{pmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & -1 \end{pmatrix}$ be a square matrix, find Adj A and A^{-1}.

8) Evaluate :- $\lim_{x \to 0} \frac{(e^x - 1)\log(1+x)}{\sin^2 x}$

9) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(\log ex)^2}$

10) If $x = e^t \sin t$ and $y = e^t \cos t$, then show that $(x+y)^2 d^2y/dx^2 = 2\left(x\frac{dy}{dx} - y\right)$

11) If G be the centroid of the triangle ABC, then prove that $\vec{GA} + \vec{GB} + \vec{GC} = \vec{0}$

12) If $\vec{a} = 4\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$ be two diagonals of a parallelogram, then find its area.

Group – C

(iii) Answer any 2 questions:- (5 X 2 = 10)

1) If $x + y + z = 0$. The show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$

2) Verify Rolle's theorem for the functions $f(x) = (x-1)(x-2)^2$ in $1 \le x \le 2$

3) If $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & -1 \\ -1 & 1 & -7 \end{pmatrix}$ find A^{-1}, hence solve the following equations $x + y - z = 3$, $2x + 3y + z = 10$ and $3x - y - 7z = 1$

4) If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ find a X b and the area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

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Part – B

Choose the correct option:- (Answers to be done in this sheet)

1) Let A = { 1 ,2 ,3 }, then the number of relations containing (1 ,2) and (1, 3)which are reflexive and symmetric but not transitive is

a) 2 b) 1 c) 3 d) 4

2) The mapping of $f : Z \rightarrow Z$ defined by $f(x) = 3x - 2$, for all $x \in Z$, then f will be
a) Onto but not one – one b) one – one but not onto c) many one and into
d) many – one and onto

3) If $g(x) = x^2 + x - 2$, and $(g \circ f)x = 2x^2 - 5x + 2$, then $f(x) =$
a) $2x^2 - 3x - 1$ b) $2x + 3$ c) $2x^2 + 3x + 1$ d) $2x - 3$

4) If $\sin^{-1} x - \cos^{-1} x = \pi/6$, state which of the following is the value of x?
a) ½ b) $1/\sqrt{2}$ c) 1 d) $\sqrt{3}/2$

5) If A be a square matrix of order 3 X 3, then $|KA|$ is equal to
a) $K^3|A|$ b) $K|A|$ c) $K^2|A|$ d) $3K|A|$

6) The value of the $\lim_{x \to 0} \dfrac{a^{2x} - 1}{2x}$ is
a) $\frac{1}{4} \log_e a$ b) 1 c) ½ d) $\log_e a$

7) The function $f(x) = |x + 1|$ is
a) Continuous at x = -1 b) Differentiable at x = 1 c) differentiable at $x = \pm 1$
d) none of these

8) If $x^2 + y^2 = a^2$, then the value of dy/dx is
a) x/y b) – x/y c) y/x d) – y/x

9) If $f(x) = \log (3x + 1)$ then the value of $f''(1)$ is
a) 9/16 b) 9/4 c) – 9/4 d) – 9/16

10) If $\vec{a} = 2\hat{i} + 4\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + m\hat{k}$ and $|\vec{a} \times \vec{b}| = 0$, then the value of m is
a) 3/2 b) -3 c) - 3/2 d) 3

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9.8.18.

ST. LAWRENCE HIGH SCHOOL

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Pre Test Examination

Sub: Mathematics **Class: XII**

Duration: 3 hrs 15mins **Date: 06.08.18**

Model Answers

Part - B

1(b) 2(b) 3(d) 4(d) 5(a) 6(d) 7(a) 8(b) 9(d) 10(c)

Part - A

Group – A

(I)

1. Let R be the relation " is congruent to " on the set T of all triangles.
   Since every triangle is congruent to itself, R is reflexive on T i.e Δ is congruent to Δ for all v $\Delta \in T$
   R is symmetric on T since for $\Delta_1 \in T$, $\Delta_2 \in T$, we have
   Δ_1 congruent to $\Delta_2 \rightarrow \Delta_2$ congruent to Δ_1
   R is transitive on T since $\Delta_1 \in T$, $\Delta_2 \in T$, $\Delta_3 \in T$,we have
   Δ_1 congruent to Δ_2 and Δ_2 congruent to $\Delta_3 \rightarrow \Delta_1$ congruent to Δ_3
   Thus R is a equivalence relation on T
2. Let x> 0 be an arbitrary element of Z. Then $f(x) = |x| = x$ and $f(-x) = = |-x| = x$
   i.e f(x) = f(-x), when x , (-x) $\in$ Z and $x \neq -x$
   Hence $f : Z \rightarrow Z$ is a many one function.

3. If the principal value of $Cot^{-1}(-1)$ be α, then $Cot\ \alpha = -1 = Cot\ (\pi - \pi/4) = 3\pi/4$

4. Let $tan^{-1}2 = \alpha$ and $Cot^{-1} 3 = \beta$
   Hence $tan\ \alpha = 2$ and $Cot\beta = 3$
   LHS = $Sec^2\ \alpha + Cosec^2\ \beta = 1 + 4 + 1 + 9 = 15$

5. The given determinant = $(x-1)(x^2 + x + 1)- x^3$
   $= x^3 - 1 - x^3 = -1$

6. We have $D = \begin{vmatrix} 1 & 2 & -1 \\ 1 & -2 & 2 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -1 \\ 0 & -4 & 3 \\ 0 & -2 & 2 \end{vmatrix}$ $\left(\begin{matrix} R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \end{matrix} \right)$

   $= \begin{vmatrix} -4 & 3 \\ -2 & 2 \end{vmatrix} = -2$. Hence the given system of equation has only trivial solution i.e x = y = z =0

7. We have $\lim_{x \to 1+} f(x) = \lim_{x \to 1+} (2x^2 + 1) = 3$

   Again $\lim_{x \to 1-} f(x) = \lim_{x \to 1-} (2x^2 + 1) = 3$ and f(1) = 3

   Therefore the function $f(x) = (2x^2 + 1)$ is continuous at x = 1.

8. Let y be the given expression and $x = \sin\theta$
   Putting the value of x as $Sin\theta$ in the given expression we get
   $y = \theta + \sin^{-1}Sin(\pi/2 - \theta) = \pi/2$
   Differentiating both sides w.r.t x we get
   $dy/dx = d/dx(\pi/2) = 0$

9. Differentiating two times successively w.r.t x, we get

$y_1 = \frac{a^2}{x^2+a^2}$ x 1/a = $\frac{a}{x^2+a^2}$

and y_2 = a. d/dx $(x^2 + a^2)^{-1}$

$= -\frac{a}{(x^2+a^2)^2}$ x 2x = - 2ax/$(x^2 + a^2)^2$

10. Since the given vectors are collinear we have

$p\hat{i} + 2\hat{j} + 6\hat{k} = m(3\hat{i} - 3\hat{j} + q\hat{k})$ where m is a scalar quantity.

Solving we get m = - 2/3 and mq = 6

Hence p = 3m = -2 and -2q/3 = 6, or q = -9

Group – B

(II) 1. Since 2X1 +1 =3 = 41 , so (1,1) $\notin$ R. Hence R is not reflexive

Again since 2X1 + 39 ≠41, so ((1,39)∈ R

But (39,1) $\notin$ R as 2X39 + 1 = 79 ≠ 4

Therefore R is not symmetric.

Further (20,1) , (1,39) ∈ R as 2X20 +1 = 41 and 2X1 + 39 = 41 but (20,39) $\notin R$ as 2X20 +39 ≠41

Thus R is not transitive.

2. We have f(x) = 5 or $2x^2 - 5x + 6 = 5$

Hence $x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 1}}{2X2} = \frac{5 \pm \sqrt{17}}{4}$ which is clearly ∈ R (domain)

3. Let LHS $\sin^{-1} x = \alpha$, therefore $\cos^{-1}x = \pi/2 - \sin^{-1}x = \pi/2 - \alpha$

Therefore LHS = $\sin^{-1}\cos\alpha + \cos^{-1}\sin(\pi/2 - \alpha)$

$= \pi/2 - \alpha + \alpha = \pi/2$ (proved)

4. Let $\sin^{-1}(3/5) = \alpha$

Therefore $\sin\alpha = 3/5$ and $\cos\alpha = \sqrt{1 - \sin^2\alpha} = 4/5$

5. By the given expression we have

x + y = t - x, or 2x = t - y--------------(1)

y - z = z - t , or 2 z - t = y-------------(2)

5 - t = z - y , or z + t = 5 + y----------(3)

And 7 + x = x + z + t , or z + t = 7---(4)

From (3) and (4) we get y = 2

Hence eq (2) reduces to 2z - t = 2 -----------(5)

Solving (4) and (5) we get z = 3 and t = 4

From (1) we get x = 1

Hence solutions are x = 1, y = 2, z = 3 and t = 4

6. Solving br Cramer's rule we get $x = D_1 / D$, $y = D^2/D$ and $z = D^3 /D$ --------------(1)

Where $D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 1 & -1 \\ 5 & -9 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 0 \\ 5 & -9 & 0 \end{vmatrix}$ = 1x(-36 - 10) = - 46

$D_1 = \begin{vmatrix} 10 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & -9 & 0 \end{vmatrix} = \begin{vmatrix} 10 & 1 & 1 \\ 10 & 2 & 0 \\ 1 & -9 & 0 \end{vmatrix}$ = 1x(-90 - 2) = -92

$D_2 = \begin{vmatrix} 3 & 10 & 1 \\ 1 & 0 & -1 \\ 5 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 10 & 1 \\ 4 & 10 & 0 \\ 5 & 1 & 0 \end{vmatrix}$ = 1x (4 - 50) = - 46

$D_3 = \begin{vmatrix} 3 & 1 & 10 \\ 1 & 1 & 0 \\ 5 & -9 & 1 \end{vmatrix}$ = 10x(-9 -5) + 1x(3 -1) = -138

Hence form (1) we get x = 2, y =1 , and z = 3

7. The determinant of matrix A = $|A| = 2\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} - 5\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$

$= 2(-1-4) - 5(-3-2) + 3(6-1) = 30$

$$\text{Now we have Adj } A = \begin{pmatrix} +\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 5 & 3 \\ 2 & -1 \end{vmatrix} & +\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} \\ +\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 11 & 7 \\ 5 & -5 & 5 \\ 5 & 1 & -13 \end{pmatrix}$$ Since $|A| \neq 0$ Hence A^{-1} exists and it is 1/30 (Adj A)

8. $$\lim_{x\to 0} \frac{(e^x - 1)/x \,.\, \log(1+x)/x}{\sin^2 x/x^2}$$

$$\frac{\lim_{x\to 0} (e^x-1)/x \,.\, \lim_{x\to 0} \log(1+x)/x}{\lim_{x\to 0} (\sin x)/x \,.\, \lim_{x\to 0} (\sin x)/x} = \frac{1 \times 1}{1 \times 1} = 1$$

9. Taking log of both sides we get
$y \log x = x - y$ (because $\log e = 1$)
or $y = \frac{x}{1+\log x}$
Differentiating both sides w.r.t x we get
$dy/dx = \frac{(1+\log x).1 - x.1/x}{(\log e + \log x)^2} = \frac{\log x}{(\log ex)^2}$

10. $dx/dt = e^t \cos t + e^t \sin t = y + x$
Again $dy/dt = e^t \cos t + e^t(-\sin t) = y - x$
Hence $dy/dx = \frac{y-x}{y+x}$

Therefore $d^2y/dx^2 = \frac{(x+y)\left(\frac{dy}{dx}-1\right) - (y-x)\left(\frac{dy}{dx}+1\right)}{(x+y)^2}$,
$= (x + y - y + x)\, dy/dx - x - y - y + x$
$= 2x\, dy/dx - 2y = 2(x\, dy/dx - y)$

11.Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be the position vectors of the vertices A , B and C resp of the triangle ABC. Then the centroid position is $1/3(\vec{a} + \vec{b} + \vec{c})$
Therefore GA = (position vector of A) – (position vector of G)

$= 1/3(2\vec{a} - \vec{b} - \vec{c})$

Similarly $\vec{GB} = 1/3(2\vec{b} - \vec{c} - \vec{a})$ and $GC = 1/3(2\vec{c} - \vec{a} - \vec{b})$

Hence $\vec{GA} + \vec{GB} + \vec{GC} = 0$

12. we have $|\vec{a} \times \vec{b}|$ modulus of $\begin{vmatrix} i & j & k \\ 4 & -1 & -3 \\ -2 & 1 & 2 \end{vmatrix}$

$= |(-2+3)\hat{i} + (6-8)\hat{j} + (4-2)\,]\hat{k}|$
$= \sqrt{9} = 3$
Therefore the reqd area of the parallelogram $= \frac{1}{2}|\vec{a} \times \vec{b}| = 3/2$ square units.

Group – C

(III)

1) From the problem we get $\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^3-y^3 & y^3-z^3 & z^3 \end{vmatrix}$

$= (x-y)(y-z)[y^2 + yz + z^2) - (x^2 + xy + y^2)]$
$= (x-y)(y-z)(z-x)(y+z+x) = 0$

2) From the given expression we have $f'(x)\ 3x^2 - 10x + 8$
Hence f(x) is continuous $1 \le x \le 2$ and $f'(x)$ exists everywhere in $1 < x < 2$
Again f(1) = 0 and f(2) = 0
So f(1) = f(2)
Hence function f(x) satisfies all the conditions of Roll's theorem. Therefore there exists at least one value of x (say x = c) between x = 1 and x = 2 such that $f'(c) = 0$
Or $3c^2 - 4c - 6c + 8 = 0$
Solving we get c = 4/3 or c = 2
Clearly 1<4/3<2 and $f'(4/3) = 0$. Hence Roll's theorem is verified.

3) $|A| = 1\begin{vmatrix} 1 & -4 \\ 3 & -4 \end{vmatrix} = -4 + 12 = 8$

And Adj A = transpose of the matrix $\begin{pmatrix} +\begin{vmatrix} 3 & -1 \\ 1 & -7 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ -1 & -7 \end{vmatrix} & +\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 1 & -7 \end{vmatrix} & +\begin{vmatrix} 1 & 3 \\ -1 & -7 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \end{pmatrix}$

= transpose of matrix $\begin{pmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{pmatrix}$ $A^{-1} = \text{Adj. } A / |A| = 1/8\begin{pmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{pmatrix}$

Now $A^T X = B$ where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} 3 \\ 10 \\ 1 \end{pmatrix}$
Or $X = (A^T)^{-1} B = (A^{-1})^T B$

$1/8\begin{pmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 10 \\ 1 \end{pmatrix} = 1/8\begin{pmatrix} 3 \\ 10 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ 1 \end{pmatrix}$

Therefore the reqd soln is x = 3 , y = 1 and z = 1

4) We have $\vec{a} \times \vec{b} = \hat{i}\begin{vmatrix} -2 & 1 \\ -3 & 4 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 1 \\ 1 & 4 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix}$

$= -5\hat{i} - 11\hat{j} - 7\hat{k}$
The area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

$= \sqrt{(-5)^2 + (-11)^2 + (-7)^2} = \sqrt{195}$ square units.