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ST. LAWRENCE HIGH SCHOOL

Pre Annual Examination – 2020

FOR GOD AND COUNTRY

Sub: Physics | **Class: XI** | **F.M.: 70**

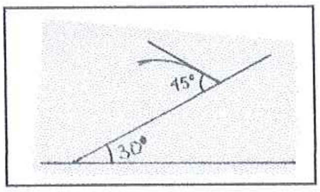
Duration: 3hrs 15 mins | **Date: 21.01.2020**

SECTION-I

Answer the following questions (Multiple Choice Questions) **(1x14=14)**

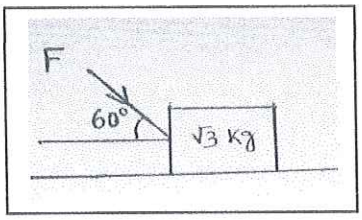
1. If area A, force F and velocity V are considered as the fundamental units then what will be the dimension of Young's modulus?
   a) $ML^{-1}T^{-2}$ b) FAV c) $F^{-1}AV$ **d) $FA^{-1}V^0$**

2. A body of mass 100g is projected with an initial velocity $2\sqrt{2}m/s$ from an inclined plane making 45^0 with that plane as shown in the figure. If the plane makes 30^0 angle with the horizontal, then what will be the speed of the body at highest poit?
   a) $(\sqrt{3}+1)m/s$ b) $(\sqrt{3}-1)m/s$ c) $\sqrt{2}m/s$ d) 2m/s



3. Two diagonals of a parallelogram are given by two vectors $2\hat{\imath}-\hat{\jmath}+3\hat{k}$ and $2\hat{\imath}+\hat{\jmath}-\hat{k}$. One area vector of this parallelogram can be
   a) $-2\hat{\imath}+8\hat{\jmath}+4\hat{k}$ **b) $\hat{\imath}-4\hat{\jmath}-2\hat{k}$** c) $2\hat{\imath}-8\hat{\jmath}+4\hat{k}$ d) None of these

4. What is the maximum value of the force F supplied on the block (as shown in the figure) such that the block does not move? Mass of the block = $\sqrt{3}\ kg$, g = $10m/s^2$, $\mu_s=\frac{1}{2\sqrt{3}}, \mu_k=\frac{1}{3\sqrt{3}}$
   a) 10 N b) 12 N c) 15 N **d) 20 N**



5. A bob is hung with the help of a mass less inextensible thread of length $'l'$ from the ceiling of a bus which is moving with a constant velocity v. If the driver suddenly press the break, the maximum angular deflection of the bob with respect to the vertical direction is
   a) $2\cos^{-1}\left(\frac{v}{2\sqrt{gl}}\right)$ b) $\sin^{-1}\left(1-\frac{v^2}{2gl}\right)$ c) $\cos^{-1}\left(1-\frac{v^2}{4gl}\right)$ **d) $2\sin^{-1}\left(\frac{v}{2\sqrt{gl}}\right)$**

6. A solid iron ball of mass m and radius r started moving from ground with initial velocity (of centre of mass) v. The maximum height (w.r.t ground) it can roll up on a curved path of radius of curvature R is
   a) $\frac{v^2}{2g}$ **b) $\frac{7v^2}{10g}$** c) $\frac{10v^2R}{7gr^2}$ d) None of these

7. If the total energy of a revolving satellite around eart is $-2E$ then what is its potential energy?
   a) $-4E$ b) $-E$ c) E d) $2E$

8. A capillary tube of radius r is immersed in water and m mass of water rises in it to a height **h**. If another capillary tube of radius $2r$ and of same material is immersed in the same water, the mass of water rise in it is
   a) $\frac{m}{2}$ b) m **c) $2m$** d) $4m$

9. Two identical drops of water are falling through air with terminal velocity v. If they coalesce, what will be the terminal velocity?
   a) v b) $2v$ c) $3v$ **d) *None of* these**

10. The change in internal energy of certain mole of ideal gas when the volume changes from V to 2V at constant pressure P is $\left(R\ is\ the\ molar\ gas\ constant\ and\ \gamma=\frac{C_p}{C_v}\right)$
   a) $\frac{R}{\gamma-1}$ b) PV **c) $\frac{PV}{\gamma-1}$** d) $\frac{\gamma PV}{\gamma-1}$

11. For a reversible cyclic process
   a) Entropy is zero b) work done is zero c) entropy always increases **d) change of entropy is zero.**

12. The internal energy of an diatomic ideal gas at 300K is 100J. In this 100J
   a) the potential energy is 40 J b) rotational kinetic energy is 60 J
   c) translational kinetic energy is 60 J d) None of these

13. A particle is executing SHM with amplitude A and time period T. If the body start moving from the mean position, then the time taken by the body to move from $+\frac{A}{2}$ $to + A$ is

a) $\frac{T}{4}$ b) $\frac{T}{6}$ c) $\frac{T}{8}$ d) $\frac{T}{16}$

14. The equation of a progressive wave is given as $y = 5sin\,\pi(t - 0.01\,x)$ along the x axis. Here, all quantities are in SI units. The phase difference between the points separated by a distance of 10m along x- axis is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{10}$ c) 2π d) None of these

SECTION -II

GROUP –A

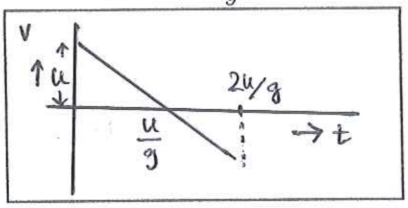
Answer the following questions in short.(Alternatives are to be noted): **(1x4=4)**

1. According to Plank's quantum theory, the energy of a photon of wavelength λ is $E = \frac{hc}{\lambda}$. If h be Plank's constant and c denotes the velocity of light in vacuum , determine the dimension of h.

<u>Ans:</u> $[h] = \left[\frac{E\lambda}{c}\right] = \frac{MLT^{-2}.L.L}{LT^{-1}} = ML^2T^{-1}$

2. A body is thrown vertically upward with initial velocity u. Draw velocity versus time graph for $\frac{2u}{g}$ time.

<u>Ans:</u> The graph will be a straight line with slope as '$-g$' as it will be satisfying the equation $v = -gt + u$. Again, the velocity will be zero at time $= \frac{u}{g}$. Hence the graph will be



Or

A ball is thrown horizontally from the top of a building while another is dropped from the same point at the same instant. Which ball will strike the ground first?

<u>Ans:</u> Both of them will touch the ground at same instant. For the downward motion, acceleration will be g in both the cases. Initial velocity for both is zero and vertical distance covered by both are also same.

3. What are the maximum and minimum theoretical values of Poisson's ratio?

<u>Ans:</u> For all substances, the theoretical value of Poisson's ratio lies between -1 and 0.5.

4. The displacement –time relationship of a system is given as $x = A\sin wt + A\sin 2wt$. Is the motion periodic or SHM or both? Justify your answer.

<u>Ans:</u> $A\sin wt$ is a periodic function with period $= \frac{2\pi}{w} = T$

$A\sin 2wt$ is a periodic function with period $= \frac{2\pi}{2w} = \frac{T}{2}$

Hence it is clear that the entire function i.e. x repeats after a minimum time $T = \frac{2\pi}{w}$. But the entire expression cannot be represented as a single sin or cosine function. Hence it is periodic but not SHM.

GROUP –B

Answer the following questions. (Alternatives are to be noted) : **(2x5=10)**

5. An object moving with a constant speed of 6.25m/s decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where '$v$' is the instantaneous velocity. What will be the time taken by the object to come to rest?

<u>Ans:</u> given $\frac{dv}{dt} = -2.5\sqrt{v}$

Or, $\frac{dv}{\sqrt{v}} = -2.5dt$ integrating we get

Or, $\int_{6.25}^{0} v^{-\frac{1}{2}}dv = -2.5\int_0^t dt$

Or, $2\left[\sqrt{v}\right]_{6.25}^{0} = -2.5\,t$

Or, $-2 \times \sqrt{6.25} = -2.5\,t$

Or, $t = 2\,sec$

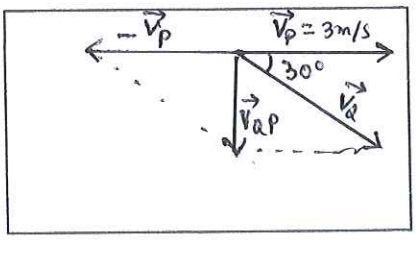
Or

A particle P is moving along a straight line with velocity 3m/s and another Q with velocity 4m/s at an angle 30^0 to the path of P. find the velocity of Q relative to velocity of P.

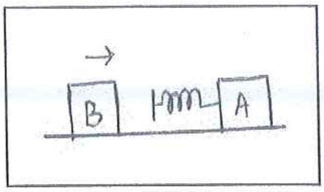
<u>Ans:</u> velocity of Q relative to P is actually relative velocity of Q w.r.t P

$= \vec{V}_{QP} = \vec{V}_Q - \vec{V}_P$

So, $|\vec{V}_{QP}| = \sqrt{9 + 16 + 2.3.4cos120^0} = \sqrt{13}\,m/s.$



6. Two blocks A and B of equal mass equal to 1kg are lying on a smooth horizontal surface as shown in the figure. A mass-less Spring of spring constant k = 200N/m is attached with A. B block starts moving with uniform velocity 2m/s and collides with A. Find the maximum compression of the spring. (Consider the compression to be within the elastic limit of the spring)



Ans: In the presence of the spring the momentum and energy both will be conserved.
If v be the velocity just at maximum compression then both the blocks will move with v.
By momentum conservation, $1 \times 2 + 1 \times 0 = (1+1)v$ or, $v = 1m/s$
If we let the maximum compression of the spring is x then -
By energy conservation, $\frac{1}{2}.1.2^2 + 0 = \frac{1}{2}(1+1).1^2 + \frac{1}{2}.200.x^2$ {As energy stored in spring $= \frac{1}{2}kx^2$}
Solving $x = \frac{1}{10}m = 10cm$

7. Define surface tension. What is the excess pressure inside a soap bubble of radius R and surface tension T when it is well inside the water? 1+1

Ans: Surface Tension: It is the property by virtue of which the free surface of a liquid at rest behaves like an elastic stretched membrane tending to contract so as to occupy minimum surface area.
Or, it is the tensile force acting per unit length of an imaginary line drawn on the free liquid surface at rest.
Inside the water soap bubble will have only one surface (inner one). Hence the excess pressure will be $P = \frac{2T}{R}$.

Or

When a spring is stretched by a length x, prove that the elastic potential energy stored in the spring of spring constant k is $E = \frac{1}{2}kx^2$.

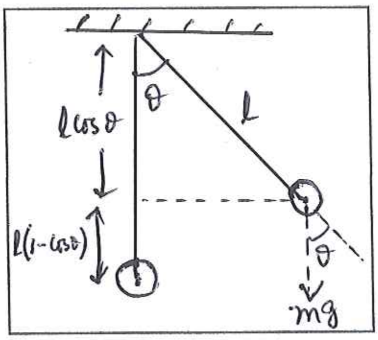
Ans: If the spring is stretched by l lengh, then the restoring force acting will be $= Kl$
Now to stretch it by dl length further, the work done will be $= Kl.dl$
Hence total work done to stretch it from $l = 0$ to $l = x$ will be $= \int_0^x Kldl = \frac{1}{2}K[l^2]_0^x = \frac{1}{2}Kx^2$
This work will be stored within the spring as the elastic potential energy.

8. Write down the law of equipartition of energy. What is the total translational kinetic energy of a diatomic molecule at absolute temperature T. (Take K_B as Boltzman constant) 1+1

Ans: The average kinetic energy associated per degree of freedom of a free gas molecule at absolute temperature T will be $\frac{1}{2}K_BT$ where K_B is the Boltzman constant.
For any gas molecule the number of translational degree of freedom is 3. Hence the total translational kinetic energy of a diatomic molecule at absolute temperature T will be $= 3 \times \frac{1}{2}K_BT = \frac{3}{2}K_BT$

9. The bob of a simple pendulum is released when the string makes an angle θ with the vertical. If m be the mass of the bob then find the tension when the bob is at the bottommost position.

Ans: let l be the length of the pendulum (including the radius of the bob)
When the bob is at θ angle, the height it is raised is $= l - lcos\theta$. If the instantaneous velocity of the bob at bottommost position be v, then, from the energy conservation, $mgl(1 - cos\theta) = \frac{1}{2}mv^2$

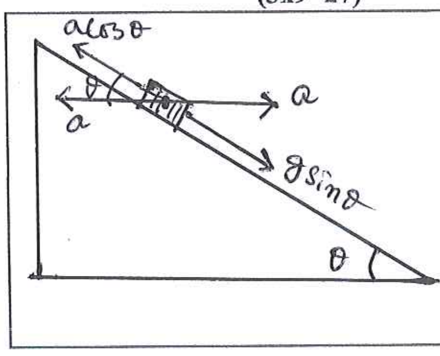


Or, $\frac{mv^2}{l} = 2mg(1 - cos\theta)$ ------(1)
The tension of the string when the bob is at bottommost position is =
$mg + \frac{mv^2}{l} = mg + 2mg(1 - cos\theta) = mg(3 - 2\,cos\theta)$

GROUP-C

Answer the following questions. (Alternatives are to be noted): **(3x9=27)**

10. A small block of mass m is placed on the smooth wedge of mass M as shown in the figure . Calculate the minimum force applied on the wedge horizontally for which the small block remains stationary w.r.t the wedge.



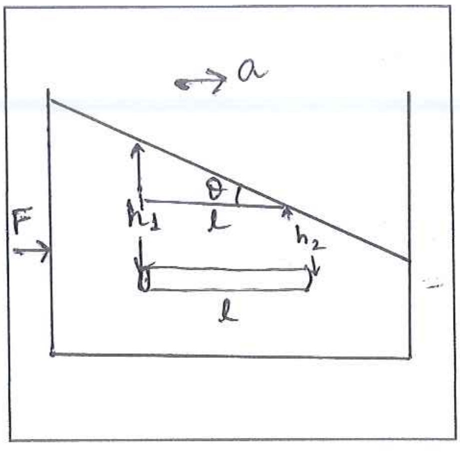
Ans: Let F force to be applied. Then the acceleration of the two block system will be $a = \frac{F}{M+m}$
Because of this acceleration in the smaller block of mass m, a pseudo force will be acting along the opposite direction of applied force as shown in the figure.
The component of this pseudo force along the inclined plane will balance the downward slipping.

Hence, $m\,a\,cos\theta = m\,g\,sin\theta$ or , $a = g\,tan\theta$

So, the required minimum force $F = (M+m)a = (M+m)g\,tan\theta$

Or

An water container is displaced horizontally with an acceleration a with the help of an external force. Calculate the angle the upper surface of the water makes with the horizontal direction.



Ans: Let the external force is applied from the left as shown in the figure and the acceleration produced is a.

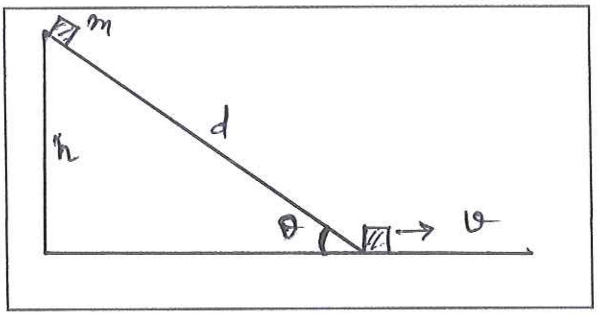
If we consider a narrow cylindrical horizontal water column as shown in the figure, then due to pressure difference, the force acting on the water column is $= (h_1\rho g - h_2\rho g)A$

Due to this force, the water column moves with acceleration a.

Hence FBD of the water column gives $(h_1\rho g - h_2\rho g)A = Al\rho\,a$

Or, $a = \frac{h_1-h_2}{l}\,g = g\,tan\theta$

11. A block of mass 'm' is released at the top (at a height h w.r.t ground) of a rough inclined plane of angle of inclination θ. If it starts sliding down from that point, then calculate the velocity of the block when it just reaches the ground level. Coefficient of kinetic friction is μ.



Ans: From the energy conservation,

Potential energy at the top of the inclined plane = work done by friction + K.E of the block at ground level

Or, $mgh = \frac{1}{2}mv^2 + \mu mg\,cos\theta\,.d$ $\quad$ [but $\frac{h}{d} = sin\theta$]

Or, $mgh = \frac{1}{2}mv^2 + \mu mg\,cos\theta\,\frac{h}{sin\theta}$

Or, $\frac{v^2}{2} = gh - \mu g\,h\,cot\theta$

Or, $v = \sqrt{2gh(1-\mu cot\theta)}$

12. State and prove the work energy theorem for a conservative force. 1+2

Ans: The work done by the external force will be equal to the change in kinetic energy of a body.

If a body experience a force F during certain time and its velocity increases to v $ftom$ u with an acceleration a, then we have $v^2 = u^2 - 2\,a\,s$

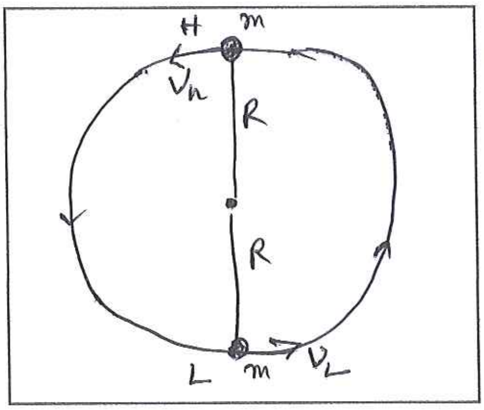
Multiplying both sides by $\frac{1}{2}$ m we get, $\quad \frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}m\,2\,a\,s$

Or, $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m\,2\,a\,s$

Or. $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = m\,a\,s = F.s = work\ done\ by\ external\ force$

Or

Calculate the minimum velocity needed by a point object at the lowest point for vertical circular motion of radius R.



Ans: *let* $v_h = minimum\ velocity\ needed\ at\ highest\ point$

$v_l = minimum\ velocity\ needed\ at\ highest\ point$

The tension of the thread at highest point is $T_h = \frac{m{v_h}^2}{R} - mg$

For VCM, $T_h \geq 0$,

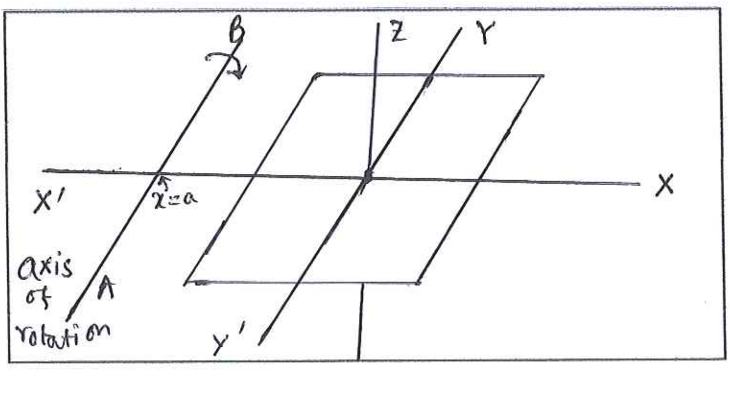
So, $\frac{m{v_h}^2}{R} \geq mg$ or, ${v_h}^2 \geq gR$ --------- (1)

Now, from energy conservation,

$$\frac{1}{2}mv_l^2 = \frac{1}{2}mv_h^2 + mg\,2R$$

By considering the critical value of $v_h = gR$ *as obtained in eqn* – (1), we get $\quad v_l \geq \sqrt{5gR}$

13. A square lamina of mass M and of sides 'a' is placed in X-Y plane such that its centre coincides with origin. What will be the moment of inertia of the lamina about an axis parallel to Y-axis and lying on X-Y plane at $x = -a$ and $z = 0$.



Ans: Refer to the figure,

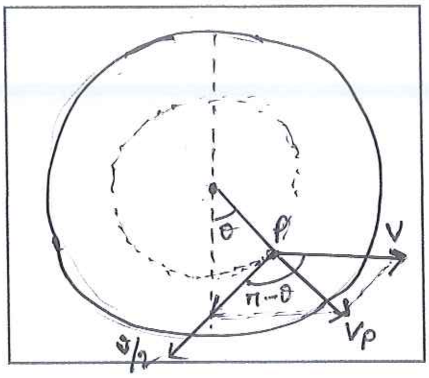
$.I_z = \frac{Ma^2}{6}$ so, $I_{yy'} = \frac{1}{2}\,I_z = \frac{Ma^2}{12}$

So, $I_{AB} = I_{yy'} + Md^2$

$= \frac{Ma^2}{12} + Ma^2 = \frac{13}{12}M\,a^2$

Or

A uniform disc of radius R is rolling without slipping on a rough horizontal plane with v as the constant linear velocity of its centre. Determine the magnitude of linear velocity (instantaneous) of a point P lying on the disc at a distance $\frac{R}{2}$ as shown in the figure.



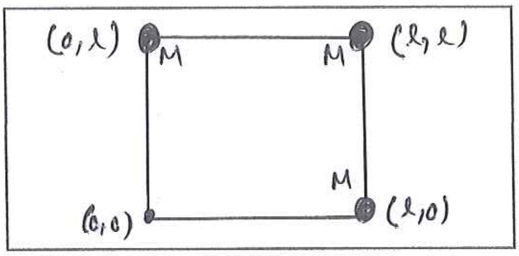
Ans: Refer to the figure, $v_p|_{tangential} = \frac{wR}{2} = \frac{v}{2}$

So, the net velocity of the point is

$$V_p^2 = \left(\frac{v}{2}\right)^2 + v^2 + 2.\frac{v}{2}.v\cos(180^0 - \theta)$$

Or, $v_p = \frac{v}{2}\sqrt{5 - 4\cos\theta}$

14. Three point masses of equal mass equal to M are placed at three vertices of a square of length l. Calculate the distance of the centre of mass of this three mass system from the fourth vertex.



Ans: Refer to the figure

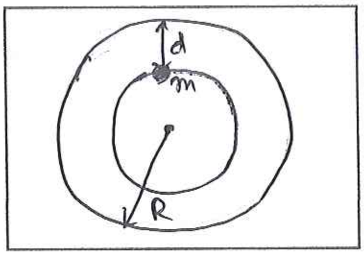
$x_{com} = \frac{Ml+Ml+0}{3M} = \frac{2}{3}l$

And $y_{com} = \frac{0+Ml+Ml}{3M} = \frac{2}{3}l$

Hence the coordinate of the COM is $\left(\frac{2}{3}l, \frac{2}{3}l\right)$

So, the required distance $= \sqrt{\left(\frac{2}{3}l - 0\right)^2 + \left(\frac{2}{3}l - 0\right)^2} = \frac{2\sqrt{2}}{3}l$

15. Derive the expression for variation of acceleration due to gravity with depth from earth's surface.



Ans: Refer to the fig

$mg_d = \frac{G\frac{4}{3}\pi(R-d)^3\rho.m}{(R-d)^2}$

Or, $g_d = G\frac{4}{3}\pi(R-d)\frac{M}{\frac{4}{3}\pi R^3}$

$= GM\frac{(R-d)}{R^3}$

$= gR^2\frac{(R-d)}{R^3} = g\left(1 - \frac{d}{R}\right)$

Or

Calculate the work done in displacing 'm' mass from a point R distance above the earth's surface to another point 3R above earth's surface. M and R are the mass and radius of earth.

Ans: ATP, the mass has to be displaced to 4R distance from 2R distance w.r.t the centre of earth.

The potential energy of the mass at 2R distance is $V_a = -\frac{GMm}{2R}$

That at 4R distance is $V_b = -\frac{GMm}{4R}$

The work done = difference in potential energy

$$= V_b - V_a = -\frac{GMm}{4R} - \left(-\frac{GMm}{2R}\right) = \frac{GMm}{4R} = \frac{gR^2m}{4R} = \frac{mgR}{4}$$

16. The ratio of angular frequencies of revolution of two identical satellites placed at two different orbits around earth is 8:1. If the first one is at a distance 'r' from the centre of earth, calculate the separation between two satellites in terms of r.

Ans: From Kepler's law, $T^2 \propto r^3$

Or, $\frac{2\pi}{w} \propto r^{\frac{3}{2}}$

Or, $w \propto \frac{1}{r^{\frac{3}{2}}}$

Now, ATP, $\frac{w_1}{w_2} = \frac{8}{1} = \left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$

Or, $r_2 = 4r_1$ but, $r_1 = r$

Hence the shortest distance of the two satellites $= r_2 - r_1 = 4r - r = 3r$

17. With the help of first law of thermodynamics prove that $c_p - c_v = R$ for 1mole ideal gas.

Ans: $dQ = dU + PdV$---------(1)

$c_v = \left.\frac{dQ}{dT}\right|_v = \frac{dU}{dT} + 0$ $[from\ 1st\ law\ and\ as\ dV = 0\ for\ constant\ volume]$ ------- (2)

$c_p = \left.\frac{dQ}{dT}\right|_p = \frac{dU}{dT} + P\left.\frac{dV}{dT}\right|_p = c_v + P\left.\frac{dV}{dT}\right|_p$ ------$by\ (1)\ and\ (2)$

Now, for 1mole ideal gas, $PV = RT$ and at constant pressure, $P\left.\frac{dV}{dT}\right|_p = R$ ------ (3)

So, c_p becomes, $c_p = c_v + P\left.\frac{dV}{dT}\right|_p = c_v + R$

Hence $c_p - c_v = R$

Or

Prove $\gamma = 1 + \frac{2}{f}$. $\gamma = \frac{c_p}{c_v}$ *and f* is the degree of freedom of gas molecules.

<u>Ans:</u> For 1 mole of ideal gas, the total internal energy $U = \frac{f}{2}RT$ [from the law of equipartition of energy]

Now, $c_v = \frac{dU}{dT} = \frac{f}{2}R$ and $c_p = c_v + R = \frac{f}{2}R + R = R\left(1 + \frac{f}{2}\right)$

So, $\gamma = \frac{c_p}{c_v} = \frac{R\left(1+\frac{f}{2}\right)}{\frac{f}{2}R} = 1 + \frac{2}{f}$

18. Keeping the temperature constant at T Kelvin, volume of n mole of ideal gas is changed from V_i to V_f. Calculate the thermodynamic work done in this process.

<u>Ans:</u> $dW = PdV$ also for n mole ideal gas, PV= nRT

So, $W = \int_{v_i}^{v_f} PdV = \int_{v_i}^{v_f} n\frac{RT}{V}dV = nRT\,[lnV]_{v_i}^{v_f} = nRT\,ln\frac{v_f}{v_i}$

<u>Group-D</u>

Answer the following questions. (Alternatives are to be noted): **(5x3=15)**

19. a)What is the importance of null vector?

b) Find out the unit vector perpendicular to the plane contained by the **Z-axis** and the vector $\hat{\imath} + \hat{\jmath}$.

c) A ball is projected from the top of a tower with an initial velocity of 10m/s at an angle of 30^0 with the horizontal. it hits the ground at a distance of 17.3m from the base of tower. Calculate the height of the tower. (take $g = 10m/s^2$ and $\sqrt{3} = 1.73$) 1+1+3

Ans: a) It is the additive identity element in the set of all vectors or in vector field .

b)The unit vector along Z-axis is $\hat{k}$. So, one vector perpendicular to the said plane will be $= \hat{k} \times (\hat{\imath} + \hat{\jmath}) = \hat{\jmath} - \hat{\imath}$

And the unit vector of this vector will be $= \frac{-\hat{\imath}+\hat{\jmath}}{|-\hat{\imath}+\hat{\jmath}|} = \frac{1}{\sqrt{2}}\,(-\hat{\imath} + \hat{\jmath})$

Note: Another unit vector can be just the anti parallel vector of this i.e. $\frac{1}{\sqrt{2}}\,(\hat{\imath} - \hat{\jmath})$

c) Refer to the diagram, the range of the projectile part is

$R = \frac{u^2 Sin 60^0}{g} = 5\sqrt{3}$ m

so, AB = $17.3m - 5\sqrt{3}\,m = (10\sqrt{3} - 5\sqrt{3}\,)m = 5\sqrt{3}\,m$

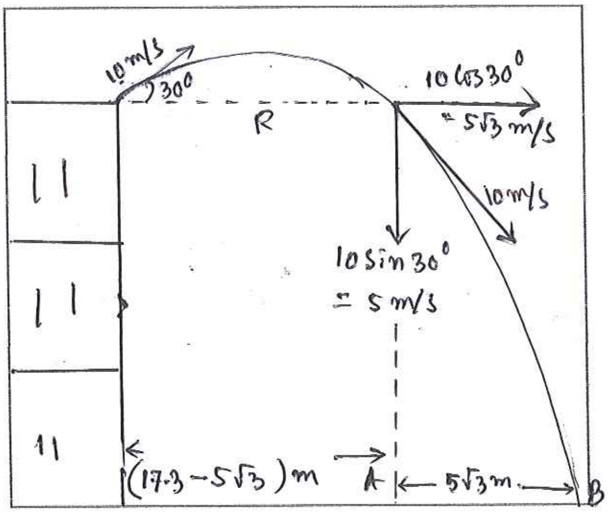
After this complete projectile motion,

Time taken to travel AB path horizontally = time taken to fall h height vertically.

Now, time taken to travel AB path $= \frac{5\sqrt{3}}{u\ Cos 30^0} = \frac{5\sqrt{3}}{\frac{10\sqrt{3}}{2}} = 1\ sec$

So the body has come down h height in 1 sec with initial velocity $u\ Sin30^0 = \frac{10}{2}m/s = 5m/s$

Hence $h = (5 \times 1) + \frac{1}{2} \times 10 \times 1^2\ m\ = 10\ m$



Or

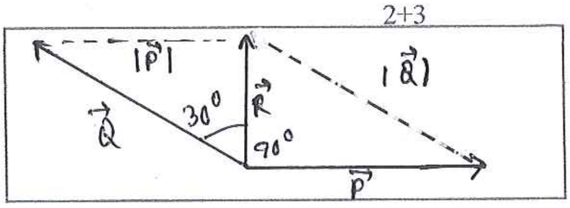
a) Resultant of $\vec{P}$ ($|\vec{P}| \neq 0$) and another unknown vector making an angle 120^0 with $\vec{P}$ is perpendicular to $\vec{P}$. Determine the magnitude of that unknown vector in terms of $|\vec{P}|$.

b) A person sees a train leaving the station from rest with a constant acceleration of 2 m/s². He started running with a constant velocity to catch the train which was 9m ahead of him at that moment and finally he catches the train. Find out his velocity. 2+3

<u>Ans:</u> a) Refer to the figure, if $\vec{Q}$ be the unknown vector,

then $\frac{|\vec{P}|}{|\vec{Q}|} = Sinn\ 30^0$

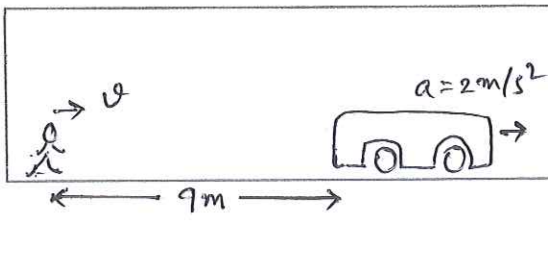
So, $|\vec{Q}| = 2|\vec{P}|$



b) Let he catches the train after t seconds running with a constant velocity $v\ m/s$.

So at that instant, the velocity of the man will be equal to the velocity of the train.

The train started from zero with acceleration $2m/s^2$. Hence after t time the speed of the train will be



$= 0 + 2t =$ velocity of the man $= v$

Also, distance travelled by man – dist travelled by train = 9 m

Or, $vt - \left(o.t + \frac{1}{2}.2.t^2\right) = 9.$

Or. $2t.t - t^2 = 9$

Or, $t^2 = 9 \quad so, \ t = 3$

Hence the velocity of the man $= 2t = 2 \times 3\, m/s = 6\, m/s$

20. a) Define stress.

b) Write down Hook's law. What do you mean by permanent set of a body?

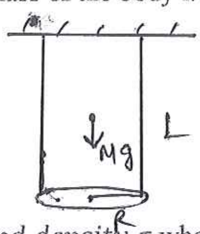
c) A cylindrical rod of length L, radius R and mass M is hung vertically with one of its ends fixed with rigid ceiling. If the rod is elongated due to its own weight, calculate the increase in length. Young's modulus of the material of the rod is Y. 1+(1+1)+2

Ans: a) The restoring force developed per unit area of a body when body undergoes external applied force that tends to deform the body, is known as stress.

b) Within the elastic limit, the stress is directly proportional to strain.

If stress is increased beyond the elastic limit, then the body will not get back its own original shape and gains a permanent deformation (strain) even if the external force is withdrawn. This stage of the body is known as the permanent set.

c)As the rod is elongated because of its own weight which acts at the centre of mass of the body i.e. $\frac{L}{2}$ distance below the supporting end, so, the Young's modulus will be



$Y = \frac{Mg}{\pi R^2} . \frac{l}{\left(\frac{L}{2}\right)}$ where , l is the elongation.

Or, $l = \frac{Mgl}{2\pi R^2 Y}$

Or

a) Derive the expression for the terminal velocity of a spherical body of radius r *and density* σ when falling under gravity through a viscous medium of density ρ and coefficient of viscosity η.

b) Two identical soap bubble of radii R and surface tension S have excess pressure P inside both. If they coalesce to form a single soap bubble, then what will be the excess pressure of this bigger bubble in terms of P? 3+2

Ans: The body achieves terminal velocity when

$$Viscous\ force + Buoyant = weight\ of\ the\ body$$

Or, $6\pi r\eta v_t + \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi r^3 \sigma$

Or, $v_t = \frac{2}{9}\frac{r^2(\sigma-\rho)}{\eta} g$

b) Excess pressure inside one bubble at initial condition is $P = \frac{4S}{R}$.

When they coalesce to form a single bubble, let the radius be R_1. Then, $2R^3 = R_1^3$

Or, $R_1 = 2^{\frac{1}{3}}R$. Hence the excess pressure inside in this case will be $P_1 = \frac{4S}{R_1} = \frac{4S}{2^{\frac{1}{3}}R} = 2^{-\frac{1}{3}}P$

21. a) What do you mean by stationary wave and how is it produced?

b) In a transverse progressive wave of amplitude A, the maximum particle velocity is 4 times its wave velocity. Determine the wave length of the wave. (1+1)+3

Ans: a) The wave which is not progressive in nature and confined within a particular boundary is known as stationary wave.

When two waves of same amplitude and same frequency superimpose travelling from opposite direction with opposite phase, then progressive wave is generated.

b)let $y = A\, Sin\,(wt - kx)$ be the wave equn.

So, the particle velocity $v = \frac{dy}{dt} = Aw\, Cos(wt - kx)$

So, maximum particle velocity $v_m = Aw$

Also the wave velocity $v_w = \frac{w}{k} = \frac{w\lambda}{2\pi}$

ATP, $v_m = 4v_w$

Or, $Aw = 4\frac{w\lambda}{2\pi}$ or, $\lambda = \frac{\pi A}{2}$

Or

a) What do you mean by beats?

b) The equation of motion of a particle executing S.H.M is $x = a\sin\left(wt + \frac{\pi}{6}\right)$ with time period T. Find out the time interval at which the velocity is being half of its maximum value.

c) Two vibrations simultaneously act on the same point and cause the disturbances as $y_1 = a\sin wt$ $and\ y_2 = a\sin\left(wt + \frac{\pi}{2}\right)$. Determine the maximum resultant displacement of the point. 1+2+2

Ans: a) When two waves of slightly different frequencies travelling along same path in the same direction in a medium superpose upon each other, the intensity of the resultant wave at any point becomes maximum and minimum periodically with a constant time interval. This periodic variation of intensity is known as beats.

b) $x = a\sin\left(wt + \frac{\pi}{6}\right)$

So, $v = aw\cos\left(wt + \frac{\pi}{6}\right)$ so, $v_m = aw$

ATP, $aw\cos\left(wt + \frac{\pi}{6}\right) = \frac{aw}{2}$

Then, $wt + \frac{\pi}{6} = \frac{\pi}{3}, \ \frac{2}{3}\pi, \ \frac{4}{3\pi}, \ \frac{5}{3}\pi$ etc

So, the interval of phase = $either\ \frac{\pi}{3}\ or\ \frac{2}{3}\pi$

Hence the interval of time = either $\frac{T}{6}$ or $\frac{T}{3}$

c) resultant $y = y_1 + y_2 = a\sin wt + a\sin\left(wt + \frac{\pi}{2}\right) = a.2.\sin\left(wt + \frac{\pi}{4}\right)\ \cos\frac{\pi}{4}$

so, $y = \frac{2a}{\sqrt{2}}\sin\left(wt + \frac{\pi}{4}\right)$

hence the maximum resultant disp is $\sqrt{2}a$