ST. LAWRENCE HIGH SCHOOL

PRE-TEST
MODEL ANSWERS

Sub: Mathematics **Class: XII** **F. M. 80**
Duration: 3 Hrs 15mins. **Date:07.08.2019**

[Relevant rough work must be done in the margin of the page containing the answers]

GROUP-A

1.a) Answer any one question: **1x2=2**

i) Let A={1,2,3}.Find a relation on A which is symmetric, transitive but not reflexive

Sol: Define a relation R={(2,3),(3,2),(2,2)}.(1,1) does not belong to R,so not reflexive

ii) Find x,if $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

Sol:$\sin^{-1}x+\cos^{-1}x=90^0$.Add the equations,x=$\sqrt{3}/2$

b) Answer any one question : **1x2=2**

i) If A=$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$,find the value of A^2-4A+3I,I is the identity matrix

Sol:$A^2=\begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$,$A^2$-4A+3I=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ii) Solve for x $\begin{vmatrix} 2-x & 2 & 3 \\ 2 & 5-x & 6 \\ 3 & 4 & 10-x \end{vmatrix} = 0$

Sol:Applying C_3-3C_1 and R_1+3R_3,x=1,8$\pm\sqrt{37}$

c) Answer any three questions : **3x2=6**

i) Examine the continuity of $f(x) = \frac{|\sin x|}{x}$ when x≠ 0

=0 , x=0 show that f(x) is not continuous at

at x=0

Sol:$\lim_{x\to 0+}\frac{\sin x}{x} = 1, \lim_{x\to 0-}\frac{-\sin x}{x} = -1$.So f(x) is not continuous at x=0

ii) Integrate : $\int e^{x^3} x^5 dx$

Sol:$\int e^{x^3} x^3\, d(x^3)\frac{1}{3}$=$\int \frac{1}{3} e^z z dz$.Integrate by parts,Ans:$\frac{(x^3-1)e^{x^3}}{3} + c$

iii) Verify Lagrange's MVT for the function $f(x) = x^2 + 4x + 1$ in [2,3]

Sol:$\frac{f(3)-f(2)}{3-2}$ =2c+4.c=5/2.Lagrange's MVT holds

iv) Evaluate : $\lim_{x\to 0}\frac{\sin|x|}{x}$

Sol:sin(-x)=-sinx.Does not exist.See c i)

v) Evaluate : $\int \frac{1}{e^x+1}\, dx$

Sol:$\int \frac{e^{-x}}{1+e^{-x}} dx$ =-log(1+e^{-x})+c

vi) If $x^m y^n = (x+y)^{m+n}$ prove that $\frac{dy}{dx} = \frac{y}{x}$ Type equation here.

Sol:mlogx+nlogy=(m+n)log(x+y).Differentiate w.r.t. x

d) Answer any two question : **2x2=4**

i) Find the unit vector perpendicular to the vector 2i-j+k

Sol:$\frac{2i-j+k}{\sqrt{4+1+1}}$

ii) Show that the vectors 3i+2j+4k,6i+4j+8k are collinear

Sol:6i+4j+8k=2(3i+2j+4k)

iii) Show that the vectors 3i,4j and 5k form the sides of a right triangle

Sol:$3^2+4^2=5^2$

GROUP : B

2.a) Answer any one question : **1x4=4**

i) If $f(x) = ax^3 + b$. Show that f(x) is a bijective mapping

Sol:f(x)=f(y) implies x=y,so f is injectiveIf f(x)=y then x=$[(y-b)/a]^{1/3}$,so f is onto

ii) Solve: $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{4}$

Sol:$\tan^{-1}\frac{1+x+1-x}{1-(1-x^2)} = \pi/4. x^2=2$

b) Answer the following questions : **4x2=8**

i) Evaluate : $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

Sol:Page 156 of textbook Example 1vii)

OR

Without expanding, prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-c)(c-a)(a-b)$

Sol:Page 167 Example 27

ii) Solve the following set of equations by Cramer's rule:x+y+z=3,2x+3y-z=4,-x-y=-2

Sol:$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 4 & 3 & -1 \\ -2 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ -1 & -1 & 0 \end{vmatrix}} = 1$.Similarly y=1,z=1

OR

If $A = \begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}$,find A^{-1} by elementary row or column operations

Sol:Ans:$\frac{1}{22}\begin{pmatrix} 5 & -3 \\ 4 & 2 \end{pmatrix}$ Similar problem page210 of textbook

c) Answer the following questions : **4x3=12**

i) If $x = acos^3\theta, y = bsin^3\theta$; find$\frac{d^2y}{dx^2}$

Sol:$\frac{dx}{d\theta} = 3acos^2\theta(-sin\theta), \frac{dy}{d\theta} = 3bsin^2\theta cos\theta, \frac{dy}{dx} = \frac{-atan\theta}{b}, \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{-atan\theta}{b}\right)\frac{d\theta}{dx} = \frac{-asec^2\theta}{b} \frac{1}{-3acos^2\theta sin\theta}$

OR

If y^2=4ax; show that $\frac{d^2y}{dx^2} \cdot \frac{d^2x}{dy^2} = \frac{-2a}{y^3}$

Sol:$\frac{dy}{dx} = \frac{2a}{y}, \frac{d^2y}{dx^2} = \frac{-4a^2}{y^3}, \frac{dx}{dy} = \frac{y}{2a}, \frac{d^2x}{dy^2} = \frac{1}{2a}$

ii) Evaluate :$\int \frac{xdx}{x^4-x^2+1}$

Sol:let x^2=z,xdx=dz/2, $\frac{1}{2}\int \frac{dz}{z^2-z+1} = \frac{1}{2}\int \frac{dz}{(z-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$ Ans:$\frac{1}{\sqrt{3}}\tan^{-1}\frac{2x^2-1}{\sqrt{3}} + c$

OR

Evaluate : $\int \sqrt{1+sin2x}dx$

Sol:$\int \sqrt{(sinx+cosx)^2}dx$=sinx-cosx+c

iii) Integrate : $\int \frac{dx}{(1+x^2)\sqrt{1+x^2}}$

Sol:let x=tan t,dx=sec^2tdt $\int \frac{sec^2 tdt}{sec^3 t} = \int cost dt = \sin(\tan^{-1} x) + c$
OR $\int \sin^{-1} x\, dx$
Sol:Let x=sinθ, $dx = cos\theta d\theta, \int \theta cos\theta d\theta$.Integrate by parts.Ans:x$\sin^{-1}$x+$(1-x^2)^{1/2}$+c

d) **Answer any one question :** **1x4=4**

i) If a=3i+j+9k and b=i+λj+3k,then find the value of λ for which the vector (a+b) and (a-b) are perpendicular to each other.

Sol.a+b=4i+($\gamma + 1$)j+12k,a-b=2i+(1-γ)j+6k.dot product is 0 i.e 4.2+$(\gamma + 1).(1-\gamma) + 12.6 = 0, \gamma = \pm 9$

ii) Find the value of λ for which the vectors a=2i-j+k,b=i+2j-3k and c=3i+λj-5k are coplanar

Sol: $\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \gamma & -5 \end{vmatrix} = 0$.For coplanarity [abc]=0. So,$\gamma = \frac{22}{7}$

e) **Answer any two questions :** **2x4=8**

i) Evaluate $\int \frac{x^6-1}{x-1} dx$

Sol:$\int \frac{(x^2)^3-1}{x-1} dx = \int \frac{(x^2-1)(x^4+x^2+1)}{x-1} dx$. Ans:$x^6/6+x^5/5+x^4/4+x^3/3+x^2/2+x+c$

ii) Find the value of $\int \frac{1-cos\theta}{1+cos\theta} d\theta$

Sol:$\int \frac{2sin^2\frac{\theta}{2}}{2cos^2\frac{\theta}{2}} d\theta = \int (sec^2\frac{\theta}{2} - 1) d\theta = 2\tan\frac{\theta}{2} - \theta + c$

iii) Evaluate : $\int \frac{dx}{\sqrt{2ax-x^2}}$

Sol:$\int \frac{dx}{\sqrt{a^2-(x^2-2ax+a^2)}} = \int \frac{dx}{\sqrt{a^2-(x-a)^2}} = \sin^{-1}\frac{x-a}{a} + c$

GROUP : C

3. **Answer any four question :** **4x5=20**

a) If y=sin(logx),prove that $x^2y_2+xy_1+y=0$

Sol:y_1=cos(logx)/x xy_1=cos(logx).Diff. w.r.t x,xy_2+y_1=-sin(logx)/x=-y/x

b) If x=a(t+sint),y=a(1-cost) find $\frac{dy}{dx}$

Sol: $\frac{dx}{dt} = a(1 + cost), \frac{dy}{dt} = asint, \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{sint}{1+cost}$

c) Find the derivative of $x^{sin^{-1}x}$ w.r.t $\sin^{-1}$x

Sol: Let $\sin^{-1}$x=u,du/dx=$\frac{1}{\sqrt{1-x^2}}$ let z= $x^{\sin^{-1}x}$,so z=x^u ,logz=ulogx,Diff.w.r.t u,1/z$\frac{dz}{du} = logx + u\frac{1}{x}\frac{dx}{du}$ .Now put the values

d) Find the equation of tangent and normal to the circle $x^2+y^2=a^2$ at (2,3)

Sol: $(\frac{dy}{dx}) = \frac{-x}{y}$ at (2,3) is -2/3 Equation of tangent is y-3=-2/3(x-2).Equation of normal is y-3=3/2(x-2)

e) Find the approximate value of $(82)^{1/4}$ by using differentials

Sol:f(x)=$x^{1/4}$,f(81+1)=f(81)+f'(81)$\times$ 1, $(82)^{1/4}=(81)^{1/4}+1/4\times(81)^{-3/4}$=3.009

f) Examine the differentiability of $f(x)=2x^2+1$ at $x=1$

Sol: $Rf'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{2h^2+4h}{h}=4$ since $f(1)=3$,similarly,$Lf'(1)=4$.So f(x) is differentiable at x=1

FOR GOD AND COUNTRY

ST. LAWRENCE HIGH SCHOOL

PRE-SELECTION TEST

Sub: Mathematics **Class: XII** **F. M. 10**

Duration: **Date: 07.08.19**

PART : B

(Choose the correct answer and write in the appropriate box)

1.a) The principal value of $\sin^{-1}$ (½)is

i)30^0 ii)60^0 iii)90^0 iv) 0^0

Ans:i)

b) If A and B are two matrices such that AA^T = I,then A is

i) orthogonal ii) symmteric iii) skew-symmetric iv) none of these

Ans:i)

c) A^{-1} exists if,A matrix is

i) singular ii) non-singular iii) diagonal iv) triangular

Ans:ii)

d) If $f(x)=(x^2-1)^{1/2}$ then the value of $f''(1)$

i) $\frac{1}{\sqrt{2}}$ ii) $-\frac{1}{\sqrt{2}}$ iii) 1 iv) undefined

Ans:iv)

e) If a.b=2 and |a|=2,|b|=1 then angle between a and b

i) 90^0 ii) 0^0 iii) 135^0 iv) 270^0

Ans:ii)

f) The slope of the tangent to the straight line 3x+2y=1 at the point (0,0)is

i)0 ii)1 iii)-3/2 iv) 2

Ans:iii)

g) If two rows of a determinant are equal,value of determinant will be

i)0 ii)1 iii)2 iv)-1

Ans:i)

h) If $f(x)=x^2$ then f(x) is
i)surjective ii) injective iii)bijective iv) none of these
Ans:iii)

i) Derivative of a^x is
i)a^x ii)loga iii)a^xloga iv) x^a
Ans:iii)

j) $\int cosecx\, cotxdx$ is
i) –cosecx+c ii)cotx+c I ii)cosecx+c iv)-cotx+c
Ans:i)