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ST. LAWRENCE HIGH SCHOOL

FOR GOD AND COUNTRY

1st Term Exam – 2019

Model Answer

Sub: Physics **Class: XI-A2** **F. M.: 70**

SECTION-I

Answer the following questions (Multiple Choice Questions) **(1x14=14)**

1. The position of a particle at time t is given by $x_t = \frac{v}{\alpha}(1 - e^{-\alpha t})$,where v is a constant and $\alpha > 0$.The dimensions of v and α are:
   a) $[M^0LT^{-1}]$ and $[T^{-1}]$ b) $[M^0LT^0]$ and $[T^{-1}]$
   c) $[M^0LT^{-1}]$ and $[LT^{-2}]$ d) $[M^0LT^{-1}]$ and$[T]$

2. A variable force F is applied on an object for certain time such that $F \propto t$, where 't' is the instantaneous time. What will be the nature of $v\ versus\ t^2$ curve for the particle? (represents instantaneous velocity)
   a) Straight line passing through origin b) Straight line with intercepts in X-axis or Y-axis
   c) Parabola d) none of these

3. Which quantity remains unchanged in case of a projectile ?
   a)Momentum b) Kinetic Energy
   c) Vertical component of velocity **d) Horizontal component of velocity**

4. The percentage errors in the measurement of mass and velocity are 2% and 3% respectively.How much will be the maximum error in the estimation of kinetic energy obtained by measuring mass and velocity:
   a) 11% **b) 8%** c)5% d) 1%

5. If two finite vectors (with non zero magnitude) are such that $|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$, then the correct relation between them will be -
   a) they are parallel to each other b) they are anti parallel to each other
   c) they are perpendicular to each other d) $\vec{P} = 2\vec{Q}$

6. A projectile can have the same range R for two angles of projection which are complementary to each other .If T_1 and T_2 be the time of flights in the two cases, then the product of the two time of flights is directly proportional to
   a)R b) 1/R c)$1/R^2$ d) R^2

7. Speeds of two identical cars are u and 4u at a specific instant .The ratio of the respective distance in which the two cars are stopped from that instant is :
   a) 1:1 b) 1:4 c) 1:8 **d) 1:16**

8. A man can swim with speed 4km/h in still water. At what angle he should project himself w.r.t the bank of the river to reach exactly opposite point of the river? (velocity of river is 2km/h)
   a) 30^0 b) 45^0 **c) 60^0** d) 90^0

9. A bullet of mass 'a' moves freely in air with uniform velocity 'b' and then hits an wooden target of mass 'c' kept on a frictionless support. It penetrates the block a little and then both of them move together. The final velocity of the system is: a) $\sqrt{\frac{ab^2}{a+c}}$ b) $\frac{a+b}{c}a$ **c) $\frac{ab}{a+c}$** d) $\frac{a+c}{a}b$

10. A lift of mass 500kg is coming down with an acceleration $7.8 m/s^2$. What will be the reading of the weighing machine (measures weight) on which a person of mass 60kg is standing inside that lift? ($g = 9.8\ m/s^2$)
   a) 60 kg b) 880 N c) 280 N **d) 120 N**

11. A cricket ball of mass 500g hits a cricket bat perpendicularly with velocity 20m/s and then gets reflected back with velocity 30m/s. The impulse exerted by ball on bat will be
   a) 50 N.s **b) 25 N.s** c) 1 N.s d) 0.5 N.s

12. Two particles having masses 'x' and 'y' are moving in a circular path having radii 'a' and 'b'. If their time periods of rotation are same, then the ratio of their angular velocities will be:

a) $\frac{a}{b}$ b) $\frac{b}{a}$ c) 1 d) ab

13. Two blocks A and B of masses m_1 *and* m_2 are kept in contact on a smooth horizontal surface. F amount of horizontal force is applied on A such a way that both move together. The force exerted on B by A will be –

a) $\frac{m_1}{m_1+m_2}F$ **b) $\frac{m_2}{m_1+m_2}F$** c) $(m_1+m_2)\frac{F}{m_1}$ d) F

14. A particle is projected making an angle of 45^0 with horizontal having kinetic energy K.The kinetic energy at the highest point will be –

a) K b) $\frac{K}{\sqrt{2}}$ c) $\frac{K}{4}$ **d) $\frac{K}{2}$**

SECTION -II

GROUP –A

Answer the following questions in short.(Alternatives are to be noted): **(1x4=4)**

(1) Given Force = α / (density +β³). What are the dimensions of α and β ?

Ans: [β³] = [density] so, [β] = $[ML^{-3}]^{\frac{1}{3}} = M^{\frac{1}{3}}L^{-1}$

[α] = [force x density] = $[MLT^{-2}.ML^{-3}] = M^2L^{-2}T^{-2}$

OR

If velocity V , acceleration A and force F are three fundamental quantities of a system, then what will be the dimensions of linear momentum in this system?

Ans: Denote linear momentum by p.

Let, [p] = $V^xA^yF^z$

Then, $MLT^{-1} = (LT^{-1})^x(LT^{-2})^y(MLT^{-2})^z$

So, z = 1, x = 1 and y = -1

Hence , [p] = $VA^{-1}F$

(2) What is the angle made by vector $\vec{A}$ = 2î +2ĵ with x axis ?

Ans: Taking dot product with î , $\sqrt{4+4}.1.Cos\theta = 2$. This gives $\theta = 45^0$.

(3) A retarding force is applied to stop a motor car. If the speed of the motor car is doubled, how much more distance will it cover before stopping under the same retarding force?

Ans: As the mass of the car and the retarding force are unchanged so the retardation in both the cases will be same. If u and 2u be the initial velocities in two cases, then $0 = u^2 - 2as_1$ and $0 = 4u^2 - 2as_2$.

So the extra path travelled in 2nd case will be = $s_2 - s_1 = \frac{3u^2}{2a}$

(4) A force of 5N changes the velocity of a body from 10m/s to 20m/s in 5sec.How much force is required to bring about the same change in 2sec?

Ans: If m be the mass of the body then, $m\frac{20-10}{5} = 5$, which gives $m = 2.5Kg$

So force required for 2nd case = mass x acceleration = $2.5 \times \frac{20-10}{2} = 12.5\ N$

GROUP –B

Answer the following questions. (Alternatives are to be noted) : **(2x5=10)**

(5) Find the component of vector $\vec{A} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ *in the direction of vector* $\vec{B} = 2\hat{\jmath} + \hat{\imath} + 2\hat{k}$.

Ans: Required component $= \vec{A}.\frac{\vec{B}}{|\vec{B}|} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}).\frac{\hat{\imath}+2\hat{\jmath}+2\hat{k}}{\sqrt{4+1+4}} = \frac{1}{9}(1-4+2) = -\frac{1}{9}$. Negative sign indicates, the angle between them is obtuse.

(6) To a diver going east in a car with velocity of 40km/hr , a bus appears to move towards north with a velocity of 40√3 km/hr.What is the actual velocity and direction of motion of the bus?

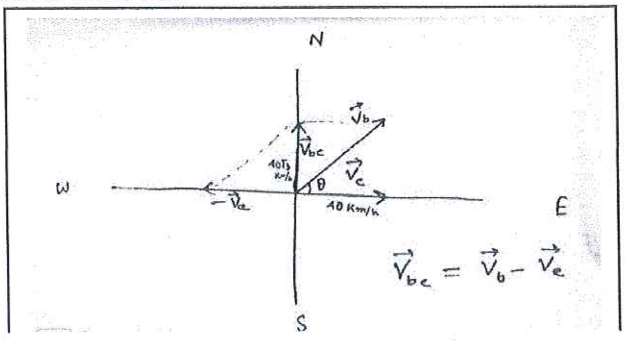
Ans: Let V_c = *velocity of the car*

V_b = *velocity of the bus*

V_{bc} = *realative velocity of bus w.r.t car*

And these velocity vectors are directed as shown in the figure. Then, $tan\theta = \frac{40\sqrt{3}}{40}$ or $\theta = 60^0$

And $V_c = \sqrt{40^2 + (40\sqrt{3})^2} = 80\ km/h$



Hence the actual velocity of the car is 80km/h along 60^0 north of east.

(7) What do you mean by limiting force of friction and angle of repose? 1+1

Ans: Refer to any standard text book.

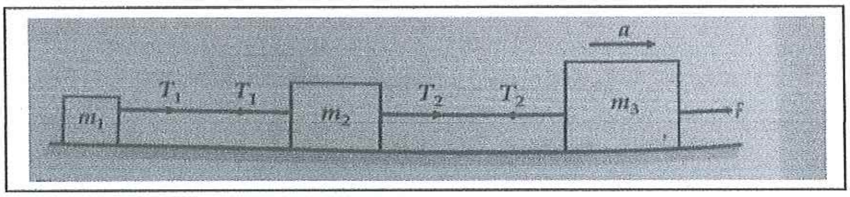
OR

A bullet of mass 100g is fired at a certain height along the horizontal direction with an initial speed of $50\sqrt{3}\ m/sec$. It hits a target at a finite distance making an angle of 30^0 with the horizontal. Calculate the kinetic energy of the bullet at that point. (g = 10 m/s^2)

Ans: let the velocity there be 'v'. As the horizontal component remains same, so $v.Cos30° = 50\sqrt{3}$

So, $v = 100m/s$. Hence the kinetic energy at that point $= \frac{1}{2}.\frac{100}{1000}.100.100\,J = 500J$

(8) As shown below, three blocks connected together lie on a horizontal frictionless table and pulled to the right with a force 50N.If m_1 =5kg ,m_2 =10kg and m_3 =15kg,find the tensions T_1 and T_2. 1+1



Ans: The common acceleration $= a = \frac{50}{5+10+15} = \frac{5}{3} m/s^2$

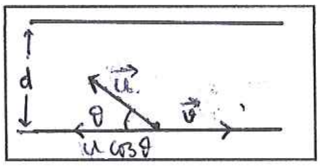
$T_1 = m_1 a = \frac{25}{3} N.\ Also,\ T_2 - T_1 = m_2 a$. So $T_2 = \frac{25}{3} + \frac{50}{3} = 25N$

OR

A man rows directly across a flowing river in time t_1 and rows an equal distance down the stream in t_2 .If u be the speed of man in still water and v that of stream then show that $t_1^2 : t_2^2 = u + v : u - v$

Ans: let the man project himself making θ angle with the bank of the river while directly crossing the river.

Then $u\ Cos\theta = v$ and $t_1 = \frac{d}{u\ Sin\theta}$ where d = width of the river. And $t_2 = \frac{d}{u+v}$



Now, $t_1 = \frac{d}{u\ Sin\theta} = \frac{d}{u\sqrt{1-Cos^2\theta}} = \frac{d}{u\sqrt{1-\frac{v^2}{u^2}}} = \frac{d}{\sqrt{u^2-v^2}} = \frac{d}{\sqrt{(u+v)(u-v)}}$

Hence $t_1^2 : t_2^2 = u + v : u - v$

(9) Prove that when the external applied force is zero, the angular momentum of a system remains conserved.

Ans: Refer to any standard text book.

GROUP-C

Answer the following questions. (Alternatives are to be noted): (3x9=27)

(10) The frequency of vibration of a stretched string 'n' depends upon the mass per unit length 'm', the length 'l' and the tension of the string 'T'. Establish the relation among them by dimensional analysis.

Ans: let $n = km^x l^y T^z$ where k is a dimensionless constant.

So, $T^{-1} = (ML^{-1})^x L^y (MLT^{-2})^z$

Solving, $x = -\frac{1}{2}, y = -1\ and\ z = \frac{1}{2}$. Hence $n = \frac{k}{l}\sqrt{\frac{T}{m}}$

(11) If two unit vectors ĝ and û are inclined at an angle ϕ then prove that $|ĝ - û| = 2sin\phi/2$.

Ans: If, ĝ and û are inclined at an angle ϕ, then angle between ĝ and (−û) will be $(180^0 - \emptyset)$.

So, $|ĝ - û| = |ĝ + (-û)| = \sqrt{|ĝ|^2 + |-û|^2 + 2.|ĝ|.|-û|.Cos(180^0 - \emptyset)} = \sqrt{1 + 1 + 2.1.1Cos(180^0 - \emptyset)}$

$= \sqrt{2 - 2\ Cos\emptyset} = \sqrt{2(1 - Cos\emptyset)} = 2\sin\left(\frac{\phi}{2}\right)$.

(12) For a particular initial velocity, determine the angle of projection for which the range will be maximum.

Ans: Refer to any standard text book.

(13) In an observation, the measurements of two resistances obtained are $r_1 = 2 \pm 0.02\ \Omega$ and $r_2 = 4 \pm 0.04\ \Omega$. Calculate the equivalent resistance (with error estimation) of the parallel combination of these two.

Ans: $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$

Differentiating, $\frac{\nabla r}{r^2} = \frac{\nabla r_1}{r_1^2} + \frac{\nabla r_2}{r_2^2}$. So, $\nabla r = r^2\left(\frac{\nabla r_1}{r_1^2} + \frac{\nabla r_2}{r_2^2}\right) = \left(\frac{8}{6}\right)^2\left(\frac{0.02}{4} + \frac{0.04}{16}\right) = 0.013\ \Omega$

Hence, equivalent resistance $r \pm \nabla r = (1.33 \pm 0.013)\ \Omega$

Or

The measurements of mass, linear velocity and radius of the circular orbit of a point mass are measured as 50 ± 0.01 g, 10 ± 0.02 cm/s and 30 ± 0.03 cm respectively. Calculate the centrifugal force acting on the particle with error estimation.

Ans: $f = \frac{mv^2}{r}$

So, $\frac{\nabla f}{f} = \frac{\nabla m}{m} + 2.\frac{\nabla v}{v} + \frac{\nabla r}{r} = \frac{0.01}{50} + \frac{0.04}{10} + \frac{0.03}{30} = 0.0052$

Then, $\nabla f = f \times 0.0052 = \frac{50\times100\times0.0052}{30} = 0.87$

Hence the centrifugal force $f = 166.67 \pm 0.87\ dyne.$

(14) What do you mean by impulse of a force? A certain amount of force acts on a moving object for certain time and stops that object. If the mass and initial velocity of the object is 100g and 5m/sec respectively, then calculate the impulse of force on the object. 1+2

Ans: Impulse = force x time duration = change in momentum = final momentum – initial momentum

$= 0 - \frac{100}{1000} \times 5\ N.m/s = -0.5\ N.m/s$

(15) What should be the minimum angle of an inclined plane for which a block of mass 500g will start sliding down? If it just starts sliding down at that angle, then what will be the acceleration with which it slides down? (Given $\mu_s = \frac{1}{\sqrt{3}}$ and $\mu_k = 0.5$) 1+2

Ans: When the body is just about to slide down, then it just balances the limiting frictional force which should be calculated considering μ_s. So that minimum angle is $\theta = \tan^{-1}\frac{1}{\sqrt{3}} = 30^0$.

Again, once it start sliding down, kinetic frictional force comes to play, and the free body analysis of the body gives, $mg\ sin30° - \mu_k\ mg\ cos30° = m.a$

Hence $a = \frac{g}{2} - 0.5 \times \frac{\sqrt{3}}{2}g = \frac{g}{2}\left(1 - \frac{\sqrt{3}}{2}\right) = 0.67\ m/s^2$ { if g= $10m/s^2$}

Or

A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t second then calculate the maximum velocity attained by the car.

Ans: Let the particle accelerates during t_1 *time and then decelerates during* t_2 time. If maximum velocity be v, then $v = 0 + \alpha t_1$ ....(1) and $0 = v - \beta t_2$ ....(2)

So total time $t = t_1 + t_2 = \frac{v}{\alpha} + \frac{v}{\beta} = \frac{v(\alpha+\beta)}{\alpha\beta}$

Or, $v = \frac{\alpha\beta}{\alpha+\beta}\ t$

(16) A ball thrown vertically upward with an initial velocity of 20m/s from the top of a building, hits the ground after 5sec. calculate the height of the building. (g = $10m/s^2$)

Ans: let the height of the building be h.

Now, for the upward motion from the top of the building, $0 = 20^2 - 2.10.s$, which gives $s = 20$ m

Time taken to travel that distance upward can be determined from, $0 = 20 - 10t$, which gives $t = 2sec$

So, while coming down, it has travelled $h + 20\ m$ path in (5-2) =3 sec. Hence we get,

$$h + 20 = (0 \times 3) + \left(\frac{1}{2} \times 10 \times 3^2\right)$$

solving, we get $h = 25m$

(17) A balloon with mass M is descending down with an acceleration 'a' where a < g, what mass 'm' of its contents must be removed so that it starts moving up with acceleration 'a'?

Ans: Let, F amount of force is applied on mass M because of the bouncy of the balloon in the upward direction. And this force remains same all the time.

Now for the downward motion , $Mg - F = Ma$ or, $F = M(g - a)$

Let, m be the mass for which it goes up. Then, $F - mg = ma$. Or , $m = \frac{F}{g+a}$

So, $m = \frac{M(g-a)}{g+a}$.

Hence the amount needed to remove = $\nabla M = M - m = M - \frac{M(g-a)}{g+a} = M\left(\frac{g+a-g+a}{g+a}\right) = \frac{2a}{g+a}M$

(18) Define angular velocity. Pulling a roller is easier than pushing it – why? 1+2

Ans: While pushing, a component of the applied force gets added up with the weight of the body, as a result the effective weight and the normal reaction increases, hence the frictional force also increases significantly.

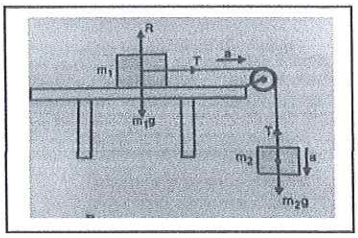
But while pulling it, a component of the applied force does the opposite work. It decreases the weight of the body and ultimately the frictional force also decreases. And it becomes easier to pull the roller.

Group-D

Answer the following questions. (Alternatives are to be noted): **(5x3=15)**

(19) Derive an expression for the maximum velocity a car can attain without skidding on a level road .

A body of mass m_1= 10kg is placed on a smooth horizontal table and a nail prevents its sliding towards right. It is connected to an inextensible mass less string which passes over a frictionless pulley and carries at the other end a body m_2 of mass 5kg .What will be the acceleration produced in the bodies and the tension in the string when the nail fixed on the table is removed? (g = 10m/s^2) 2+1+2



__Ans:__ The force causing the motion here is nothing but the weight of 5kg mass.

So, the common acceleration of the bodies $a = \frac{5\times10}{5+10} = \frac{50}{15} m/s^2$

Simply, $T = m_1 a = 10 \times \frac{50}{15}\ N = 33.33\ N$

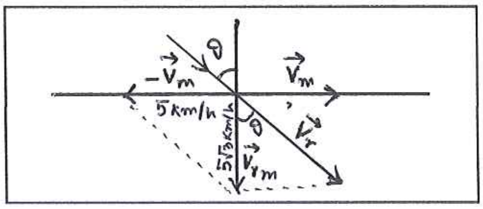
Or

Can a particle with nonzero acceleration have uniform speed – explain? To a person moving eastwards with a velocity 5km/h, rain appears to fall vertically downward with a speed 5√3km/h. Find the actual speed and direction of the rain. 1+2+2

__Ans:__ Yes, for uniform circular motion the body in motion will have constant speed but it will have acceleration due to continuous change in the direction of velocity vector.

Let, $V_m = velocity\ of\ man,\ V_r = velocity\ of\ rain\ and\ V_{rm} = relative\ velocity\ of\ rain\ w.r.t\ man.$

Then, from the diagram it is clear that, $tan\theta = \frac{5}{5\sqrt{3}}\ or, \theta = 30^0$



Also, $V_r = \sqrt{5^2 + (5\sqrt{3})^2} = 10km/h$

Hence rain drops are falling with a speed 10km/h making 30^0 angle with vertical direction as shown in the figure.

(20) What is the significance of slope of i) Displacement versus time curve and ii) velocity versus time curve? Draw the velocity versus time curve of a particle in 1D motion whose acceleration is infinite. The displacement of a body undergoing a force $\vec{F} = \hat{\imath} - 0.5\hat{\jmath} + 3\hat{k}$ is $\vec{S} = 2\hat{\imath} - 2\hat{\jmath} - \hat{k}$. What is the work done on the body by the force? What can you conclude on your result? 1+1+1+1+1

__Ans.__ i) slope indicates velocity ii) slope indicates acceleration.

It will be a straigt line parallel to velocity axis and not passing through origin.

Work done $W = \vec{F}.\vec{S} = 2 + 1 - 3 = 0$

As the work done is zero, but neither the force nor the displacement, hence the force is work less force and applied perpendicularly with the displacement.

Or

A body falling freely under gravity passes two points 30m apart in 1sec. Find the height of the point (w.r.t the upper point) from where it began to fall.

A train moves 1st half of its total path in 't_1' time with speed 'v_1' and next half in 't_2' time with speed 'v_2'. Find average speed.

Coordinates of two points P and Q are (3,4) and (7,2). Determine the vector $\overrightarrow{PQ}$. 3+1+1

__Ans:__ Let the body is dropped h height above the upper point.

Then velocity at upper point $v^2 = 0 + 2.10.h$ or $v = \sqrt{20h}$

For the next 30m, this velocity will be the initial velocity. Hence for this 30m path, $30 = vt + \frac{1}{2}gt^2$

Or, $\sqrt{20h}.1 + \frac{1}{2}.10.1^2 = 30$. Solving, $h = 31.25m$

Hence the body was released 31.25m above the upper point.

Average speed $= \frac{total\ distance}{total\ time} = \frac{v_1t_1+v_2t_2}{t_1+t_2}$

$\overrightarrow{PQ} = (7-3)\hat{\imath} + (2-4)\hat{\jmath} = 4\hat{\imath} - 2\hat{\jmath}$.

(21) Determine the angle vector $\vec{A} = \sqrt{6}\hat{\imath} + \sqrt{6}\hat{\jmath} + 2\hat{k}$ makes with X-Y plane.

Calculate the area of the triangle whose two adjacent sides are represented by two vectors $3\hat{\imath} + 4\hat{\jmath}$ and $7\hat{\jmath} - 3\hat{\imath}$.

For any three vectors $\vec{A}, \vec{B}$ and $\vec{C}$, prove that $\vec{A} \times (\vec{B} + \vec{C}) + \vec{B} \times (\vec{C} + \vec{A}) + \vec{C} \times (\vec{A} + \vec{B}) = \vec{0}$.

2+2+1

Ans: The unit vector normal to X-Y plane is $\hat{k}$. $\vec{A}.\hat{k} = 2 = |\vec{A}|.|\hat{k}|.cos\theta$

So, $cos\theta = \frac{2}{4}, \quad or \ \theta = 60^0$

Hence the vector makes $90^0 - 60^0 = 30^0$ angle with the X-Y plane.

Area of triangle $= \frac{1}{2}(3\hat{\imath} + 4\hat{\jmath}) \times (-3\hat{\imath} + 7\hat{\jmath}) = 33\hat{k}$. Hence the magnitude of the area is 33 sq unit.

L.H.S $= \vec{A} \times (\vec{B} + \vec{C}) + \vec{B} \times (\vec{C} + \vec{A}) + \vec{C} \times (\vec{A} + \vec{B})$

$= \vec{A} \times \vec{B} + \vec{A} \times \vec{C} + \vec{B} \times \vec{C} + \vec{B} \times \vec{A} + \vec{C} \times \vec{A} + \vec{C} \times \vec{B}$

$= \vec{A} \times \vec{B} - \vec{C} \times \vec{A} + \vec{B} \times \vec{C} - \vec{A} \times \vec{B} + \vec{C} \times \vec{A} - \vec{B} \times \vec{C}$

$= \vec{0}$