Indranil Ghosh
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ST. LAWRENCE HIGH SCHOOL

FOR GOD AND COUNTRY

MODEL SOLUTIONS SELECTION TEST 2019

Sub: Mathematics | **Class: XII** | **F. M. 80**
Duration: 3 Hrs 15mins. | **Date: 25.11.2019**

1.a) Answer any one question: 1x2=2

i) Prove that f(x)= sinx is neither one-one nor onto
Sol: Range of f is [-1,1] not equal to R and $\sin 120^0=\sin 60^0$,

ii) Prove $4(2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7})=\pi$

Sol: $4(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7})=4\tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{74}}=\pi$

b) Answer any one question: 1x2=2

i) If A=$\begin{bmatrix}1 & -0\\ -1 & 7\end{bmatrix}$, find the value of k, if A^2=8A+kI,I is the identity matrix

Sol: $A^2=8A+kI=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}\times\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}=\begin{bmatrix}8 & 0\\ -8 & 56\end{bmatrix}+\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}=\begin{bmatrix}8+k & 0\\ -8 & 56+k\end{bmatrix}8+k=1,56+k=49,k=-7$

ii) Prove $\begin{vmatrix}0 & a & -b\\ -a & 0 & c\\ b & -c & 0\end{vmatrix}=0$

Sol: $\Delta=(-1)^3\begin{vmatrix}0 & -a & b\\ a & 0 & -c\\ -b & -c & 0\end{vmatrix}=-\begin{vmatrix}0 & a & -b\\ -a & 0 & c\\ b & -c & 0\end{vmatrix}=-\Delta, 2\Delta=0 i.e\ \Delta=0$

c) Answer any three questions: 3x2=6

i) Examine the continuity of f(x) $=\frac{|x-1|}{x}$ when x≠ 1
=0 , x=1

Sol: $f(x)=\frac{(x-1)}{x}, x>1\ \ f(x)=\frac{1-x}{x}, x<1\ \ RHL=\frac{1-1}{1}=0,\ LHL=0$, so continuous

ii) Integrate: $\int sin^3x\,dx$
Sol: $\frac{1}{4}\int(3\sin x-\sin 3x)dx=\frac{-3}{4}\cos x+\frac{1}{12}\cos 3x+c$

iii) Verify Rolle's theorem for the function $f(x)=(x-1)(x-2)^2$ in [1,2]
Sol: $\frac{dy}{dx}=3x^2-10x+8\ exists\ in (1,2), f(x) is\ continuous, f(1)=f(2), 3c^2-10c+8=0, c=4/3,2$.so f satisfies the theorem

iv) Evaluate: $\lim_{x\to 0}\frac{|\sin x|}{x}$
Sol: RHL=$\lim_{x\to 0+}\frac{sinx}{x}=1, LHL=-1$,so limit does not exist

v) If the radius of a spherical balloon increases by 0.1%, find the percentage increase in volume.
Sol: $V=\frac{4}{3}\pi r^3, \frac{dv}{dr}=4\pi r^2, \Delta r=0.001r, dv=\frac{dv}{dr}\Delta r=0.003v, \%=\frac{0.003v}{v}\times 100=0.3\%$

vi) Find the range of values of x for which f(x)=$2x^3-9x^2-24x+5$ increases
Sol: $\frac{dy}{dx}=6(x-4)(x+1), if\ x>4\ or\ x<-1\ then\ \frac{dy}{dx}>0$

d) Answer any one question : 2x1=2

i) If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$,show that the vectors $\vec{a},\vec{b},\vec{c}$ are coplanar

Sol: $\vec{a}$, $\vec{a}.(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}\times\vec{c}\times\vec{a})=\vec{a}.0=0,(\vec{a}\times\vec{a}).\vec{b}+\vec{a}.(\vec{b}\times\vec{c})-(\vec{a}\times\vec{a}).\vec{c}=0, so\ \vec{a}.(\vec{b}\times$ $\vec{c}$)=0 which is condition for coplanarity

ii) Find the equation of the plane which passes through the points (1,2,3),(2,3,1) and (3,1,2)

Sol: $\begin{vmatrix} x-1 & y-2 & z-3 \\ 2-1 & 3-2 & 1-3 \\ 3-1 & 1-2 & 2-3 \end{vmatrix}=0\,, x+y+z=6$

e) Answer any one question: 2x1=2

i) If A and B are independent events, prove that A^C and B^C are also independent

Sol: $P(A^C\cap B^C)=1-P(A\cup B)=1-P(A)-P(B)+P(A)P(B)=P(A^C)P(B^C)$

ii) State Baye's Theorem in probability

Sol: see page 858 of textbook

2.a) Answer any one question : 1x4=4

i) A relation R on the set of natural numbers N is defined as, (x, y) ϵR→(x-y) is divisible by 5,for all x, y ϵ N. Prove that R is an equivalence relation

Sol: (x-x) =0 is divisible by n, so R is reflexive.-(y-x) is divisible by n i.e. y-x is also divisible, so (y, x) belongs to R, so R is symmetric on N. y-z is divisible by n, so (x-y)+(y-z)=x-z is divisible by n, so (x, z) belongs to R, so R is transitive

ii) Solve: $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$ and $\cos^{-1}x-\cos^{-1}y=\frac{\pi}{3}$

Sol: Let $\sin^{-1}x=a$, $\sin^{-1}y=b$ so $a+b=\frac{2\pi}{3}$ so from 2nd equation using $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$, $-a+b=\frac{\pi}{3}$ so $a=\frac{\pi}{6}, b=\frac{\pi}{2}, x=\frac{1}{2}, y=1$

b) Answer the following questions : 4x2=8

i) Solve : $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix}=0$

Sol: apply $c''_1=c_1+c_2+c_3$ $(3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix}=0, R'_2=R_2-R_1, R'_3=R_3-R_1, x=$ $0,3a$

OR

Without expanding, evaluate $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

Sol: let, logx=a, logy=b, logz=c, $\log_x y=\frac{b}{a}, \log_x z=\frac{c}{a}\ etc, so \begin{vmatrix} 1 & b/a & c/a \\ a/b & 1 & c/b \\ a/c & b/c & 1 \end{vmatrix}=$ $\frac{1}{abc}\begin{vmatrix} a & b & c \\ a & b & c \\ a & b & c \end{vmatrix}=0$

ii) Solve the following set of equations by matrix method:x+2y-3z=-4,2x+3y+2z=2,3x-3y-4z=11

Sol:|A|=67 AdjA=$\begin{bmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ 15 & 9 & -1\end{bmatrix}$ $X = A^{-1}B, \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{67}\begin{bmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ 15 & 9 & -1\end{bmatrix} \times \begin{bmatrix}-4\\ 2\\ 11\end{bmatrix} = \begin{bmatrix}3\\ -2\\ 1\end{bmatrix}, x = 3, y = -2, z = 1$

OR

If $A = \begin{bmatrix}1 & 2 & 1\\ 1 & 0 & 3\\ 2 & -3 & 0\end{bmatrix}$,find A^{-1} by elementary row or column operations

Ans: $\begin{bmatrix}1/2 & 1/6 & -1/3\\ 1/3 & -1/9 & -1/9\\ -1/6 & 7/18 & -1/9\end{bmatrix}$ see page 210 similar problem

c) Answer the following questions: **4x3=12**

i) If $y = (\sin^{-1} x)^2 + (\cos^{-1} x)^2$; show that$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 4$

Sol:$\frac{dy}{dx} = 2\sin^{-1} x\frac{1}{\sqrt{1-x^2}} + 2\cos^{-1}\frac{-1}{\sqrt{1-x^2}}, \sqrt{1-x^2}\frac{dy}{dx} = \pi - 4\cos^{-1} x. diff.w.r.t\, x, \sqrt{1-x^2}\, y_2{+}y_1\frac{-x}{\sqrt{1-x^2}} = 4\frac{1}{\sqrt{1-x^2}}$

OR

If y=$\tan^{-1}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$; show that $\frac{dy}{dx} = \frac{x}{\sqrt{1-x^4}}$

Sol: let $x^2 = \cos 2\theta, y = \tan^{-1}\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \tan^{-1}\tan(\frac{\pi}{4}-\theta), so\ y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x^2, \frac{dy}{dx} = \frac{2x}{2\sqrt{1-x^4}}$

ii) Evaluate:$\int\frac{(3x-5)dx}{x^2-2x+10}$

Sol: let$3x-5 = l.\frac{d}{dx}(x^2-2x+10) + m, l = \frac{3}{2}, m = -2, \frac{3}{2}\int\frac{2x-2}{x^2-2x+10}dx - \frac{2}{3}\int\frac{dx}{(x-1)^2+3^2} = \frac{3}{2}\log(x^2-2x+10) - \frac{2}{3}\tan^{-1}\frac{x-1}{3} + c$

OR

Evaluate: $\int\cos(\log x)dx$

Sol: let $I = \cos(\log x)\int dx - \int[\frac{d}{dx}\cos(\log x)\int dx]dx, I = x[\cos(\log x) + \sin(\log x)] - I + c$

iii) Solve: cosy dx+(1+e^{-x})sinydy=0 given y=$\frac{\pi}{4}$ when x=0

Sol:$\int\frac{e^x dx}{e^x+1} - \int(-\tan y)\,dy = 0, \log\frac{e^x+1}{cosy} = \log c, x = 0, y = \frac{\pi}{4}\ so\ c = 2\sqrt{2}, (e^x+1) = 2\sqrt{2}\cos y$

OR

Iv) Solve: xdy-ydx = $2\sqrt{y^2-x^2}$ dx

Sol: Put y=vx,$\frac{dy}{dx} = v + x\frac{dv}{dx}, \int\frac{dv}{\sqrt{v^2-1}} = 2\int\frac{dx}{x} + \log c, |v + \sqrt{v^2-1}| = x^2 c, put\ v = y/x$

d) Answer any one question: **1x4=4**

i) If $\vec{a} = \vec{\imath} + \vec{\jmath} + \vec{k}$ and $\vec{b} = \vec{\jmath} - \vec{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

Sol: Let $\vec{c} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}, \vec{a} \times \vec{c} = \begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 1 & 1 & 1\\ x & y & z\end{vmatrix} = (z-y)\vec{\imath} + (x-z)\vec{\jmath} + (y-x)\vec{k} = \vec{\jmath} - \vec{k}, so\ y = z; x - z = 1; y - x = -1, now(\vec{\imath} + \vec{\jmath} + \vec{k}).(x\vec{\imath} + y\vec{\jmath} + z\vec{k}) = 3 implies\ x + y + z = 3, so x = \frac{5}{3}, y = z = 2/3$

ii) If G be the centroid of the triangle ABC, then prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=0$

Sol: Let $\vec{a},\vec{b},\vec{c}$ be the p.v's of vertices A, B, C with arbitrary origin. The p.v of centroid G is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$ $\overrightarrow{GA}=\vec{a}-\frac{\vec{a}+\vec{b}+\vec{c}}{3}=\frac{2\vec{a}-\vec{b}-\vec{c}}{3}$ $\overrightarrow{GB}=\frac{2\vec{b}-\vec{c}-\vec{a}}{3}$ $\overrightarrow{GC}=\frac{2\vec{c}-\vec{a}-\vec{b}}{3}$ so put the values

e) Answer any one question: **1x4=4**

i) Evaluate $\int_{-5}^{0} f(x)dx$ where $f(x)=|x|+|x+2|+|x+5|$

Sol: $\int_{-5}^{0}-xdx+\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{0}(x+2)dx+\int_{-5}^{0}(x+5)dx=\frac{25}{2}+\frac{13}{2}+\frac{25}{2}=63/2$

ii) Find the value of $\lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{n}\sqrt{\frac{n+r}{n-r}}$

Sol: $\lim_{n\to\infty}\sum_{r=0}^{n-1}h\sqrt{\frac{1+rh}{1-rh}}-\lim_{n\to\infty}\frac{1}{n},nh=1,\int_0^1\sqrt{\frac{1-x}{1+x}}dx=\int_0^1\frac{1}{\sqrt{1-x^2}}dx+\int_0^1\frac{x}{\sqrt{1-x^2}}dx=\frac{\pi}{2}+1$

f) Answer any one question: **4×1=4**

i) A speaks the truth in 60% of cases and B in 90% of cases. In what percentage of cases they are likely to contradict each other in stating the same fact.

Sol: Let X and Y be events of speaking truth by A & B,P(X)=0.6,P(B)=0.9,P(X^C)=0.4,P(Y^C)=0.1 so $Z=(X\cap Y^C)\cup(X^C\cap Y),P(Z)=0.6\times 0.1+0.4\times 0.9=0.42$,so 42%

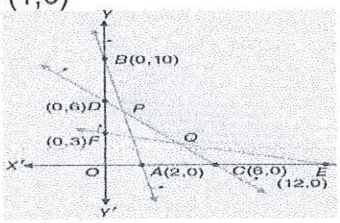
ii) Let x and y be two independent random variables having variances k and 2 respectively. If the variance of z=3y-x be 25, find k

Sol: Var(Z)=3^2Var(Y)+$(-1)^2$Var(X),25=9×2+k,k=7

3. Answer any one question: **5x1=5**

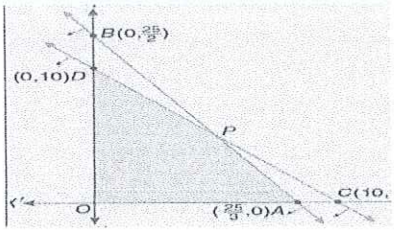
i) Represent geometrically the following LPP and find the optimal solution
Minimize Z=3x+2y subject to the constraints
5x+y≥10, x+y≥6, x+4y≥12 and $x\geq 0, y\geq 0$

Sol: $\frac{x}{2}+\frac{y}{10}=1,\frac{x}{6}+\frac{y}{6}=1,\frac{x}{12}+\frac{y}{3}=1$ solving we get E(12,0),Q(4,2),P(1,5),B(0,10),Z_{min}=13 at (1,5)



ii) A man has Rs.1500 to purchase rice and wheat. A bag of rice and a bag of wheat cost Rs.180 and Rs.120 respectively. He has a storage capacity of 10 bags only. He earns a profit of Rs.11 and Rs.8 per bag of rice and wheat respectively. How many bags of each must he buy to make maximum profit?

Sol: let x bags of rice and y bags of wheat, total cost 180x+120y≤1500,x+y≤10,so Max z=11x+8,$\frac{x}{\frac{25}{3}}+\frac{y}{\frac{25}{2}}=1,\frac{x}{10}+\frac{y}{10}=1$ solving O(0,0),A(25/3,0),P(5,5),D(0,10),Z_{max}=95 at (5,5)



b) Answer any two questions: $5\times 2 = 10$

i) Solve: $(x^2y^3+2xy)dy=dx$

Sol: $\frac{1}{x^2}\frac{dx}{dy}-\frac{2y}{x}=y^3, -\frac{1}{x}=y, \frac{1}{x^2}\frac{dx}{dy}=\frac{dz}{dy}, \frac{dz}{dy}+2yz=y^3 I.F=e^{\int 2ydy}=e^{y^2}, so\ ze^{y^2}=$

$\int y^3 e^{y^2}dy=\frac{1}{2}\int ve^v\,dv, (v=y^2)=\frac{1}{2}(ve^v-e^v)+k$, Ans: $1/x=\frac{1}{2}(1-y^2)+ce^{-y^2}$, c=-k

ii) If the line lx+my=1 be a normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, show that

$\frac{a^2}{l^2}-\frac{b^2}{m^2}=(a^2+b^2)^2$

Sol: $\frac{dx}{dy}=\frac{a^2x}{b^2y}$ equation of normal at $(a\sec\theta, b\tan\theta)$ $y-b\tan\theta=-\frac{dx}{dy}(x-a\sec\theta), xa\tan\theta+$ $yb\sec\theta=(a2+b2)\sec\theta\tan\theta$, comparing with

lx+my=1, $\sec\theta=\frac{a}{l(a^2+b^2)}, \tan\theta=\frac{b}{m(a^2+b^2)}$, putting the value in $sec^2\theta-tan^2\theta=1$, we get the result

iii) Find the area included between $y^2=9x$ and $y=x$

Sol: See textbook pg563 for figure. The curves intersect at (0,0) and (9,9) $\int_0^9\sqrt{9x}dx-$ $09xdx=54-812=272$ square units

iv) Find the coordinates of a point on the parabola $y=x^2+7x+2$ which is closest to the straight line $y=3x-3$

Sol: Let the point $P(h,h^2+7h+2)$ on the parabola be closest to straight line, let s be perpendicular distance of P from y-3x+3=0, $s=\frac{h^2+7h+2-3h+3}{\sqrt{1+9}}, \frac{ds}{dh}=\frac{2h+4}{\sqrt{10}}, \frac{ds}{dh}=$ $0, so\ h=-2, \frac{d^2s}{dh^2}=\frac{2}{\sqrt{10}}>0$ so h is minimum, so point is $[-2,(-2)^2+7(-2)+2]$, Ans: (-2,-8)

c) Answer any one question: $5\times 1 = 5$

i) Show that the line whose vector equation is $\vec{r}=(2\vec{\imath}-2\vec{\jmath}+3\vec{k})+\gamma(\vec{\imath}-\vec{\jmath}+4\vec{k})$ is parallel to the plane whose vector equation is $\vec{r}.(\vec{\imath}+5\vec{\jmath}+\vec{k})=5$. Also, find the distance between them

Sol: $\sin\theta=\frac{(\vec{\imath}-\vec{\jmath}+4\vec{k}).(\vec{\imath}+5\vec{\jmath}+\vec{k})}{|(\vec{\imath}-\vec{\jmath}+4\vec{k}).(\vec{\imath}+5\vec{\jmath}+\vec{k})}=0, \theta=0$, so parallel, d=5, $\vec{a}.\vec{n}=(2\vec{\imath}-2\vec{\jmath}+3\vec{k}).(\vec{\imath}+5\vec{\jmath}+\vec{k})=$ $-5\neq 0$, so it does not line on the plane. Now $p=\frac{|\vec{a}.\vec{n}-d|}{|\vec{n}|}=\frac{10}{27}$ units, p is distance

ii) Find the distance between the straight lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x}{2}=\frac{y-5}{3}=\frac{z+1}{4}$

Sol: P(0,5,-1) lies on 2nd line. L be foot of perpendicular on 1st line from P, so L is(1+2t,2+3t,3+4t).d.r.s of PL IS 1+2t,3t-3,4t+4, PL is perpendicular to 1st line, so (1+2t)2+(3t-3)3+(4t+4)4=0 i.e. t=-9/29. L is(11/29,31/29,52/29), PL=$\frac{\sqrt{19517}}{29}=4.817$units

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SELECTION TEST 2019

Sub: Mathematics **Class: XII** **F. M. 10**

(Choose the correct answer and write in the appropriate box)

1.a) The diagonal elements of a skew-symmetric matrix are all

i)0 ii)1 iii)-1 iv) 2

Ans: i)

b) If all elements of a row or column of a determinant are zero,then value of determinant is

i) 1 ii) 1/2 iii) 0 iv) -2

Ans: iii)

c) $\tan\frac{1}{3}(\tan^{-1}x+\tan^{-1}\frac{1}{x})$ is equal to

i) $\sqrt{3}$ ii) $\pi/2$ iii) 2π iv) $1/\sqrt{3}$

Ans:iv)

d) If x=acost, y=asint, $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ is equal to

i) t ii) -1 iii) π iv) a

Ans:ii)

e) If two vectors 2i+pj-3k and qi-4j+2k are parallel,then p,q is equal to

i)6,-4/3 ii) 2,3 iii)1,1 iv)0,-3

Ans:i)

f) The equation of the tangent to the straight line ax-by=c at the point (0,0)is

i)ab=xy ii)ay=bx iii)ax=by iv) x=y

Ans:iii)

g) The maximum value of y=(x-1)(3-x) is

i)1,3 ii)does not exist iii)10 iv)2

Ans: 1

h) The integrating factor of $\frac{dy}{dx}$+y=x

i)x ii) e^{-x} iii)e^{x} iv) logx

Ans:iii)

i) If P(A∪B)=1/3,then $P(A^C \cap B^C)$ is equal to

i)1 ii)0 iii)1/ 3 iv) 2/3

Ans:iv)

j) $\int \sin 2019x dx$ is

i) –cos2019x+c ii)cos2019x+c I ii)cosec2019x+c iv)-cos2019x/2019+c

Ans:iv)