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ST. LAWRENCE HIGH SCHOOL

Selection Test Exam – 2019

FOR GOD AND COUNTRY

Sub: physics **Class: XI** **F. M. : 70**

Duration: 3hrs 15 mins

Section – I
(MCQ)

Chose the correct option for the following questions. (Answer all) **1x14=14**

i. A force $(2\hat{\imath} + \hat{\jmath} - \hat{k})$ N acts on a body of mass 10kg. If the body is initially at rest, then the velocity(in m/s) of the body at the end of 20s will be – **Ans: a)** $2\sqrt{6}$

ii. A particle is projected making an angle of 45^0 with horizontal having kinetic energy K.The kinetic energy at the highest point will be – **Ans: b)** $\frac{K}{2}$

iii. Two blocks A and B of masses m_1 *and* m_2 are kept in contact on a smooth horizontal surface. F amount of horizontal force is applied on A such a way that both moves together. The force exerted on B by A will be –

Ans: b) $\frac{m_2 F}{m_1+m_2}$

iv. A mass 1kg is suspended by a thread. It is i) lifted with an acceleration of $4.9 m/s^2$, ii) lowered with an acceleration of $4.9 m/s^2$. The ratio of the tension in the thread for two cases will be – **Ans: a) 3:1**

v. A uniform metal chain is placed on a rough table such that one end of it hangs down over the edge of the table. When one-third of its length hangs over the edge, the chain starts sliding. What is the coefficient of static friction?

Ans: d) $\frac{1}{2}$

vi. A block of mass 1kg slides down a rough inclined plane of inclination 60^0 starting from its top. If co-efficient of kinetic friction is 0.5 and the length of the plane is 1m, then the work done against friction is – **Ans: b) 2.45 J**

vii. A stone of mass 16kg is attached to a string 144 m long and is whirled in a horizontal circle. The maximum tension the string can bear is 16 N. The maximum speed that can be given to the stone without breaking it will be –

Ans: a) 12 m/s

viii. The change in potential energy when a body of mass m is raised to a height nR from the earth's surface is(R is radius of earth) – **Ans: c)** $mgR\frac{n}{n+1}$

ix. Two solid spheres of same metal but of mass M and 8M falls simultaneously on a viscous medium and their terminal velocities are v and nv. Then value of n is – **Ans: c) 4**

x. Excess pressure of one soap bubble is four times the other soap bubble. Then the ratio of the volume of first bubble to the other is – **Ans: a)1:64**

xi. A sample of gas expands from volume V_1 to V_2. The amount of work done by the gas is greatest when the expansion is – **Ans: b) isobaric**

xii. The r.m.s velocity of molecules an ideal gas at -153 °C is v. At 207°C , the r.m.s velocity of the same gas molecules will be – **Ans: b)** $2v$

xiii. A particle of mass 10^{-2}kg is executing S.H.M with an amplitude of 0.5m and a time period of $\frac{\pi}{5}$ sec. The maximum value of force acting on the particle is – **Ans: d) 0.5 N**

xiv. A wave is represented by $y = A\,Sin(x - 2t)$ m. Velocity of propagation of wave is – **Ans: d) 2 m/s**

Section – II

Group – A

Answer the following question in one sentence (Alternatives are to be noted) **1 x 4 = 4**

1. **The length and breadth of a rectangular field are 1.05289 m and 13.20 m respectively. How many significant digits will be there in its area?**
   Ans: For multiplication, the number of significant digits in the answer is equal to the least number of significant digits the measurements are having. So, here it will be 4.
2. **The displacement(cm) - time (sec) relation is given as $x^2 = 4t$. Calculate the velocity at $t = 9$ sec.**
   Ans: $x^2 = 4t$
   Differentiating w.r.t 't' we get
   $$2x\frac{dx}{dt} = 4$$
   Or, $v = \frac{dx}{dt} = \frac{2}{x} = \frac{2}{\sqrt{4t}} = \frac{1}{\sqrt{t}}$
   Or, at $t = 9sec, the\ velocity\ will\ be\ v = \frac{1}{\sqrt{9}} = \frac{1}{3} cm/sec$

   Or

   A particle is displaced from a point (0,6) to $(-2, 6)$. What will be the displacement vector?
   Ans: The required vector will be $\vec{r} = (-2 - 0)\,\hat{\imath} + (6 - 6)\hat{\jmath} = -2\hat{\imath}$
3. **How will the time period of oscillation of a simple pendulum be changed if it is raised up vertically with some acceleration?**
   Ans: If it is raised up with acceleration $'a'$ then the time period becomes $T = 2\pi\sqrt{\frac{l}{g+a}}$. So the time period decreases.

   Or

   Why does the temperature of a large water droplet decrease when it breaks into many small droplets?
   Ans: The total energy of droplet = Surface energy + Thermal energy.
   When it breaks into many droplets, the total surface area and hence the surface energy increases. To balance that change in energy, the temperature decreases.
4. **Can the entropy of a system decrease? Give example.**
   Ans: Yes, the entropy of a particular system may decrease, e.g. conversion of water to ice. The randomness of the water molecules decrease, hence the entropy. But that decrease in entropy cause increase in entropy in the rest of the universe. So, entropy of a particular system may decrease, but the entropy of the universe will increase always.

Group – B

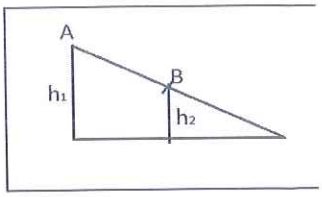
Answer the following question in short (Alternatives are to be noted) **2 x 5 = 10**

5. **Find the unit vector which is perpendicular to the plane contained by two vectors $2\hat{\imath} - 2\hat{\jmath}$ and $2\hat{\jmath} - 2\hat{k}$.**
   Ans: The cross product of the given two vectors will be the vector perpendicular to the plane contained by given two.
   Let that vector be $\vec{A}$. So $\vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -2 & 0 \\ 0 & 2 & -2 \end{vmatrix} = 4\hat{\imath} + 4\hat{k}$
   So, the required unit vector is $\hat{A} = \frac{4\hat{\imath}+4\hat{k}}{\sqrt{16+16}} = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{k})$

   Or

   The displacement (in m) – time (in sec) graph of a particle is a parabola with focus at (4,0) and vertex at (0,0). What is the velocity of the particle at time 4sec?
   Ans: The displacement of the particle will be $x^2 = 4.4.t$
   Or, $x^2 = 16t$
   So, the velocity will be $2x\frac{dx}{dt} = 16$. So the velocity will be $v = \frac{8}{x} = \frac{8}{4\sqrt{t}} = \frac{2}{\sqrt{t}}$
   So, the velocity at t = 4sec will be, 1m / sec.
6. **A man of mass 60kg jumps from a trolley of mass 20kg standing on a smooth surface with absolute velocity 3m/s. Find the velocity of the trolley and the total energy produced by man.** 1+1
   Ans: Let the velocity of trolley is v. Then from the conservtion of linear momentum, $60 \times 3 = 20 \times v$.
   So, $v = 9\ m/s$.
   The total energy produced by man = K.E of man + K.E of trolley = $\left(\frac{1}{2} \times 60 \times 9 + \frac{1}{2} \times 20 \times 81\right) J = 1080J$

Or



A block of mass is released at the top of a rough inclined surface (making 60^0 with horizontal) at point A. The block slides down by 8m and comes to rest at point B. Find out the difference of height $(h_1 - h_2)$. $[\mu_k = 0.5, g = 10m/s^2]$

Ans: From the conservation of energy,

the potential energy at A = potential energy at B + the work done against friction.

So, $mgh_1 = mgh_2 + \mu mg\ Cos60^0 \times 8$

So, $h_1 - h_2 = 0.5 \times 0.5 \times 8 = 2m$

7. **Two identical wires of Young's modulus Y_1 *and* Y_2 are joined back to back. Determine the new equivalent Young's modulus.**

Ans: let the length and area of cross-section of one such wire are L and A respectively.

If the elongation for 1st wire be l_1 then, $Y_1 = \frac{F/A}{l_1/L} = \frac{PL}{l_1}$ [let F/A = P]

Similarly $Y_2 = \frac{P}{l_2}$ [where l_2 *is the elongation of 2nd wire.*]

So, $l_1 + l_2 = PL\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)$

If they are connected back to back and given same force F, and the equivalent Young's modulus be Y then,

$$Y = \frac{P}{\frac{(l_1 + l_2)}{2L}} = \frac{2PL}{PL\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)} = \frac{2Y_1Y_2}{Y_1 + Y^2}$$

Or

A small iron ball has terminal velocity v in water. What will be the terminal velocity if the radius is doubled?

Ans: we kknow, $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$. So, $v \propto r^2$ [*when all other parameters are unchanged.*]

Therefore, $\frac{v_1}{v_2} = \frac{r_1^2}{r_2^2}$

If we take $v_1 = v,\ r_1 = r\ then\ r_2 = 2\,r$. Hence the corresponding terminal velocity becomes, $v_2 = \left(\frac{r_2}{r_1}\right)^2 . v_1 = 4v$

8. **Establish the relationship $c_p - c_v = R$**

Ans: Refer to any standard text book.

Or

Calculate the efficiency of a Carnot engine working between ice point and steam point.

Ans: $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{373} = 0.2681$. Hence the efficiency is 26.81%

9. **Two stretched strings of same material, of equal diameter and of length 1m and 2m vibrate with frequencies 100Hz and 150Hz respectively. Calculate the ratio of their tension.**

Ans: For 1st harmonic, the expression of frequency is, $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$

So, $T \propto f^2 l^2$

Or, $\frac{T_1}{T_2} = \frac{(f_1 l_1)^2}{(f_2 l_2)^2}$. Here, $f_1 = 100Hz, l_1 = 1m, f_2 = 150\ Hz\ and\ l_2 = 2m$

Hence $\frac{T_1}{T_2} = \left(\frac{100}{300}\right)^2 = \frac{1}{9}$

Or

The equation of motion of a particle executing SHM is $x = a\,Sin\left(\omega t + \frac{\pi}{6}\right)$. Find the minimum time at which the velocity becomes half of its maximum value.

Ans: The maximum velocity is $v_{max} = a\omega$.

According to the problem, $\frac{dx}{dt} = \frac{a\omega}{2}$

Or, $a\omega\ cos\left(\omega t + \frac{\pi}{6}\right) = \frac{a\omega}{2}$

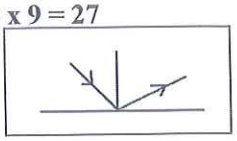
Or, $cos\left(\omega t + \frac{\pi}{6}\right) = \frac{1}{2}$

So, $\left(\omega t + \frac{\pi}{6}\right) = \frac{\pi}{3}$

Or, $t = \frac{\pi}{6\omega}$

Group – C

Answer the following question in short (Alternatives are to be noted) 3 x 9 = 27



10. A cricket ball hits the ground making θ angle with the vertical direction and then rebounds along a direction making φ angle with vertical as shown in the figure. If the coefficient of restitution is e, find the value of φ in terms of θ.

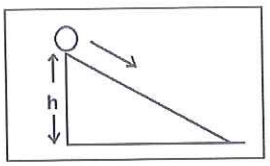
Ans: Consider the magnitude of velocity of the ball before collision is u the components of which are $uCos\theta$ and $u\,Sin\theta$.

If the magnitude of velocity of the ball becomes v after collision, then –

The vertical component $= vCos\varphi = e \times vertical\ component\ before\ collision = e.uCos\theta$.....(1)

And, the horizontal component remains same with value $vSin\varphi = u\,Sin\theta$.....(2)

Then, (2) ÷ (1) gives, $tan\varphi = \frac{tan\theta}{e}$ $or, \varphi = \tan^{-1}\left(\frac{tan\theta}{e}\right)$ (Ans)



11. A solid sphere of radius r and mass m is released on the top of a rough inclined plane of inclination θ. Calculate the velocity of its centre of mass when it just reaches the ground.

Ans: From the energy conservation –

$The\ P.E\ at\ the\ top = P.E\ at\ ground + K.E\ at\ the\ ground + Rotational\ energy\ at\ ground$

Let the velocity of the sphere when it just touches the ground be v, then the above equation becomes

$mgh = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$or, \frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mr^2.\left(\frac{v}{r}\right)^2 = mgh$

Or, $v = \sqrt{\frac{10gh}{7}}$ $[Ans]$

Or

What do you mean by angle of repose? A piece of stone comes to rest after being displaced by 20m on a rough surface. If the initial velocity of the stone was 1m/s, then determine the coefficient of kinetic friction of the stone-surface system. (g=10m/s^2) 1+2

Ans: Refer to book.

The initial K.E of the stone is getting balanced by the frictional force. Hence ,

$\frac{1}{2}mv^2 = \mu mg \times s$

Or, $\mu = \frac{1}{400}$ $[Ans]$

12. w_1 amount of work is done to stretch a spring by x amount. It is then cut into two equal pieces. If w_2 amount of work is done to stretch one piece by x amount then find $w_1 : w_2$.

Ans: Let the spring constant be k.

Then, $w_1 = \frac{1}{2}kx^2$

If it is cut into two equal pieces, spring constant of each becomes 2k.

So, $w_2 = \frac{1}{2}.2k.x^2$

So, $w_1 : w_2 = 1:2$

Or

What is the significance of the area under force versus displacement curve? Prove the work-energy theorem for a constant force.

Ans: It gives the measurement of work-done under that force.

Refer to text book.

13. State the Parallel axis theorem of moment of inertia. Calculate the moment of inertia of a square lamina (of side x and mass M) about an axis passing through a vertex and perpendicular to the plane of it. 1+2

Ans: Refer to text book.

For square lamina, $I_{cm} = \frac{M}{6}x^2$

Now, for the given axis, the perpendicular distance of that axis from the axis passing through centre of mass will be half of the diagonal of the square $= \frac{\sqrt{2}}{2}x$

So, $I = I_{cm} + Md^2 = \frac{M}{6}x^2 + M\frac{\sqrt{2}}{2}x.\frac{\sqrt{2}}{2}x = \frac{Mx^2}{6} + \frac{Mx^2}{2} = \frac{2}{3}Mx^2$ $[Ans]$

14. At what condition the angular momentum of a rotational system remains conserved? Establish the relation between torque and moment of inertia. 1+2

Ans: If there is no net external torque acting on the system, the angular momentum remains conserved.

Refer to text book.

Or

For a rigid body rotating about a fixed axis, establish the relation between the kinetic energy of the body and moment of inertia.

Ans: Refer to text book.

15. **Find out the expression of acceleration due to gravity at a depth d from the surface of earth.**

Ans: Refer to text book.

Or

Prove that a freely falling body will execute SHM in a tunnel through centre of earth about centre of earth.

Ans: The acceleration of the body at the centre of earth will be zero. So, momentarily the gravitational force on the body will be zero. But due to inertia, the body will cross the centre. As it crosses the centre, gravitational pull will be exerted on it along the opposite direction and the body will move back towards the centre. So this restoring force is always along the opposite direction of the displacement of the body.

The gravitational acceleration i.e. the acceleration of the freely falling body when the body is at a distance x from the centre of earth, is $g' = g\left(1 - \frac{R-x}{R}\right) = g\frac{x}{R}$

So, the restoring force $F = mg' = mg\frac{x}{R}$

Hence, $F \propto x$

From the above two discussion, we can conclude, $\vec{F} \propto -\vec{x}$. Hence the motion will be one SHM.

16. **Calculate the kinetic energy of a satellite which is moving with velocity v at a distance r from the centre of earth. Mass of the satellite is m and that of earth is M.**

Ans: Refer to text book.

Or

State and prove Kepler's third law. 1+2

Ans: Refer to text book.

17. **The temperature of 1 mole of a diatomic ideal gas, is increased from 35^{0}C to 135^{0}C adiabatically. Calculate the work-done. (R=8.31J /mol-K).**

Ans: For diatomic ideal gas at low temperature, $f = 5$ so, $\gamma = 1 + \frac{2}{5} = \frac{7}{5}$

So, $W_{ad} = \frac{R}{\gamma - 1}(T_2 - T_1) = \frac{8.31}{\frac{7}{5}-1} \times 100 = 2077.5\,J$

18. **Prove that $\gamma = 1 + \frac{2}{f}$. Symbols have usual meaning.**

Or

State the law of equipartition of energy. Give the kinetic interpretation of pressure and internal energy of a gas. 1+1+1

Ans: refer to book.

Group – D

Answer the following question in short (Alternatives are to be noted) **5 x 3 = 15**

19. **i) The position vectors of the points (2,2) and (-4,4) represent the velocity of a car and a train respectively. What will be the magnitude of relative velocity of car w.r.t train?**

ii) A ball of mass 100g is dropped from a height of 125m. If the coefficient of restitution is 0.4 then what will be the height attained by the ball after first rebounce? (g = 10m/s^2) 3+2

Ans: i) $\overrightarrow{v_c} = 2\hat{\imath} + 2\hat{\jmath}$ and $\overrightarrow{v_t} = -4\hat{\imath} + 4\hat{\jmath}$

Now, the relative velocity of car w.r.t train will be $\vec{v}_{ct} = \overrightarrow{v_c} - \overrightarrow{v_t} = (2\hat{\imath} + 2\hat{\jmath}) - (-4\hat{\imath} + 4\hat{\jmath}) = 6\hat{\imath} - 2\hat{\jmath}$

So, the magnitude of the relative velocity is $= \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10}\ unit$

ii) If, u be the velocity with which the ball just touches the ground, then –

$u^2 = 0 + 2.10.125$

Or, $u = 50\ m/s$

After first rebounce the velocity = e u = 0.4 x 50 m/s = 20 m/s

So, the ball starts moving up with initial velocity 20 m/s.

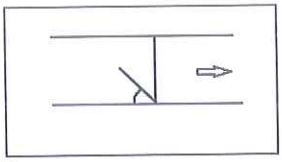
If it reaches a height h then, $0 = 20^2 - 2.10.h$

Or, $h = 20\ m$

Or

i)At what direction (w.r.t bank of the river) a man should project himself, in order to cross the river in shortest path? What will be the time taken by him to cover the river along that path?(velocity of man at still water is 8km/h, velocity of river is 4km/h and the width of the river is $8\sqrt{3}\ km$) 1+2

ii)A boy throws his toy through the window horizontally with a velocity 2m/s. The toy touches the ground at a perpendicular distance of 10m from the bottom of the building. Calculate the height of the window w.r.t the bottom of the building. (g = 10m/s^2). 2



Ans: i) Let he should project himself along θ direction.

Then, $8\ cos\theta = 4$ or, $\theta = 60^0$

Shortest path will be the width of the river. So, time taken to cross the river $= \frac{8\sqrt{3}}{8\ sin 60^0} = 2\ hrs$

ii) The time of flight here $= \frac{10}{2}\ sec = 5\ sec$ [as the horizontal component of velocity remains unchanged]

so, during 5 sec the ball was also having vertically downward motion.

Then, height of the window $= ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}.10.25\ m = 125\ m$

20. Define surface tension. What do you mean by angle of contact? Find out the expression of capillary rise of liquid in a capillary tube. **1+1+3**

Or

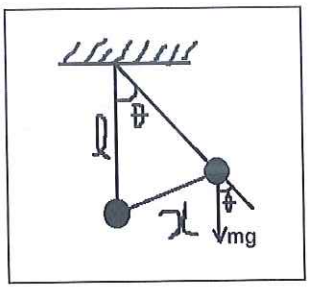
Define poisson's ratio. Calculate the elastic potential energy stored per unit volume of a substance. Write down Stoke's law. **1+1+3**

Ans: Refer to text book.

21. Can a periodic motion be non-oscillatory? Give example. Find the expression for the time period of oscillation of a simple pendulum. **1+1+3**

Ans: Yes. Every oscillatory motion is periodic but not all periodic motions are oscillatory. E.g. revolution of earth about sun.

Let the bob of the pendulum is displaced by a very little amount x from the mean position.



Then the retarding force F = a component of the weight = $mg\ sin\theta$

Now for small value of x, θ will also be very small.

Then, $sin\theta = \theta = \frac{x}{l}$

So, $F = mg\ sin\theta = mg\frac{x}{l} = kx(say)$

So, $\frac{k}{m} = \frac{g}{l}$ or, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{l}}$

So the time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{l}}} = 2\pi\sqrt{\frac{l}{g}}$

Or

What do you mean by stationary wave? Calculate the frequency of the 3rd harmonic for stationary wave in both end open pipe. A plane progressive wave equation is represented by $y = a\ Sin(100t + 2x)$. Find out the condition for which the maximum particle velocity becomes half of the wave velocity. **1+2+2**

Ans: The given wave equation is $y = a\ Sin(100t + 2x) = a\ sin2(50t + x)$

So, the wave velocity = 50 m/s

The maximum particle velocity = $50a$ m/s

So the required condition is $50a = \frac{50}{2}$

Or, $a = \frac{1}{2}$