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Solution

Full Marks : 80

Sub: MathematicsClass: XI **Time : 3 Hrs.**
Date : 08/08/2019

GROUP : A

Select the correct alternatives : **1x8=8**

1.a) Which one is true?

i) $2+3i>1+4i$ ii) $3+3i>6+2i$ iii) $3+9i>5+6i$ **iv) None of these**

b) The least positive integral value of n so that $\left(\frac{1+i}{1-i}\right)^n=1$ is

i) 2 **ii) 4** iii) 3 iv) none of these

c) If $\sin 49^0=\frac{3}{4}$; then the value of $\sin 581^0$ is

i) $\frac{\sqrt{7}}{4}$ **ii)** $-\frac{\sqrt{7}}{4}$ iii) $\frac{7}{2}$ iv) none of these

d) If $A\subseteq B$ *and* $B\subseteq A$ then

i) $A=\phi$ ii) $A\cap B=\phi$ **iii)** $A=B$ iv) none of these

e) $A-B=\phi$ iff

i) $A\neq B$ **ii)** $A\subset B$ iii) $B\subset A$ iv) $A\cap B=\phi$

f) If $\tan x=\frac{b}{a}$; then the value of $\left(a^2+b^2\right)\sin 2x$ is

i) ab ii) 2ab iii) $\frac{a}{b}$ iv) $\frac{2a}{b}$

g) The acute angle between the lines $x+y\sqrt{3}+7=0$ and $x\sqrt{3}-y+8=0$ is

a) 45^0 **ii)** 90^0 iii) 30^0 iv) 60^0

h) The slope of the straight line joining the points $\left(\sqrt{3},1\right)$ and $\left(-3,-\sqrt{3}\right)$ is

i) 1 **ii)** $\sqrt{3}$ iv) $\frac{1}{\sqrt{3}}$ iv) $-\frac{1}{\sqrt{3}}$

GROUP:B

Answer any six questions : 6x4=24

2. For any two sets A and B; prove that $(A \cap B)' = A' \cup B'$.

Refer page 14.

3. If $A = \{x / -1 \le x \le 2\}$ and $B = \{x / 0 \le x \le 4\}$ find

a) $A - B$ b) $(A \cup B) - (A \cap B)$

Refer page 22, example -14.

4. Prove that $(a + b\omega + c\omega^2)^3 + (a + b\omega^2 + c\omega)^3 = 27abc$ if $a + b + c = 0$ and ω is an imaginary cube root of unity.

$x = a + b\omega + c\omega^2,\ y = a + b\omega^2 + c\omega$.

L.H.S, $x^3 + y^3 = (x+y)(x^2 + xy + y^2) = [2a + b(\omega + \omega^2) + c(\omega^2 + \omega)]$

$[a(1+\omega) + b(1-\omega^2) + 2c\omega^2][a(1+\omega^2) + 2.b\omega + c(\omega^2 + 1)]$.

$= (2a - b - c)\,\omega^2 (2c - a - b) \cdot \omega (2b - a - c)$

$= (2a - b - c)(2c - a - b)(2b - a - c) = 3a \cdot 3c \cdot 3b = 27abc$ = R.H.S.

5. If show that $amp\left(\frac{z_1}{z_2}\right) = amp\, z_1 - amp\, z_2$

$\frac{z_1}{z_2} = \frac{1 + i\sqrt{3}}{\sqrt{3} - 1} = \frac{(1 + i\sqrt{3})(\sqrt{3} + i)}{(\sqrt{3} - i)(\sqrt{3} + i)} = i$

$\arg \frac{z_1}{z_2} = \tan^{-1}\left(\frac{1}{0}\right) = \frac{\pi}{2}$, $\arg z_1 - \arg z_2 = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

$\therefore \arg \frac{z_1}{z_2} = \arg z_1 - \arg z_2$

6. Solve by Sridhar Acharyya's formula : $6x^2 - (18 + 5i)x + 18 + i = 0 (i = \sqrt{-1})$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{(18 + 5i) \pm \sqrt{(18 + 5i)^2 - 4 \times 6 \times (18 + i)}}{2 \times 6}$.

$= \frac{(18 + 5i) \pm \sqrt{324 + 180i + 25i^2 - 432 - 24i}}{12}$

7. The sum of n terms of two A.P's are in the ratio $(4n - 13) : (3n + 10)$. Find the ratio of their 9th terms. Refer page 393, example- 11.

8. Find the sum of the series upto n terms: $1 + 5 + 12 + 22 + 35 + \ldots$ to n term.

$S_n = 1 + 5 + 12 + 22 + 35 + \cdots + t_n$.

$S_n = 1 + 5 + 12 + 22 + \cdots + t_{n-1} + t_n$.

$0 = 1 + 4 + 7 + 10 + 13 + \cdots$ to n terms $- t_n$.

$\therefore t_n = \frac{3n^2}{2} - \frac{n}{2}$ $\therefore S_n = \sum_{n=1}^{n} t_n = \frac{1}{2} n^2 (n+1)$

9. Find, graphically, the solution set of the following linear inequation :

$$2x+5y \le 40$$
$$x+y \le 11$$
$$x \ge 0, y \ge 0$$

Refer page 322, example- 43.

GROUP : C

Answer an six questions : **6x4=24**

10. Evaluate $\tan 70^0 - \tan 50^0 + \tan 10^0$

Refer page 980, problem 13

11. If $p\sin\theta = q\sin(120^0+\theta) = r\sin(240^0+\vartheta)$; find the value of $pq+qr+rp$.

12. If α, β, γ are in A.P.; show that $\cot\beta = \dfrac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$

$\alpha+\gamma = 2\beta$.

$$\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha} = \frac{2\sin\frac{\alpha-\gamma}{2}\cos\frac{\alpha+\gamma}{2}}{2\sin\frac{\alpha+\gamma}{2}\sin\frac{\alpha-\gamma}{2}} = \cot\frac{\alpha+\gamma}{2} = \cot\beta.$$

13. If $\tan\beta = \dfrac{\sin\alpha\cos\alpha}{2+\cos^2\alpha}$; prove that $3\tan(\alpha-\beta) = 2\tan\alpha$

Refer page 125, example-13

14. Show that $\sin^3\theta + \sin^3(120^0+\theta) + \sin^3(240^0+\theta) = -\dfrac{3}{4}\sin 3\theta$

Refer page 143, example-11

15. If $\tan(A+B) = 3\tan A$; show that $\sin(2A+2B) + \sin 2A = 2\sin 2B$.

Refer page 145, example- 20

16. Find the value of θ for which $\sin\theta\sin\left(\theta - \dfrac{\pi}{3}\right)$ is maximum.

Refer page 135, example- 13

17. If $\tan(\alpha-\beta) = 1; \sec(\alpha+\beta) = \dfrac{2}{\sqrt{3}}$; find positive magnitudes of α *and* β.

Refer page 113, example -13

GROUP : D

Answer any six questions : **6x4=24**

18. The ratio of the distances of a moving point from the points (3,4) and (1,–2) is 2:3; find the locus of the moving point. Refer page 544, example 33.

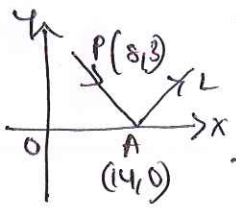
19. Find the ratio in which the straight line $3x+4y=21$ divides the join of $(-9,5)$ and(7,9). Refer page 468, example-13.

20. Find the equation of the straight line which passes through the intersection of the straight lines $2x+3y=5$ and $3x+5y=7$ and makes equal positive intercepts upon the co-ordinate axes. Refer page 469, example- 16.

21. Find the equation of the straight line passing through the point $(2,-3)$ and parallel to the straight line $3x-4y=0$. Refer page 539, example-4.

22. Find the distance of the point (3,5) from the line $2x+3y=14$ measured parallel to the line $x-2y=1$. Refer page 542, example-12.

23. A ray of light through (8,3) is reflected at (14,0) on the x-axis. Find the equation of the reflected ray. Slope of $AP = \frac{3-0}{8-14} = -\frac{1}{2}$ $\angle OAP = \angle xAL$.



$\Rightarrow \left|\frac{-\frac{1}{2}-0}{1+(-\frac{1}{2})\cdot 0}\right| = \left|\frac{m-0}{1+m\cdot 0}\right| \Rightarrow \frac{1}{2} = |m| \therefore m = \frac{1}{2}$ $(\because m \neq -\frac{1}{2})$

$\therefore$ Equation $\Rightarrow y-0 = \frac{1}{2}(x-14) \Rightarrow x-2y=14$.

24. The co-ordinates of one vertex of an equilateral triangle are (2,3) and the equation of its opposite side is $x+y=2$. Find the equation of its other sides.

Equation of AB, AC with slope m passing through (2,3) is

$y-3 = m(x-2) \Rightarrow mx-y-2m+3=0$.

$\because \triangle ABC$ is equilateral, $\therefore \tan 60^\circ = \left|\frac{m+1}{m-1}\right| \Rightarrow m = 2+\sqrt{3}, 2-\sqrt{3}$.

$\therefore (2+\sqrt{3})x-y-2(2+\sqrt{3})+3=0$

and $(2-\sqrt{3})x-y-2(2-\sqrt{3})+3=0$.

25. Find the equation of the straight line that passes through (–1,2) and perpendicular to the line $2x-3y+1=0$. Refer page 540, example-5.

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