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# ST. LAWRENCE HIGH SCHOOL

**Selection Test**

FOR GOD AND COUNTRY

**Sub: Mathematics** **Class: XII** **FM: 80**

**Duration: 3hrs 15 mins** **Part- A & B** **Date: 15.11.18**

## Model Answers :-

Part - A

1.c 2.d 3.b 4.a 5.c 6.a 7.b 8.b 9.b 10.b

Part - B

(1) Answer any One question :- ( 2 X 1 = 2 )

(a) Let A = { 1,2,3,4,6 }. Let R be the relation on A defined by { (a,b) : a , b ∈ A, b is exactly divisible by a }. Find the domain and range of R.

Ans: We have 1/1,1/2,1/3,1/4,1/6,2/2,2/4,2/6,3/3,3/6,4/4,6/6

Therefore 1R1,1R2,1R3,1R4,1R6,2R2,2R4,2R6,3R3,3R6,4R4,6R6

Therefore R={ (1.1),(1.2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)}

Domain: { 1,2,3,4,6}

Range: { 1,2,3,4,6}

(ii)Show that $9\pi/8 - 9/4 \sin^{-1} 1/3 = 9/4 \sin^{-1} 2\sqrt{2}/3$ ( 2 X 1 = 2)

Ans: L.H.S = $9/4 ( \pi/2 - \sin^{-1} 1/3) = 9/4 \cos^{-1} 1/3$

Let $\cos^{-1}1/3 = \theta$ , or $\cos\theta = 1/3$

Hence $\sin\theta = 2\sqrt{2}/3$

Hence L.H.S = $9/4 \sin^{-1} 2\sqrt{2}/3$ = RHS

(b) Answer any One question:-

(i) If $A = 1/3 \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Show that $A A^T = I$

Ans: $A^T = 1/3 \begin{bmatrix} -1 & -2 & 2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix}$

Hence $A A^T = 1/3 \begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \times 1/3 \begin{pmatrix} -1 & -2 & 2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$

(ii) Evaluate:- $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Ans: Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ . Expanding $\Delta$ along $R_1$ we get

$(b+c)(ca+cb+a^2+ab-bc)-a((ba+b^2-bc)+a(bc-c^2-ca)$

Solving we get 4abc

(c) Answer any **Three** questions :- **( 2 X 3 = 6 )**

(i)If $x = \text{Cos}^{-1} 1/\sqrt{1+t^2}$ , $y = \text{Sin}^{-1} t/\sqrt{1+t^2}$ , find $\frac{dy}{dx}$

Ans: Let $t = \tan\theta$

$x = \text{Cos}^{-1}(1/\sec\theta) = \theta = \tan^{-1}t$

hence $dx/dt = 1/(1+t^2)$

Similarly $dy/dt = 1/(1+t^2)$

Hence $dy/dx = \frac{dy/dt}{dx/dt} = 1$

(ii)Verify Rolle's theorem for the function $f(x) = |x^2 - 4|$ on $[-1, 1]$

Ans: We have $x \in [-1,1] \Rightarrow 0 \leq x^2 \leq 1 \Rightarrow x^2 - 4 < 0 \Rightarrow |x^2 - 4| = -(x^2-4)$

Therefore $f(x) = -(x^2 - 4) = 4 - x^2$ on $[-1,1]$

f(x) being a polynomial is continuous on [-1.1]

$f'(x) = -2x$ and this exists uniquely on (-1.1)

$f(-1) = 3$ and $f(1) = 3$

Therefore $f(-1) = f(1)$

(iii)Evaluate $\int \frac{\text{Cos}x dx}{\text{Sin}^2x + 4\text{Sin}x + 5}$

Let z = sinx, hence dz = cos x dx

$I = \int dz/(z^2 + 4z + 5) = \int dz/(z+2)^2 + 1$

Let t = z=2, hence dt=dz

Therefore $I = \int dt/(t^2+1) = \tan^{-1}(z+2) + C = \tan^{-1}(\sin x + 2) + C$

(iv) The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 secs it is 6 units, find the radius of the balloon after t secs.

Ans: Let dV/dt=k

$\Rightarrow d/dt(4\pi r^3/3) = k$

$\Rightarrow 4\pi r^2 dr = kdt$

By integrating we get $4\pi r^3 = 3kt + 3C$

Now r = 3, when t = 0, and r = 6 when t = 3

Therefore, solving $r = (63t + 27)^{1/3}$

(v) Find the interval where $f(x) = \frac{x}{x^2+1}$ is strictly decreasing

Ans: We have $f(x) = x/(x^2+1)$, So $f'(x) = (1-x)(1+x)/(x^2+1)^2$

So, $f'(0) = 0$ , $(1-x)(1+x)=0$, or x = 1 , -1

Therfore values in ascending order is x = -1,1

For x<-1, f'(x)=-ve , so f(x) is strictly decreasing

For -1<x<1, f'(x) = +ve, so f'(x) is strictly increasing

For x>1, f'(x) = -ve, so f'(x) is strictly decreasing

So f'(9x) is strictly decreasing.

(vi) Find the differential equation of all circles touching the x axis at the origin.

Ans: The equation of all circles touching the x axis at the origin is given by

$(x - 0)^2 + (y - a)^2 = a^2$ , or $x^2 + y^2 - 2ay = 0$ ---------(1)
Now differentiating both sdes w.r.t x we get
$(x^2 + y^2)\, dy/dx = 2xy + 2y^2\, dy/dx$
Or, $(x^2 - y^2)dy/dx = 2xy$, which is the reqd. differential equation of all circles touching the x axis at origin.

(d) (i) If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, find the value of $\lambda$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$. **( 2 X 1 = 2 )**

Ans: Since perpendicular $(\vec{a} + \lambda\vec{b}).\vec{c} = 0$
$\Rightarrow (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k} . (3\hat{i} + \hat{j}) = 0$
$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0 = 0$
Or $\lambda = 8$

(ii) Find the equation of the plane through the point ( 3 , 4 , -1 )and parallel to the plane $(2\hat{i} - 3\hat{j} + 5\hat{k}) \bullet \vec{r} + 7 = 0$.
Ans: The given plane is
$\Rightarrow x(2) + y(-3) + z(5) + 7 = 0$---------------(i)
Let $2x - 3y + 5z = k$------------(ii)
(ii) passes through the point (3,4,-1)
$\Rightarrow k = -11$.
$\Rightarrow (2\hat{i} - 3\hat{j} + 5\hat{k}) \bullet \vec{r} = -11$

(e) (i) ) If A and B are events associated with a random experiment such that **( 2 X 1 = 2 )**
P ( A ) = 1/3 , P ( B ) = 1 / 4 , P ( A ∩ B ) = 1/5 find P ( A ∪ B ).
Ans: P(AUB)=1/3+1/4-1/5= 23/60

( ii ) If the sum of the mean and variance of a binomial distribution for 5 trials is 1.8, find the distribution.
Ans: Let n and p be the two parameters of the binomial distribution. Then mean = np and variance = npq, where q = 1-q
np+npq=1.8
Solving we get p = 9/5 or 1/5
Therefore reqd. Distribution = $^5C_x\,(1/5)^x(4/5)^{5-x}$, where x = 0,1,2,3,4,5

2(a)(i) Let * be a binary operation on A = N X N defined by ( a , b ) * ( c , d ) = ( ad + bc , bd ) for all ( a,b) , ( c,d ) ∊ A. Prove that A = N X N has no identity element. **( 4 X 1 = 4 )**
Ans: Let us assume the (x,y) be the identity element in A. Then by definition of identity element (ax+bx , by) = ( a,b ) for all a , b∈ N.
Hence x = 0 , y = 1 [ because b∈ N and b ≠ 0]
Since 0 ∉ N, Hence ( 0,1 ) ∉ A = N X N
Therefore there is no identity element in A = N X N w.r.t binary equation.

( ii ) Solve :- $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 3x$
Ans:By the given expression we have $\tan^{-1}\frac{(x-1)(x+1)}{1-(x-1)(x+1)} = \tan^{-1}\frac{3x-x}{1+(3x)(x)}$
Or, $\tan^{-1}\frac{2x}{2-x^2} = \tan^{-1}\frac{2x}{1+3x^2}$
Or, solving we get $x = [4x^2 - 1] = 0$
Or x = 0 , ½ , -1/2

2(b)(i)When $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ then find ½( A + A′ ) and ½ ( A – A′ ) **( 4 X 2 = 8 )**

Ans: $A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

Hence ½( A + A′) = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

And ½( A - A′) = $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

OR

Solve by Cramer's rule :

$x + y + z = 1$ ; $ax + by + cz = k$ , $a^2x + b^2y + c^2z = k^2$ ( $a \neq b \neq c$)

Ans: Solving the given eq by Cramer's rule we get

$x = D_1/D$ , $y = D_2/D$ , $z = D_3/D$ ----------(1)

where D = $\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-b \\ a^2 & b^2-a^2 & c^2-b^2 \end{vmatrix}$ by replacing the 2nd column by $C_2 - C_1$ and 3rd column by $C_3 - C_1$

= ( a - b )(b -c )( b + c - a -b )

= ( a -b )( b - c ) ( c - a )

$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ k & b & c \\ k^2 & b^2 & c^2 \end{vmatrix}$ = ( k - b ) ( b - c ) ( c - k )

Similarly $D_2$ = ( a- k)(k-c)(c-a)

And $D_3$ = (a- b )(b – k )( k – a )

Hence from (1) $x = \frac{(k-b)(c-k)}{(a-b)(c-a)}$, $y = \frac{(a-k)(k-c)}{(a-b)(b-c)}$, $z = \frac{(b-k)(k-a)}{(b-c)(c-a)}$

(ii) Prove that $\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$ = 16( 3x + 4 )

Ans: Operating $R_1 \rightarrow R_1 + R_2 + R_3$

( 3x + 4 ) $\begin{vmatrix} 1 & 0 & 0 \\ x & 4 & 0 \\ x & 0 & 4 \end{vmatrix}$ , Operating $C_2 \rightarrow C_2 - C_1$ , $C_3 \rightarrow C_3 - C_1$ and expanding along $R_1$ we get 16(3x + 4)

OR

Find the value of $\begin{vmatrix} (a+x)^2 & a-y & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix}$

Ans: Let Δ be the given expression,

=( $a^2$ - 2ax + $x^2$) [ ($b^2$ - 2by + $y^2$) ( $c^2$ - 2cz + $z^2$) - ( $c^2$ - 2cy +$y^2$) ( $b^2$- 2bz + $z^2$ )] - (a-y)[($b^2$ - 2bx + $x^2$)( $c^2$-2cz + $z^2$) - ($c^2$ - 2cx + $x^2$)( $b^2$-2bz+$z^2$)] +($a^2$-2az+$z^2$)[($b^2$-2bx + $x^2$)($c^2$-2cy + $y^2$ ) – ( $c^2$-2cx + $x^2$)( $b^2$-2by + $y^2$)]

( c) (i) If $y = \frac{1+\sin x-\cos x}{1+\sin x+\cos x}$, show that $\frac{dy}{dx} = \frac{1}{1+\cos x}$ **(4 X 3 = 12)**

Ans : $y = \frac{(1-\cos x)+\sin x}{(1+\cos x)+\sin x}$ or y = tanx/2

Differentiating w.r.t x we get

$dy/dx = \frac{1}{2\cos^2\frac{X}{2}} = \frac{1}{1+\cos x}$

OR

If x = f(t) and y = g(t), prove that $d^2y/dx^2 = \frac{f1g2-g1f2}{f1^3}$, where suffixes denote differentiations w.r.t t.

Ans:We have $dx/dt = d/dt\, f(t) = f_1(t)$

Again $y = g(t)$ , hence $dy/dt = g_1(t)$

$dy/dx = \frac{dy/dt}{dx/dt} = g_1(t)/f_1(t)$

Hence $d^2y/dx^2 = d/dt[g_1(t)/f_1(t)]\, x\, dt/dx = (f_1g_2-g_1f_2)/f_1^2\, x 1/f_1 = (f_1g_2-g_1f_2)/f_1^3$

(ii) Evaluate $\int \frac{\tan x}{a+b\tan^2 x}\, dx$

Ans: Let $I = \int \frac{\tan x\, dx}{a+b\tan^2 x} = \int \frac{\sin x \cos x\, dx}{a\cos^2 x + b\sin^2 x}$

We put $a\cos^2 x + b\sin^2 x = z$

Differentiating we get $\sin x \cos x\, dx = \frac{dz}{2(b-a)}$

Or, $I = \frac{1}{2(b-a)}\int dz/z$

$= \frac{1}{2(b-a)}\log\left| a\cos^2 x + b\sin^2 x + k\right|$

OR Integrate $\int x e^x \cos x\, dx$.

Ans: Integrating by parts we get $x\ e^x \cos x\, dx - dx/dx\,[\ e^x \cos x\, dx]\, dx$

$= x .1/2\ e^x(\cos x + \sin x) - ½\int e^x(\cos x + \sin x)dx$------------(i)

Now $\int e^x(\cos x + \sin x)dx = e^x \sin x - \int e^x \sin x\, dx + \int e^x \sin x\, dx = e^x \sin x$

Therefore from (i) we get $\int x e^x \cos x\, dx = ½ x e^x(\cos x + \sin x) - ½ e^x \sin x + c$

(iii)Solve :- $\tan x dy - \tan y dx = 0$, given $y = \pi/2$ when $x = \pi/4$

Ans: We have $dy/\tan y - dx/\tan x = 0$

Or, $\log|\sin y| - \log|\sin x| = \log k$

Or $|\sin y| = k|\sin x|$

Y question we have $y = \pi/2$ when $x = \pi/4$

Therefore we get $\sin \pi/2 = k \sin \pi/4$

Or $k = \sqrt{2}$

Hence the solution is $|\sin y| = \sqrt{2}|\sin x|$

OR

A particle starts from the origin with a velocity u and moves in a straight line, its acceleration being always equal to its displacement . If v be the velocity when its displacement is x, then show that $v^2 = u^2 + x^2$.

Ans: Let x be the displacement of the particle at time t, then if v and a be the velocity and acceleration resp. we have $v = dx/dt$ and $a = dv/dt$

Or, $(dv/dx).(dx/dt) = x$ or $vdv = xdx$

Or $v^2 = x^2 + k$----------(1)

By question v = u when x =0. Hence form (1) we get $v^2 = x^2 + u^2$

(d ) Answer any **One** question : - **( 4 X 1 = 4 )**

(i) If α, β, γ be the unit ventors satisfying the condition $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = 0$, show that $\vec{\alpha}\vec{\beta} + \vec{\beta}\vec{\gamma} + \vec{\alpha}\vec{\gamma} = -\frac{3}{2}$

Ans: Since $\alpha + \beta + \gamma = 0$

Therefore $\vec{\alpha}.\vec{\beta} + \vec{\gamma}.\vec{\alpha} = -\vec{\alpha}.\vec{\alpha} = -1$------(i)

Similarly $\vec{\beta}.\vec{\gamma} + \vec{\alpha}.\vec{\beta} = -1 - -(ii)$ and $\vec{\gamma}.\vec{\alpha} + \vec{\beta}.\vec{\gamma} = -1$------(iii)

Adding (i), (ii) and (iii) we get -3/2

(ii) In the following statement find p if vectors $\hat{i} + 2\hat{j} - 3\hat{k}$, $p\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$ are coplanar

Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = p\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

Since $\vec{a}, \vec{b}, \vec{c}$ are coplanar hence $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

Or $\begin{vmatrix} 1 & 2 & -3 \\ p & -1 & 1 \\ 3 & -4 & 5 \end{vmatrix} = 0$

Or p = 2

( e ) (i) Evaluate:- $\int_0^{\pi} x \sin^2 x\, dx$. **( 4 X 1 = 4 )**

Ans: Let I = $\int_0^{\pi} x\sin^2 x\, dx$

Or I = $\pi \int_0^{\pi} \sin^2 x\, dx - \int_0^{\pi} x\sin^2 x\, dx$

Or 2I = $\pi/2 \int_0^{\pi} (1 - \cos 2x)dx = \pi/2\, [x - \sin 2x/2]_0^{\pi} = \pi^2/4$

(ii) $\int_0^{\pi} \frac{dx}{5+4 \cos x}$

Ans: We have $\frac{1}{5+4\cos x} = \frac{1+\tan^2 x/2}{5(1+\frac{\tan^2 x}{2})+4(1-\frac{\tan^2 x}{2})} = \frac{\sec^2 x/2}{9+\tan^2 x/2}$

Now we put tanx/2 = z, so we have $\sec^2 x/2\, dx = 2dz$

From tanx/2 = z, we get

| x | 0 | π |
|---|---|---|
| z | 0 | ∞ |

Hence $\int_0^{\pi} \frac{dx}{5+4\cos x} = \int_0^{\infty} \frac{2dz}{9+z^2} = 2 \times 1/3\, [\tan^{-1} z/3]_0^{\infty} = \pi/3$.

( f ) Answer any **One** question : **( 4 X 1 = 4 )**

( i ) Twelve cards numbered 1 to 12 are placed in a box mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is an even number.

Ans: S ={ 1,2,3,4,5,6,7,8,9,10,11,12}. Let A be the event that the number on the drawn card is greater than 3, then A = {4,5,6,7,8,9,10,11,12,]. Again if B be the event that the number on the drawn card is even, then B = { 2,4,6,8,10,12 }

Therefore $A \cap B$ = {4,6,8,10,12}. Clearly number of equally likely event points contained in the events A and $A \cap B$ are 9 and 5. Hence P(A) = 9/12 and P($A \cap B$) = 5/12.

Now by theorem of compound probability we get

P($A \cap B$)= P(A)P(B/A)

Or P(B/a) = 5/9, is the probability that the number on the drawn card is even.

(ii)For a binomial distribution , the mean and S.D are respectively 4 and $\sqrt{3}$. Calculate the probability of getting a non - zero value from this distribution.

Ans: If the parameters of binomial distribution be n and p then its mean and S.D are np and $\sqrt{npq}$. resp.

By problem np = 4 and npq = 3.

Because np=4, we get q = ¾. Hence p = ¼.

Therefore we get n = 16.

Hence the probability distribution of f(x) of the "number of successes" x of binomial distribution is given by $f(x) = {}^nC_x\, p^x q^{n-x}$ , whre x = 0,1,2,3 ......16

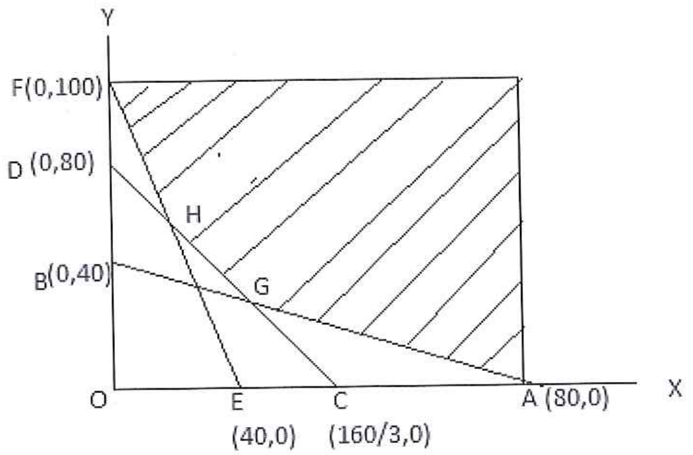
Therefore the reqd probability of getting a non zero value is

$1-f(0) = 1 - {}^{16}C_0\,(1/4)^0(3/4)^{16} = 1-(3/4)^{16}$

3 ( a) Answer any **One** question :- **( 5 X 1 = 5 )**

(i)A company owns mines, mine A produces 1 tonne of high grade ore, 3 tonnes of medium grade ore and 5 tonnes of low grade ore each day, and mine B produces 2 tonnes of each of the three grades of ore each day. The company needs 80 tonnes of high grade ore, 160 tonnes of medium grade ore and 200 tonnes of low grade ore. If it costs ₹ 200 per day to work each mine, using corner point method find the number of days each mine has to be operated for producing the required output with minimum total cost.

Ans:




Let x and y be the no of days of operations of the mines A and B resp. Then the total cost in Rs. For operating 2 mines is Z = 200x + 200y. Since total amt of ore produced is ( x + 2y ) tones of high grade and the minimum of high grade ore is reqd is 80 tons, we have

$x + 2y \geq 80$.

Similarly we have the following inequalities for the medium grade ore and low grade ore

$3x + 2y \geq 160$ and $5x + 2y \geq 200$

Thus the problem can be formulated as Minimize Z = 200x + 200y

The graphs are drawn for the following straight line

x+2y=80 , 3x+2y=160 and 5x+2y=200

| Corner Point | The value of objective function Z = 200x + 200y |
|---|---|
| A(80,0) | 200X80+200X0=16000 |
| G(40,20) | 200X40+200X20=12000 |
| H(20,50) | 200X20+200X50=14000 |

| F(0,100) | 200X0+200X100=20000 |
|---|---|

Thus the min. value of Z occurs at the corner points G(40,20) and the minimum value of Z is 12000.

For minimum cost mine A for 40 days and mine B for 20 days si Rs. 12000.

( ii)Solve graphically :- Maximize Z = - 4 x + 6 y subject to the constraints : - x + y < 3 , - x + 3y < 15 and x , y > 0

Ans: With ref to the set of rectangular Cartesian co ordinates axes O and OY the graph is drawn. The region satisfied by the lines is shown by the shaded lines. Here the region of the feasible solution in unbounded. The lines of objective function Z = -4x + 6y for different values of Z = (0,12,24) are shown in the fig. We see that the max, value of Z occurs at a single point A(3,6). Though the region of the feasible solution is unbounded, the given problem has unique primal solution at x = 3 and y = 6. The max. value of Z = 24.

( b ) Answer any **Two** questions :- **( 5 X 2 = 10 )**

(i)Show that, the function $\text{Sin}^3$ x Cos x has a maximum value at x = π/3.

Ans: Let f(x) = $\text{Sin}^3$x Cosx

= 1/8( 2Sin2x – sin4x)

Hence f'(x) = ½(cos2x – cos4x) and f''(x) = -sin2x + 2sin4x

Now we have f'(π/3) = ½[cos 2π/3 – cos 4π/3] = ½(-1/2+1/2) = 0

And f''(π/3)= -sin2π/3 + 2sin4π/3 = -3 x √3/2 which is <0

Since f'(π/3) = 0 and f''(π/3) <0.

Therefore it is evident that f(x) has max value at x = π/3

(ii)Find the condition that the straight line x Cos Θ + y Sin Θ = p may touch the parabola $y^2 = 4ax$

Ans: xcos$\theta$ + ysin $\theta$ = p---------(1)

$y^2$ = 4ax--------(2)

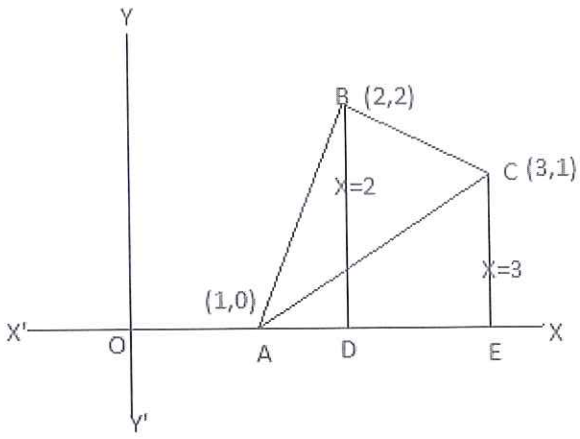
The eq of the tangent to the parabola (2) at the point ($at^2$, 2at) is x-yt = - $at^2$--------(3)

Let us assume that the straight line (1) is a tangent to the parabola (2) at the point ($at^2$, 2at). Then (1) and (3) are identical. Hence

$$\text{Cos}\frac{\theta}{1} = \frac{\sin\theta}{-t} = p/-at^2$$

Hence tan $\theta = -t$ and t = p/asin $\theta$, or p = - asin $\theta$tan $\theta$, which is the reqd condition for the line(1) to be a tangent to the parabola (2).

(iii)Using integration find the area of the triangle whose vertices are A ( 1,0 ) , B ( 2 , 2 ) and C ( 3 , 1 )




Ans : Eq. of line AB is y=2(x-1)--------------(i)

Eq. of line BC is $\frac{y-2}{x-2} = \frac{2-1}{2-3}$ = -1, or y = -x + 4------------(ii)

Eq. of line CA is $\frac{y-0}{x-1}$ = ½ or y = ½(x-1)--------(iii)

$\int_1^2 2(x-1)dx + \int_2^3 2(-x+4)dx - \int_1^3 1/2(x-1)dx$

= 2[(2-2)-(1/2-1)]+[(-9/2+12)-(-2+8)]-1/2[(9/2-3)-(1/2-1)]

= 3/2 sq.units.

(iv)Solve:- $(x+y+1)\frac{dy}{dx} = 1$

Ans: The given eq is (x+y+1)dy/dx =1 or dx/dy –x= y+1--------(i) which is of the form dx/dy +Px +Q, where Q = y+1

Multiplying both sides of eq(i) by $e^{-y}$ we get

$e^{-y}dx/dy - e^{-y}.x = (y+1)e^{-y}$

or $d/dy(xe^{-y}) = (y+1)e^{-y}$-------(ii)

Integrating both sides of (ii) we get

$xe^{-y} = -(y+1)e^{-y} - e^{-y} + c$

or $x = -y-2+ce^{y}$ which is the reqd. general solution of the given differential eq.

( c ) Answer any **One** question :- **( 5 X 1 = 5 )**

( i ) Find the equation of the plane which passes through the point ( 2, 1, 4) and is perpendicular to each of the planes 9x - 7y + 6z + 48 = 0 and x + y - z = 0

Ans:

Eq. of any plane which passes through (2,1,4) is given by a(x-2) +b(y-1)+c(z-4) = 0-------(i)

If this plane is perpendicular to 9x-7y+6z+48=0, we have

9a-7b+6c=0-------------(ii)

And since it is perpendicular to x+y-z=0 then a+b-c=0----------(iii)

From (ii) and (iii) we get by cross multiplication

A=k, b=15k and c=16k

So eq. of the plane is x+15y+16z=81 [ k $\neq$ 0]

(ii)Find the shortest distance between the following lines:- $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{4}$ and $\frac{2x+3}{5} = \frac{2y-3}{-2} = \frac{z+1}{1}$

Ans: $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{4}$ -----------------$L_1$

Or, $\frac{x+\frac{3}{2}}{\frac{5}{2}} = \frac{y-\frac{3}{2}}{-1} = \frac{z+1}{1}$ ------------$L_2$

$$S.D = \left| \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{(\overrightarrow{b_1} \times \overrightarrow{b_2})} \right|$$

$\overrightarrow{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$ , $\overrightarrow{a_2} = -\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} - \hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ \frac{5}{2} & -1 & 1 \end{vmatrix} = \hat{i} - \hat{j}\,8 + \hat{k} \cdot \frac{11}{2}$$

$\overrightarrow{a_2} - \overrightarrow{a_1} = -\frac{5}{2}\hat{i} + \frac{7}{2}\hat{j} - 4\hat{k}$

Hence $(\overrightarrow{a_2} - \overrightarrow{a_1}) . (\overrightarrow{b_1} \times \overrightarrow{b_2}) = -\frac{5}{2} - 50 = -\frac{105}{2}$

Therefore $|b_1 \times b_2| = \sqrt{1 + 64 + \frac{121}{4}} = \sqrt{381}/2$

Therefore S.D = $(-\frac{105}{2}) / (\sqrt{381}/2) = 105 / \sqrt{381}$ units.