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ST. LAWRENCE HIGH SCHOOL

FOR GOD AND COUNTRY

PRE -ANNUAL EXAMINATION-2019

SOLUTION

MATHEMATICS

Class: XI

GROUP : A (MCQ)

1. (a) (i) or (iii)

(b) (ii)

(c) (ii)

(d) (ii)

(e) (iv)

(f) (ii)

(g) (ii)

(h) (iii)

(i) (ii)

(j) (i)

2.a) i) LHS = $\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin 16^0 + \frac{1}{\sqrt{2}}\cos 16^0\right]$

$= \sqrt{2}\cos 29^0 = \sqrt{2}\cos(30^0 - 1^0) = \sqrt{2}(\cos 30^0 \cos 1^0 + \sin 30^0 \sin 1^0)$

$= \sqrt{2}\left[\frac{\sqrt{3}}{2}\cos 1^0 + \frac{1}{2}\sin 1^0\right]$

$= \frac{1}{\sqrt{2}}(\sqrt{3}\cos 1^0 + \sin 1^0)$

ii) $\tan x + \tan y = 2 \Rightarrow \sin(x+y) = 2\cos x \cos y$

$\Rightarrow \sin(x+y) = 1 \Rightarrow x + y = \frac{\Pi}{2}$

$\therefore 2\cos x \cos y = 1 \Rightarrow \cos(x+y) + \cos(x-y) = 1$

$\Rightarrow \cos(x-y) = 1 = \cos 0^0 \Rightarrow x = y$

$\therefore x = y = \frac{\Pi}{4}$ (Ans)

iii) LHS = $(A \cup B) \cap (A \cup B')$

$= (A \cap A) \cup (B \cup B')$

$= A \cup \Phi = A$ = RHS

iv) $R = \{(x, y) / x + 2y = 12 \ \& \ x, y \in A\}$ $A = \{1,2,3,4,....,12\}$

$x = 2; \quad y = 5$

$x = 4; \quad y = 4$ $\therefore R = \{(2,5), (4,4), (6,3), (8,2), (10,1)\}$

$x = 6; \quad y = 3$ Range $R = \{1,2,3,4,5\}$

$x = 8; \quad y = 2$

$x = 10; \quad y = 1$

b) i) By the problem, $3 + ix^2 y = x^2 + y - 4i$

$\therefore \ x^2 + y = 3$

$x^2 y = -4 \Rightarrow y = -\frac{4}{x^2}$

$\therefore x^2 - \frac{4}{x^2} = 3 \Rightarrow x^4 - 3x^2 - 4 = 0$

$\Rightarrow x^4 - 4x^2 + x^2 - 4 = 0$

$\Rightarrow x^2(x^2 - 4) + 1(x^2 - 4) = 0$

$\Rightarrow (x^2 + 1)(x^2 - 4) = 0$

$\left. \begin{array}{l} x = \pm i; \ x = \pm 2 \\ y = \pm 4 \ \ y = -1 \end{array} \right\}$ (Ans)

ii) Number of 7-digit numbers formed with the given digits = $\frac{7!}{3!2!} = 420$

Remaining 6 digits can be arranged among themselves = $\frac{6!}{3!2!} = 60$

$\therefore$ Reqd. no. of numbers = $420 - 60 = 360$ (Ans)

iii) $\because t_5 = 24t_3 \Rightarrow {}^{11}C_4 x^4 = 24. \ {}^{11}C_2 x^2$

$\Rightarrow x^2 = \frac{24 \times {}^{11}C_2}{{}^{11}C_4} = \frac{24 \times 11!}{2! \ 9!} \cdot \frac{4! \ 7!}{11!}$

$= 24 \times \frac{3 \times 4}{8 \times 9} = 4$

$\therefore x = \pm 2$ (Ans)

iv) $t_p = x \Rightarrow a + (p-1)d = x$ $\quad t_p = x \Rightarrow a.r^{p-1} = x$

$t_q = y \Rightarrow a + (q-1)d = y$ $\quad t_q = y \Rightarrow a.r^{q-1} = y$

$t_r = z \Rightarrow a + (r-1)d = z$ $\quad t_r = z \Rightarrow a.r^{r-1} = z$

LHS = $x^{y-z} . y^{z-x} . z^{x-y}$

= $(a.r^{p-1})^{(q-r)d} . (a.r^{q-1})^{(r-q)d} . (a.r^{r-1})^{(p-q)d} = a^0 . r^0 = 1$

c) i) $y = x + 2 \ and \ y = (2 - \sqrt{3})x + 1$

$m_1 = 1;\ m_2 = 2-\sqrt{3}$

$\therefore \tan\theta = \dfrac{(2-\sqrt{3})-1}{1+(2-\sqrt{3})}$

$= -\dfrac{\sqrt{3}-1}{\sqrt{3}(\sqrt{3}-1)} = -\dfrac{1}{\sqrt{3}} = \tan 150^0$

$\therefore \theta = 150^0$

ii) Let, any point on x-axis be $(\alpha, 0, 0)$

By the problem, $\sqrt{(\alpha-2)^2+1+9} = \sqrt{(\alpha+3)^2+4+16}$

$\Rightarrow (\alpha-2)^2+10 = (\alpha+3)^2+20$

$\Rightarrow \alpha^2+4-4\alpha+10 = \alpha^2+6\alpha+9+20$

$\Rightarrow 10\alpha = -15 \Rightarrow \alpha = -\dfrac{3}{2}$

$\therefore$ Reqd. pt is $\left(-\dfrac{3}{2}, 0, 0\right)$ (Ans)

d) i) $\lim\limits_{x\to 0}\dfrac{(a^x-1)\ln(1+x)}{\sin^2 x} = \lim\limits_{x\to 0}\dfrac{\dfrac{a^x-1}{x}.\dfrac{\ln(1+x)}{x}}{\left(\dfrac{\sin x}{x}\right)^2} = \ln a$ (Ans)

ii) $y = \sqrt{3x} - \sqrt{\dfrac{3}{x}} + \dfrac{x+6}{6-x} \Rightarrow \dfrac{dy}{dx} = \sqrt{3}.\dfrac{1}{2}x^{-1/2} - \sqrt{3}.\left(-\dfrac{1}{2}\right)x^{-3/2} + \dfrac{(6-x)+(x+6)}{(6-x)^2}$

$= \dfrac{\sqrt{3}}{2\sqrt{x}} + \dfrac{\sqrt{3}}{2x^{3/2}} + \dfrac{12}{(6-x)^2} \qquad \therefore \left.\dfrac{dy}{dx}\right|_{x=3} = 2$ (Ans)

e) i) $\because P(A\cup B) \le 1 \Rightarrow P(A)+P(B)-P(A\cap B) \le 1 \Rightarrow \dfrac{3}{4}+\dfrac{5}{8}-1 \le P(A\cap B)$

$\Rightarrow \dfrac{3}{8} \le P(A\cap B)$

$\because B = (B\cap A)\cup(B\cap A')$

$\Rightarrow P(B) = P(A\cap B)+P(B\cap A') \Rightarrow P(A\cap B) = \dfrac{5}{8} - P(B\cap A')$

$\because P(B\cap A') \ge 0 \quad \therefore P(A\cap B) \le \dfrac{5}{8} \qquad \therefore \dfrac{3}{8} \le P(A\cap B) \le \dfrac{5}{8}$ (Ans)

ii) Let x denote even natural number.

$\sum x = 2+4+6+....+2n = n(n+1)$

$\sum x^2 = 2^2+4^2+6^2+.....+(2n)^2 = 4.\dfrac{n(n+1)(2n+1)}{6} = \dfrac{2}{3}n(n+1)(2n+1)$

$\because \dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2 = 65 \Rightarrow \dfrac{2}{3}(n+1)(2n+1) - (n+1)^2 = 65$

$\Rightarrow n^2 = 196\ (\because n > 0) \quad \therefore n = 14$ (Ans)

Group- C

3(a) Answer any TWO questions: (4 X 2 = 8)

(i) If A = {1 ,2 ,3 } , B = { 2 ,3 ,4 } , C ={ 1 , 3 , 4 } and D = { 2 , 4 , 5 }, show that

(A X B) ∩ (C X D) = (A ∩ C) X (B ∩ D)

By definition of Cartesian product of two sets we get

A X B = {(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3,), (3,4)} ---------------------(1)

And C X D = {(1,2), (1,4), (1,5), (3,2), (3,4), (3,5), (4,2,), (4,4), (4,5)} ----------------(2)

Now by definition of intersection of two sets from (1) and (2) we get

{(1,2) , (1,4) , (3,2), (3,4)}-------------------(3)

Again A ∩ C = {1,3}

And B ∩ D =) 2,4 }

Therefore (A ∩ C) X (B ∩ D) = {(1,2), (1,4), (3,2), (3,4)}--------------(4)

From (3) and (4) it readily follows that (A X B) ∩ (C X D) = (A ∩ C) X (B ∩ D)

(ii) In any triangle ABC, prove that $(b^2 - c^2)$ Cot A + $(c^2 - a^2)$ Cot B + $(a^2 - b^2)$ Cot C = 0

Because a/SinA = b/SinB = c/Sinc = 2R

We have a = 2rSinA , b = 2RSInB , c = 2RSinC

$(b^2 - c^2)$ Cot A = $2R^2$(1 – Cos2B – 1 + Cos2C)CotA

= $4R^2$ SInA Sin (B – C)CosA/ SinA

= - $2R^2$ (Sin 2B – Sin 2C)

Similarly $(c^2 - a^2)$ Cot B = - $2R^2$ (Sin 2CB – Sin 2A)

And $(a^2 - b^2)$ Cot C = - $2R^2$ (Sin 2A – Sin 2B)

L.H.S = 0 Proved

(iii) Solve: 4 Sin^4x + Cos^4x = 1

4 Sin^4x + Cos^4x = Sin^4x + Cos^4x + 2Sin^2x Cos^2x

Or Sin^2x(3 Sin^2x - 2 Cos^2x) = 0

Or, either Sin^2x = 0 , Sin x= 0 or x = nπ

Or 3 Sin^2x - 2 Cos^2x = 0

Or3(1- cos 2x) - 2(1+ cos 2x) = 0

Or, Cos2x = 1/5 = Cosα (say)

Or x = nπ $\pm\frac{1}{2}\alpha$ where n is an integer and α = Cos^{-1}1/5

(b) Answer any TWO questions: (4 X 2 = 8)

(i) The sum of three numbers in A.P is 18 ; if 2 ,4 and 11 be added to them respectively, the resulting numbers are in G.P. Determine the numbers.

Let the reqd numbers be a-b , a and a + b

So by the problem we have a = 6

Again the numbers a-b+2 , a+4 , and a+b+11 are in G.P

So, $\frac{a+4}{a-b+2} = \frac{a+b+11}{a+4}$

By putting the value of a=6 and solving we get b = -12 or 3

Hence the reqd numbers are either 18,6 and -6 or 3 ,6 and 9

(ii) By mathematical induction prove that $(2^{2n} - 1)$ is divisible by 3 where n ≥ 1 is an integer.

Let P(n) be the statement $(2^{2n} - 1)$ is divisible by 3

Or P(n) : $(2^{2n} - 1)$ is divisible by 3

By putting n = 1 we get P = 3 which is divisible by 3. So P(1) is true

Now let us assume P(m) is true

Now $2^{2(m+1)}$ -1 = 2^{2m+2} – 1 = 4. 2^{2m} – 1 = 3. 2^{2m}+(2^{2m} – 1) ------------(1)

From(1) it readily follows that $2^{2(m+1)}$ -1 is divisible by 3

Therefore P(m+1) is true when P(m) is true.

Since P(1) is true and P(m+1) is true whenever P(m) is true, therefore by mathematical induction it follows that P(n) is true when n≥ 1, is an integer.

(iii) If a , b are real and $a^2 + b^2 = 1$, then show that the equation $\frac{1-ix}{1+ix}$ = a - ib is satisfied by a real value of x.

We have 1 - ix = (a - ib)(1+ ix)

$x = \frac{(1-a+ib)\{b-i(a+1)\}}{\{b+i(a+1)\}\{b-i(a+1)\}}$

or, $x = \frac{2b}{b^2+(a+1)^2}$

Therefore if a , b are real and $a^2 + b^2 =1$, then the equation is satisfied by a real value of x

(iv) Find the rank of the word LATE when its letters are arranged as in a dictionary.

Ans : 34. Process – refer to ex- 26

(v) If the third, fourth and fifth terms in the expansion of $(x + a)^n$ be 84, 280 and 560 respectively, find x , a and n.

By the condition of the problem we have

3rd term= $t_3 = {}^nC_2\, x^{n-2} a^2 = 84$-----------(1)

Similarly 4th term = $t_4 = 280$----------(2) and 5th term = $t_5 = 560$-----------(3)

Dividing (2) by (1) we have $\frac{n-2}{3} \cdot \frac{a}{x} = \frac{10}{3}$ -------------(4)

Dividing (3) by (2) we get $\frac{n-3}{4} \cdot \frac{a}{x} = 2$ -----------(5)

Now by dividing (4) by (5) we get n = 7

Putting the value of n in (4) we get

a = 2x -----------------(6)

Finally putting the value of n and a in (1) we get x = 1 and from (6) we get a = 2

(c) Answer any TWO questions: (4 X 2 = 8)

(i) Find the equations of the two straight lines through the point (0,2), which are at a distance of 2 units from the point (4,4).

Equation of any straight line through point (0,2) is y – mx – 2 = 0 ----------(1)

The perpendicular distance of line (1) form the point (4,4) is 2 unit

So, $\frac{4 - m.4 - 2}{\sqrt{1+m^2}}$, solving we get m = 0 or 4/3

Therefore the equation of the two required straight lines are

y - 2 = 0 and 3y - 4x = 6

(ii) Find the equation to the locus of mid points of chords drawn through the point (a,0) on the circle $x^2 + y^2 = a^2$.

Let $\overline{AB}$ be any chord of the given circle drawn through the point A(a,0) on it. If (α , β) be two co ordinates of B and (h, k) the co ordinates of the mid point of the chord AB n then we get $h = \frac{\overline{\alpha}+a}{2}$ and $k = \frac{\beta+0}{2}$

Now the point B (, β) lies on the given circle then we have $\alpha^2 + \beta^2 = a^2$

Or $h^2 + k^2 = ha$. Hence the reqd. equation to the locus of (h,k) is $x^2 + y^2 = ax$

(iii) Find the equation of the parabola whose focus is (3,4) and whose directrix is 3x+4y+25=0. Also find the length of the latus rectum of the parabola.

Let P(x,y) be any pont on the reqd. parabola. Now distance of P form the focus S(3,4) is SP = $\sqrt{(x-3)^2 + (y-4)^2}$

Let $\overline{PM}$ be the perpendicular distance of P form the directrix 3x + 4y +25 = 0 , then

$\overline{PM} = \pm \frac{3x+4y+25}{5}$

Since P lies on the reqd parabola hence $SP^2 = PM^2$

Simplifying $16x^2 + 9y^2 - 24xy - 300x - 400y = 0$ which is the reqd. equation of the parabola. Length of the perpendicular distance from the focus S (3,4) upon the directrix $3x + 4y + 25 = 0$ is 10 units by putting x = 3 and y =4

Hence reqd length of latus rectum is 20 units

(d) Answer any ONE question: (4 X 1 = 4)

(i) If $y = \frac{x-2}{x+2}$, show that $2x.\frac{dy}{dx} = 1- y^2$

Differentiating bth sides w.r.t x we get

$dy/dx = \frac{d}{dx}(\frac{x-2}{x+2})$

or, $\frac{4}{(x+2)^2}$

or $2x\ dy/dx = \frac{8x}{(x+2)^2} = \frac{(x+2)^2 - (x-2)^2}{(x+2)^2}$

or, $1- y^2$ (proved)

(ii)Evaluate: $\lim_{x\to 0}\frac{(e^x-1)\log(1+x)}{\sin^2 x}$

$$\lim_{x\to 0}\frac{\frac{e^x-1}{x}.\log(1+x)/x}{Sin^2x/x^2} = \frac{\lim_{x\to 0}\left(\frac{e^x-1}{x}\right).\lim_{x\to 0}\log(1+x)/x}{\lim_{x\to 0}\frac{Sin\,x}{x}.\lim_{x\to 0}\frac{Sin\,x}{x}} = \frac{1.1}{1.1} = 1$$

(e)) Answer any ONE question: (4 X 1 = 4)

(i)(a) Find the negation of each of the following statements:

p: The integer 4 is greater than the integer 6

q: All rational numbers are integers

Let p denote the given statements i.e p : the integer 4 is greater than the integer 6. Then the negation of the statement p is

~ p: The integer 4 is not greater than the integer 6.

~ p : It is false that the integer 4 is greater than the integer 6

~ p : It is not true that the integer 4 is greater than the integer 6.

Let q be the given proposition i.e r: All rational numbers are integers. Then the negation of the proposition q is

~ q : It is not the case that all the rational numbers are integers

~ *q*: It is false that all rational numbers are integers

~ q: There is at least one rational number which is not an integer.

(b) Examine whether the following sentence is a statement: x^2 -11 |x| +24 = 0 has four real roots.

x^2 -11 x +24 = 0 when x>0

(x-3)(x-8) = 0 ⇒ x = 3 or x=8

Again x^2 +11 x +24 = 0 when x<0

(x+3)(x+8) = 0 ⇒ x = - 3 or x = - 8

Clearly the equation has four real roots and hence the given sentence is always true.

(ii) Using contrapositive method show that the following compound statement is true

If x is an integer and x^2 is odd, then x is also odd.

Clearly the given " if then implication " is composed of simple statements p and q where

p : x is an integer and x^2 is odd

q : x is an odd integer

Then the given conditional compound statement is " If p then q " (i.e p = q)

To show the validity of " if p then q " by contrapositive method we assume q is false(i.e –q is true)

⇒ x is not an integer
⇒ x is an even integer ⇒ x = 2n where n ∈ Z
$x^2 = 4n^2$ ⇒ x^2 is an even integer
⇒ p is not true i.e – p is true
Thus q is false ⇒ p is false i .e – q ⇒ - p
Therefore the given " if p then q implication : is true.

(f) Answer any ONE question: (4 X 1 = 4)
(i) A bag contains 7 red and 5 white balls. 4 balls are drawn at random. What is the probability that two of them would be red and two white?

There are 7+5 = 12 balls in the bag and 4 balls from these 12 balls can be drawn in $^{12}C_4$ ways. Therefore the number of equally likely event points in the sample space of the experiment = $^{12}C_4$
Let A denote the event that all the 4 drawn balls are red. Clearly the number of equally likely event points contained in A = number of selections of 4 red balls from the 7 red balls = $^{7}C_4$
Therefore by classical definition of probability we get
$P(A) = \frac{7.6.5.4}{12.11.10.9} = \frac{7}{99}$

Let B be the event that two of the drawn balls are red and two are white.
Clearly 2 red balls can be selected from 7 red balls in $^{7}C_2$ ways and for each such selection 2 white balls can be selected from 5 white balls in $^{5}C_2$ ways. Therefore 2 red and 2 white balls can be drawn in $^{7}C_2 \times {}^{5}C_2$ ways.
Therefore by classical definition of probability we get
$(^{7}C_2 \times {}^{5}C_2) / {}^{12}C_4 = 14/33$

(ii) The mean of 5 observations is 4.4 and the variance is 8.24. If three of the five observations are 1 ,2 and 6, find the other two.

Let the other two observations be a and b. Since the mean of 5 observations 1 ,2 , 6 , a , and b is 4.4, hence we get $\frac{1+2+6+a+b}{5}$ = 4.4 or a+b = 13 -----------(1)
Again the variance of the 5 observations is 8.24. Hence
$\frac{\Sigma x^2}{5} - (\frac{\Sigma x}{5})^2 = 8.24$
Or $\frac{1^2+2^2+6^2+a^2+b^2}{5} - (4.4)^2 = 8.24$
Or $a^2 + b^2 = 97$
Putting the value of b from eq (1) and solving we get
A = 4 or a = 9
When a = 4 we get b = 9 and when a = 9 we get b = 4
Therefore the other two observations are 4 and 9.

Group- D

4(a) Answer any ONE question: (5 X 1 = 5)
(i) If tanx - tany = a and Coty - Cotx = b, prove that $\frac{1}{a} + \frac{1}{b}$ = Cot(x - y)
We have $a = \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y}$
Or $a = \frac{\sin(x-y)}{\cos x \cos y}$ or $1/a = \frac{\cos x \cos y}{\sin(x-y)}$ ---------------(1)

Again $b = \frac{\sin x \cos y - \cos x \sin y}{\sin y \sin x} = \frac{\sin(x-y)}{\sin y \sin x}$ or $1/b = \frac{\sin y \sin x}{\sin(x-y)}$ -------------(2)
Now (1) + (2) gives 1/a + 1/b = cot(x-y) proved

(ii) If a Sin α = b Sin β , then show that b Cotα + a Cot β = ($a + b$)Cot $\frac{\alpha+\beta}{2}$
$\frac{a}{\sin\beta} = \frac{b}{\sin\alpha}$ = k (say)
So a = k Sinβ and b = k sin α

L.H.S = k($\cos\alpha + \cos\beta$)= k . 2 . $\cos\frac{\alpha+\beta}{2} . \cos\frac{\alpha-\beta}{2}$

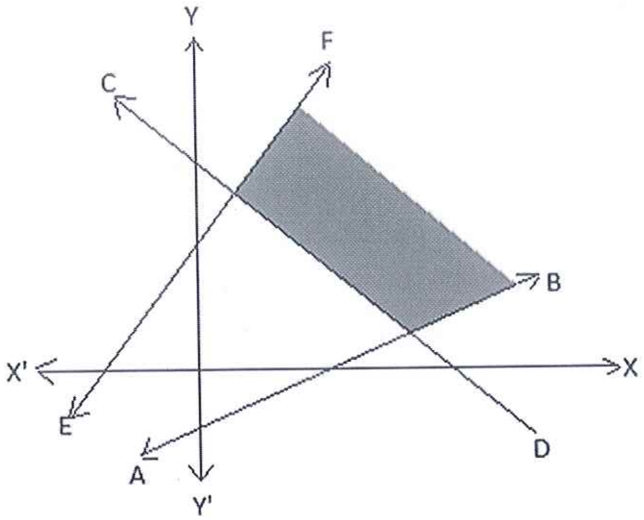
= k(2 $\sin\frac{\alpha+\beta}{2} . \cos\frac{\alpha-\beta}{2}$) $\frac{\cos\frac{\alpha+\beta}{2}}{\sin\frac{\alpha+\beta}{2}}$

(a+b) $\cot\frac{\alpha+\beta}{2}$

(b) Answer any TWO questions: (5 X 2 = 10)

(i) Exhibit graphically the solution set of the following system of linear inequations:

$x - 2y \leq 2$, $x + y \geq 3$, $-2x + y \leq 4$, $x \geq 0$, $y \geq 0$



We have $x - 2y \leq 2$ -------------(1) , $x + y \geq 3$ ---------------(2), $-2x + y \leq 4$ -----------(3)

The linear equation corresponding to linear inequations (1), (2) and (3) are

$x - 2y = 2$ ---------(4) , $x + y = 3$ ----------(5) , $-2x + y = 4$ ----------(6) resp. Each of the equations (4) , (5) and (6) represents a straight line and divides the xy plane in two half spaces. From the graphs the straight lines are

$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{EF}$ resp. We see that the origin does not lie on line (4) and it satisfies the inequation (1). Therefore the solution region of the inequation (1) consists of line AB and the half space containing the origin. Again the origin does not lie on the line (5) and it does not satisfy the inequation (2). Therefore the solution region of the inequation (2) consists of line CD and the half space containing the origin. Further the origin does not lie on the line (6) and it satisfies the inequation (3). Therefore the solution region of the inequation (3) consists of line EF and the half space containing the origin.

$x \geq 0$ represents the y axis and the half space on its right while $y \geq 0$ represents the x axis and the half space above the x axis.

The intersection of all the half spaces stated above is the shaded region and represents the solution region of the given system of inequations.

(ii) Find the number of combinations and number of permutations in the letters of the word ACCOUNTANCY taken four at a time.

Clearly there are 11 letters of 7 different kinds in the word ACCOUNTANCY

C , C , C ; A , A ; N , N ; O ; U ; T ; Y

Now 4 letters can be selected out of the above 11 letters in the following different ways.

(a) three like letters , one different

(b) two pairs of like letters.

(c) one pair of like letters and other two different

(d) four different letters.

In case (a) three like letters can be selected in 1 way and one different letter can be selected from 6 different letters A , N , O , U , T and Y in ${}^{6}C_{1}$ ways. Therefore number of selections in case (a) is 6.

In case (b) two pairs of like letters can be selected from the three pairs C , C ; A , A ; N , N in ${}^3C_2$ ways. Therefore number of selections in case (b) is 3

In case (c) one pair of like letters can be selected from the three pairs of like letters in ${}^3C_1$ ways and for each of such selections two different letters can be selected form the remaining 6 different letters in ${}^6C_2$ ways. Therefore number of selections in case (c) is ${}^3C_1 X\ {}^6C_2 = 45$

In case (d) 4 different letters can be selected from 7 different letters C , A , N , T , U , O , Y in ${}^7C_4$ ways. Therefore the number of selections in case (d) is 35.

Therefore the required total number of combinations= 6+3+45+35= 89

In finding the permutations of the letters in the word ACCOUNTANCY taken 4 at a time we are to arrange in all possible ways each of the above selections. Clearly (a) gives rise to 6 X $\frac{4!}{3!}$ = 24 permutations. (b) gives rise to 3 X $\frac{4!}{2!X\ 2!}$ = 18 permutations. (c) gives rise to 45 X $\frac{4!}{2!}$ = 540 permutations. (d) gives rise to 35 X 4! = 840 permutations. Hence the total number of permutations = 24+18+540+840= 1422

(iii) Show that the roots of the equation $9x^2 - 24x + 25 = 0$ are complex numbers; solving the equation show that the complex roots are conjugate complex numbers.

$9x^2 - 24x + 25 = 0$ --------(1). Discriminant of eq (1) $D = (-24)^2 - 4x9 = -324 < 0$.

Since D< 0 hence the roots are complex numbers.

Again $9x^2 - 24x + 25 = 0$

$(3x - 4)^2 - (3i)^2 = 0$

$3x - 4 - 3i = 0 \Rightarrow x = 4/3 + i$

$3x - 4 + 3i = 0 \Rightarrow x = 4/3 - i$

Which clearly states that the roots are conjugate complex numbers.

(iv) If Z = x + iy and $\left|\frac{z-3}{z+3}\right| = 2$, find the position of the point Z in the Argand diagram.

$|z - 3| = 2|z + 3|$

$|x + iy - 3| = 2|x + iy + 3|$

$(x - 3)^2 + y^2 = 4[(x + 3)^2 + y^2]$

Or $(x + 5)^2 + y^2 = 16$

Therefore the point z in the Argand diagram lies on a circle, the centre of the circle is at (-5,0) and radius is 4 units.

(c)) Answer any ONE question: (5 X 1 = 5)

(i) The co ordinates of the two ends of latus rectum of a parabola are (3,4) and (3,0). Find the equation of the parabola .

As the x co ordinates of the two ends of latus rectum are equal, latus rectum is parallel to y axis. Again latus rectum and axis of parabola are perpendicular to each other. Hence axis of parabola is parallel to x axis. Eq. of the reqd parabola is $(y - \beta)^2 = 4(x - \alpha)$--------(1)

The co ordinates of the end of the latus rectum of the parabola are(α + a , β + 2a) and (α + a , β - 2a).

We have $\alpha + a = 3$ ---------(2)

$\beta + 2a = 4$ ---------(3)

$\beta - 2a = 0$ -------- (5)

Solving eq (3) and (4) we get $\beta = 2$ and a = 1.

Putting a = 1 in (2) we get $\alpha = 2$

Hence eq of the parabola is $(y - 2)^2 = 4.1(x-2)$

Or $y^2 - 4(x + y) + 12 = 0$

(ii) (a) (5 , -4) and (-3 , 2) are two foci of an ellipse whose eccentricity is $\frac{2}{3}$. Find the length of the minor axis.

Let 2a and 2b be the lengths of major and minor axes resp and e be the eccentricity of the ellipse. Then by eq we have e = 2/3 and 2ae = distance between foci = $\sqrt{64+36}$ = 10

Solving we get a = 15/2.

Now $b^2 = a^2(1 - e^2)$. We have ae = 5 and solving we get b $\frac{5\sqrt{5}}{2}$

Hence the reqd length of minor axis = 2b = $5\sqrt{5}$

(b) Show that the difference of the focal distances at any point on the hyperbola $9x^2 - 16y^2 = 144$, is equal to its transverse axis.

iF 2a and 2b are the lengths of transverse and conjugate axes resp of the hyperbola $x^2/16 - y^2/9 = 1$---------(1). Then we have a = 4 and b = 3

Hence the eccentricity e = 5/4

Foci of the hyperbola (1) are S(ae,0) and S'(- ae ,0) i.e S (5 , 0) and S' (-5 , 0)

Now taking any point P(a secØ , btanØ)

Then $\overline{SP}$ = distance between the points S(5,0) and P(4 secØ , 3tanØ)

$= \sqrt{5 - 4sec\emptyset^2 + (0 - 3\tan\emptyset)^2}$

Solving we get 5 secØ - 4.

Similarly $\overline{S'P}$ = 5 secØ + 4.

Hence $\overline{S'P}$ - $\overline{SP}$ = 5 secØ + 4. – (5 secØ - 4) = 2.4 = length of transverse axis of hyperbola.