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ST. LAWRENCE HIGH SCHOOL

FOR GOD AND COUNTRY

Pre Test Examination

Model Answer

Subject: Statistics **Class: XII** **F. M. 70**

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PART : B

Q1. Answer the following questions **1x10=10**

a. State which of the following pairs of variables can be studied using the correlation :
(i) religion and family size (ii) caste and annual income
(iii) hair colour of wife and hair colour of husband **(iv) none of these**

b. (b) First three moments (μ_1', μ_2', μ_3') are same for
(i) binomial distribution **(ii) Poisson distribution**, (iii)uniform distribution, (v) none of these

c. Mean is always greater than the variance for
(i) binomial distribution (ii) uniform distribution (iii) normal distribution, (iv) none of these

d. If $r_{xy}=0$ then x and y are
(i)independent (ii)not independent **(iii)linearly independent** (iv)none of these

e. If in binomial distribution $p=\frac{1}{2}$, then
(i) mean > mode, (ii) mean < mode, **(iii) mean = mode** (iv) we cannot comment anything about the relation between its mean and mode.

f. Poission distribution is
(i) positively skewed (ii) negatively skewed (iii) symmetric (iv) none of these

g. The correlation coefficient depends upon the change of
(i) origin (ii) scale (iii) both origin and scale **(iv) only sign of scale**

h. Identify the most appropriate distribution for the following random variable X: Number of calls received at a fire service station.
(i) Binomial **(ii) Poisson** (iii) Uniform (iv) none of these

i. The correlation coefficient for two bivariate points (12 , 25) and (10 , 20)
(i) -1 **(ii) +1** (iii) 0 (iv) 0.5

j. The covariance between number of success and failure in a bernoullian trial is
(i) –V(X) (ii) V(X) (iii) V(nX) (iv) none of these

Q2.. Answer the following questions. **1X8=8**

a. Where do two regression lines coincide?
Ans. When they are parallel. r = -1 or +1.

b. Find Mean and standard deviation of a binomial distribution are respectively, 4 and $\sqrt{\frac{8}{3}}$. Find the values of n and p.

Ans. P = 2/3.

c. Define Distribution function.

Ans. F(x) = P ($X \leq x$)

d. Write the conditions of pmf.

Ans. i. $f(x) \geq 0$. For all x. and ii. $\sum_x f(x) = 1$.

e. Find the CV of Poisson distribution.

Ans. $\sqrt{\lambda}$.

f. Write the unbiased estimator of μ in N(μ , σ^2).

Ans. Sample mean.

g. Find the maximum and minimum value of y, given y = $x + \frac{1}{x}$.

Ans. $\frac{dy}{dx} = 0 \Rightarrow x = \pm 1 . \frac{d^2y}{dx^2} < 0 \; when \; x = -1 \; and \; \frac{d^2y}{dx^2} > 0 \; when \; x = 1.$

Max value of y is -2 and min value of y is 2.

h. If a binomial distribution has mode X = 3 and 4, then find C.V. of the distribution.

Ans. $\frac{1}{\sqrt{5}}$

PART: A

Q1. Answer the following questions. **2X4=8**

a. When does the random variable follow Poisson distribution?

Ans. When i. the no of trials are countably infinite,

ii. In each trail there exist only two possible outcomes , viz, success and failure and

iii. In every single trial the probability of success remains same.

b. Write the mathematical models in time series data. OR Give an example of increasing trend and decreasing trend.

Ans. Y = T+S+C+I (Additive model) and Y= T.S.C.I (multiplicative model).

OR, increasing trend: population in India decreasing trend: volume of glacier.

c. Find E(X) in case of Rectangular distribution. OR Write three properties of distribution function.

Ans. $E(x) = \int_a^b \frac{x}{b-a} dx = \frac{a+b}{2}$

OR. $i. F(-\infty) = 0 \quad ii. F(\infty) = 1 \quad iii. F(x) \; is \; right \; continuous.$

d. Find the median of binomial distribution (n , p) when p =$\frac{1}{2}$

Ans. $\frac{n}{2}$

Q2. Answer the following questions. **3X8=24**

a. A person tosses an unbiased coin m+n (m>n) times. Find the probability that the person gets exactly m consecutive heads.

Ans. Sample space..

H H .... H T X X X

T H .... H H T X X

X X ......T H H ....H

Required probability = $\frac{n+3}{2^{n+2}}$

b. If $P(X=3) = P(X=4)$ for a Poisson random variable X, then find (i) the mean of the distribution, (ii) P(X=0), (iii) $P(1 \leq X \leq 3)$.

Ans (i) the mean of the distribution =4. (ii) P(X=0) = e^{-4}.
(iii) $P(1 \le X \le 3)$ = f(1) +f(2) + f(3) = 0.41515

c. Derive mean deviation about mean of a random variable X ~ Binomial (n,p).

Ans MD(x) = $2\sum_{i=k+1}^{n}(x-np).f(x), where\ k = [np]$

$= 2\sum_{i=k+1}^{n}(\gamma_x - \gamma_{x+1}) = 2.\gamma_{k+1}$

OR Discuss the skewness of X ~ Binomial (n, p).

d. An unbiased coin is tossed 3n times. Find the probability of getting number of heads as multiple of three. Given that n is odd.

Ans. In the binomial expansion of $(1+x)^{3n}\ putting\ x = 1, w, w^2$ and adding we get the required probability as $\frac{1}{3}(2^{3n} + 2\cos\frac{3n\pi}{3}) = \frac{1}{3}(2^{3n} - 2)$

e. Find the expected number of throws required to get r success in case of infinite independent trials.

OR Write a short note on moving average method.

Ans In moving average method we can not find the secular trend values for first n and last n years for 2n or (2n+1) yearly moving average and for this reason we can not forecast in this method which can be done using curve fitting method. In case of inclusion or rectification of some values moving average is better.

f. Derive the expression of standard error of estimate of y in regression equation y on x.

OR

Derive the expression of coefficient of determination in regression equation y on x.

Ans $V(e) = \frac{1}{n}\sum e_i^2 = \frac{1}{n}\sum_{i=1}^{n}\{(y_i - \bar{y}) - r.\frac{s_y}{s_x}((x_i - \bar{x})\}^2$

$= s_y^2 - 2r\frac{s_y}{s_x}.r\, s_x\, s_y + r^2\frac{s_y^2}{s_x^2}.s_x^2 = s_y^2(1 - r^2).$

So the standard error of estimate of y in regression equation y on x is $= s_y\sqrt{(1 - r^2)}$

OR. $V(Y) = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \bar{y})^2 = \frac{1}{n}\sum_{i=1}^{n}\{\bar{y} + r.\frac{s_y}{s_x}((x_i - \bar{x}) - \bar{y}\}^2 = r^2 s_y^2$

Hence $r^2 = \frac{s_Y^2}{s_y^2}$ which is known as coefficient of determination.

g. Show that Spearman's rank correlation coefficient lies between -1 and +1.

Ans Case1: perfect agreement. $u_i - v_i = 0 \Rightarrow \sum d^2{}_i = 0 \Rightarrow r_R = 1$

Case2: perfect disagreement $u_i + v_i = n + 1 \Rightarrow \sum d^2{}_i = \frac{n(n^2-1)}{3} \Rightarrow r_R = -1$

h. Each of two persons tosses an unbiased coin n times. Find the probability that they get same number of heads.

Ans A: 1st person gets r heads. P(A) = $n_{c_x}(\frac{1}{2})^n$

B: 2nd person gets r heads P(B) = $n_{c_x}(\frac{1}{2})^n$

P(Both get r heads) = $n_{c_x}(\frac{1}{2})^n . n_{c_x}(\frac{1}{2})^n$. so the required probability= $(\frac{1}{2})^{2n}\sum_{x=0}^{n} n_{c_x}.n_{c_x} = (\frac{1}{2})^{2n}.2n_{c_n}$

Q3. Answer the following questions. **5X4=20**

a. Derive the equation of regression line y on x for n bivariate observations.

Ans In a scatter diagram let ith plotted point be(x_i, y_i) and the predicted best fitted line be Y=a+bX . $e_i = (y_i - Y_i) = (y_i - a - bx_i)$

Sum of square of errors E = $(y_i - a - bx_i)^2$

Minimizing E by differentiating partially w.r.t. a and b we get the normal equations

$\sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i$ and $\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2$

Solving we get, $\hat{b} = \frac{cov(x,y)}{V(x)} = b_{xy}$. Hence the regression equation is ,y - $\bar{y} = b_{xy}$(x- $\bar{x}$).

OR Describe the components in time series analysis.

Secular trend(T): Long term, smooth ,monotonic curve which shows the basic pattern of data.

Seasonal variation (S): It gives a periodic curve with period of oscillation one year. During every period the curve attains its crest and trough at the same point but may differ in magnitude

Cyclical variation (C): It gives a periodic curve with period of oscillation is more than one year. During every period the curve attains its crest and trough at the same point but may differ in magnitude

Irregular fluctuation(I): It has no specific pattern. It happens due to catastrophical disorders like flood, earth quake, war etc.

b. Derive the expression of geometric mean a random variable

Ans X~ Binomial (n,p).

ln G = E(ln(x)) = E $(\ln \mu + \ln(1+\frac{x-\mu}{\mu})) = \ln\ \mu + E(\frac{x-\mu}{\mu} - \left(\frac{x-\mu}{\mu}\right)^2)$ (neglecting the higher order terms, as $\mu \gg \sigma$) $= \ln \mu + 0 - \frac{\sigma^2}{\mu^2}$

$\Rightarrow \ln\frac{G}{\mu} = -\frac{\sigma^2}{\mu^2} \Rightarrow G = \mu . e^{-\frac{\sigma^2}{\mu^2}} = \mu.\left(1-\frac{\sigma^2}{\mu^2}\right).$

c. For a discrete random variable X, f(x) is the pmf. x= 0(1)15. Given that $f(x) = \frac{16-x}{x}.\frac{1}{2}\ f(x-1), x > 0$. Find the probability distribution.

Ans $f(x) = \frac{16-x}{x}.\frac{17-x}{x-1} \dots\dots.\frac{15}{1}.\left(\frac{1}{2}\right)^x.f(0) = 15_{c_x}\left(\frac{1}{2}\right)^x.f(0)$

$\sum_{x=0}^{15} f(x) = 1 \Rightarrow f(0) = \left(\frac{2}{3}\right)^{15} \Rightarrow f(x) = 15_{c_x}\left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{15-x}$

$\Rightarrow$ X ~ Binomial (15, $\frac{1}{3}$).

d. Derive the expression of rth order central moment of binomial distribution with parameters n and p.

Ans $f(x) = n_{c_x}\ p^x (1-p)^{n-x} \Rightarrow \frac{d}{dx} f(x) = \frac{1}{p(1-p)} f(x)$

$\Rightarrow \frac{d}{dx}\mu_r = \frac{d}{dx}\sum_{x=0}^{n}(x-np)^r f(x) = -nr\ \mu_{r-1} + \frac{1}{p(1-p)}\mu_{r+1}$

OR

Derive the expression of Spearman's rank correlation coefficient.

Ans Let u_i *and* v_i are the ranks of ith candidate given by judge1 and judge2 respectively.

$d_i = u_i - v_i$ *and* $\sum_{i=1}^{n} u_i = \sum_{i=1}^{n} v_i = \frac{n(n+1)}{2}$ *Also* $\sum_{i=1}^{n} {u_i}^2 = \sum_{i=1}^{n} {v_i}^2 = \frac{n(n+1)(2n+1)}{6}$

$\frac{1}{n}\sum {d_i}^2 = {s_u}^2 + {s_v}^2 - 2cov(u,v) \Rightarrow cov(u,v) = \frac{n^2-1}{12} - \frac{1}{2n}\sum {d_i}^2$

$r_{uv} = \frac{cov\ (u,v)}{s_u s_v} = \frac{\frac{n^2-1}{12} - \frac{1}{2n}\sum {d_i}^2}{\frac{n^2-1}{12}} = 1 - \frac{6\sum {d_i}^2}{n(n^2-1)} = r_R\ .$