ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

First Term Examination – 2018

Class : 10

SUB : Mathematics F.M.: 75

DURATION: 2 Hrs30Mins DATE:28.04.2018

Group-A

1.Choose the correct answer. 1x4=4

i)The length of the radius of a circle is 13 cm. and length of a chord of the circle is 10 cm. The distance of the chord from the centre of the circle is

a)12.5cm b)12cm c)24 cm d)$\sqrt{69}$ cm

ii)If the products of two roots of the equation $x^2 - 3x + k = 10$ is -2 then value of k is

a)-2 b)-8 c)8 d)12

iii)The mean proportional of 16 and 25 is

a)400 b)100 c)20 d)40

iv) The end point of the minute hand of a clock rotates in 1 hr

a) $\frac{\pi}{4}$ radian b)$\frac{\pi}{2}$ radian c) π radian d) 2π radian

2. Answer the following. 1x5=5

i)Find out compound ratio of $ab:c^2$, $bc:a^2$ and $ca:b^2$.

ii)Find out circular value of 22°30'

iii)If $x = 2+\sqrt{3}$ then find the value of $x+\frac{1}{x}$=?

iv) If two roots of the equation $3x^2 + 8x + 2 = 0$ be α and β then the value of $(\frac{1}{\alpha} + \frac{1}{\beta})$=?

v)If A:B=2:3 , B:C=4:5 and C:D=6:7 Find out A:D=?

3.Write down True or False. 1x5=5

i) $\sqrt{75}$ and $\sqrt{147}$ are similar surds.

ii)Angle in a semicircle is always a right angle.

iii) a is positive number and if $a:\frac{27}{64} = \frac{3}{4}:a$, then value of a is 9

iv)$\sin^2 45° + \cos^2 45° = 1$

v)Sum of opposite angles of a cyclic quadrilateral is complementary.

Group-B

4.Answer the following. 2x8=16

i)If the surface area of a right circular cylindrical pillar is 264 sq metre and volume is 92 cubic metre. Find out the height and length of the diameter of the pillar.

ii)If x:a=y:b=z:c,Prove that $\frac{x^3+y^3+z^3}{a^3+b^3+c^3}=\frac{xyz}{abc}$

iii)What is the rate of simple interest per annum, when the interest of some money in 4 yrs will be $\frac{8}{25}$ part of the principal—determine it.

iv)Two equal circles of radius 10 cm intersect each other and the length of their common chord is 12 cm. Determine the distance between two centres of the two circles.

v) The length of the dimensions of two cuboids are 4 , 6 , 4 units and 8 , (2h-1), 2 units respectively .If the volume of two cuboids are equal. Find the value of h.

vi)If $m+\frac{1}{m}=\sqrt{3}$ find the value of $m^2+\frac{1}{m^2}=?$

vii)If the height of a right circular cylinder is 14 cm. and lateral surface area is 264 sq cm. determine the volume of the cylinder.

viii)Express cotθ and secθ interms of sinθ.

Group-C

5. Answer the following. (any 9) 5x9=45

i)If two roots of the quadratic equation $5x^2+2x-3=0$ are α and β then determine the value of i) $\alpha^2+\beta^2$ ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ 2+3

ii)Rothin babu deposits the money for each of his two daughters in such a way that when the ages of each of his daughters will be 18 years. Each one will get Rs1,20,000. The rate of simple interest in the bank is 10% per annum and the present ages of his daughters are 13 yrs and 8 yrs respectively. Determine the money he had deposited separately in the bank for each of his daughters.

iii)If $x=\frac{8ab}{a+b}$, determine the value of $\left(\frac{x+4a}{x-4a}+\frac{x+4b}{x-4b}\right)$

iv)calculate the amount on Rs 5000 at the rate of 8% compound interest per annum for 3 years.

v)If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$ calculate $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

vi)Prove that the opposite angles of a cyclic quadrilateral are supplementary.

vii)ABCD is a parallelogram. A circle passing through the points A and B intersects the sides AD and BC at the points E and F respectively. Prove that E,F,C,D are concyclic.

viii) Draw a right angled triangle having two sides 4cm and 8 cm length,containing right angle. Then draw the circumcircle and point out the circumcentre.

ix)Determine the value : $\frac{\tan 60°-\tan 30°}{1+\tan 60°\tan 30°}+\cos 60°\cos 30°+\sin 60°\sin 30°$

x)If $\sin\alpha=\frac{a^2-b^2}{a^2+b^2}$, then show that $\cot\alpha=\frac{2ab}{a^2-b^2}$

Aparajita Mondal
3/5/18

FOR GOD AND COUNTRY

ST. LAWRENCE HIGH SCHOOL

A Jesuit Christian Minority Institution

First Term Examination– 2018

Sub: Mathematics Model Answer **Class: X** **F. M. 75**

Duration:3 hrs **Date: 28/04/2018**

Group-A

1.Choose the correct answer. **1x4=4**

i)The length of the radius of a circle is 13 cm. and length of a chord of the circle is 10 cm. The distance of the chord from the centre of the circle is

b)12cm

ii)If the products of two roots of the equation x^2- 3x +k =10 is -2 then value of k is

a) 8

iii)The mean proportional of 16 and 25 is

c)20

iv) The end point of the minute hand of a clock rotates in 1 hr

d) 2π radian

2. Answer the following. **1x5=5**

i)Find out compound ratio of ab:c^2, bc:a^2 and ca:b^2.

Ans: 1:1

ii)Find out circular value of 22°30'

Ans.$\frac{\Pi}{8}$ radian

iii)If x= $2+\sqrt{3}$ then find the value of x+ $\frac{1}{x}$=?

Ans.4

iv) If two roots of the equation $3x^2$ +8x +2=0 be α andβ then the value of ($\frac{1}{\alpha}$+ $\frac{1}{\beta}$)=?

Ans. -4

v)If A:B=2:3 , B:C=4:5 and C:D=6:7 Find out A:D=?

Ans. $\frac{16}{35}$

3.Write down True or False. 1x5=5

i) $\sqrt{75}$ and $\sqrt{147}$ are similar surds. True

ii)Angle in a semicircle is always a right angle. True

iii) a is positive number and if a:$\frac{27}{64}$= $\frac{3}{4}$:a , then value of a is 9, False

iv)$Sin^2 45^\circ + cos^2 45^\circ = 1$ True

v)Sum of opposite angles of a cyclic quadrilateral is complementary. False

Group-B

4.Answer the following. 2x8=16

i)If the surface area of a right circular cylindrical pillar is 264 sq metre and volume is 92 cubic metre. Find out the height and length of the diameter of the pillar.

Ans.According to the problem

$\frac{Volume}{lateral\ surface\ area} = \frac{\Pi r^2 h}{2\Pi r h} = \frac{92}{264}$ or r= $\frac{23}{33}$m

h= $\frac{264}{2\Pi r} = \frac{1386}{23}$ m

ii)If x:a=y:b=z:c,Prove that $\frac{x^3 + y^3 + z^3}{a^3 + b^3 + c^3} = \frac{xyz}{abc}$

Ans.let $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ =k

X=ak, y=bk, z=ck

LHS- $\frac{x^3 + y^3 + z^3}{a^3 + b^3 + c^3} = \frac{k^3(a^3 + b^3 + c^3)}{a^3 + b^3 + c^3} = k^3$

RHS- $\frac{xyz}{abc} = \frac{ak.bk.ck}{a.b.c} = k^3$

therfore LHS=RHS

iii)What is the rate of simple interest per annum, when the interest of some money in 4 yrs will be $\frac{8}{25}$ part of the principal—determine it.

Ans.Let the principal be P and rate of interest be r

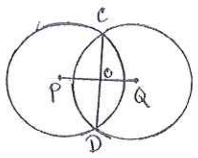
According to the problem

$\frac{4pr}{100} = \frac{8}{25} \times P$

Therefore r=8%

iv)Two equal circles of radius 10 cm intersect each other and the length of their common chord is 12 cm. Determine the distance between two centres of the two circles.

Ans



CO is perpendicular on PQ. Radius=PC=10 cm. OC=12/2=6 cm

OP^2= 100-36=64 Therfeore OP=8 cm ,OP+OQ= 8cm+8cm=16 cm

v) The length of the dimensions of two cuboids are 4 , 6 , 4 units and 8 , (2h-1), 2 units respectively .If the volume of two cuboids are equal. Find the value of h.

Ans. According o the problem

8 x (2h-1) x 2= 4 x 6 x 4

Or, 2h-1=6

Or h= 7/2= 3.5

vi)If $m+\frac{1}{m} = \sqrt{3}$ find the value of $m^2 + \frac{1}{m^2}$=?

Ans. $m^2 + \frac{1}{m^2} = (m+\frac{1}{m})^2 - 2 \times m \times \frac{1}{m} = (\sqrt{3})^2 - 2 = 1$

vii)If the height of a right circular cylinder is 14 cm. and lateral surface area is 264 sq cm. determine the volume of the cylinder.

Ans.h=14 cm ,Lateral surface area= 264 sq cm

According to the problem

2Πrh=264

Or, $r = \frac{264 \times 7}{2 \times 22 \times 14} = 3$ cm

viii)Express cotθ and secθ interms of sinθ.

Ans $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-\sin^2\theta}}$

Group-C

5. Answer the following. (any 9) 5x9=45

i)If two roots of the quadratic equation $5x^2+2x-3=0$ are α and β then determine the value of i) $\alpha^2+\beta^2$ ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ 2+3

Ans.$\alpha+\beta=-\frac{2}{5}$, $\alpha\beta=-\frac{3}{5}$, i) $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = (-\frac{2}{5})^2 - 2 \times (-\frac{3}{5}) = \frac{34}{25}$

$$\frac{\alpha^3+\beta^3}{\beta\alpha} = \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\beta\alpha} = \frac{(-\frac{2}{5})^3-3\times-\frac{3(-\frac{2}{5})}{5}}{-\frac{3}{5}} = \frac{98}{75}$$

ii)Rothin babu deposits the money for each of his two daughters in such a way that when the ages of each of his daughters will be 18 years. Each one will get Rs1,20,000. The rate of simple interest in the bank is 10% per annum and the present ages of his daughters are 13 yrs and 8 yrs respectively. Determine the money he had deposited separately in the bank for each of his daughters.

Ans. let Rothin babu deposit Rs x for his first daughter(13 yrs old) and Rs y for his 2nd daughter(8 yrs old).

Time for 1st daughter (18-13)=5 yrs

Her amount= $x + \frac{x \times 5 \times 10}{100} = 120000$

Or, $15x = 120000 \times 10$

Or, x=80,000

Time for 2nd daughter is (18-8)=10 yrs

Her amount=$y + \frac{x \times 10 \times 10}{100} = 120000$

Or, 2y= 120000

Or,y= 60,000

iii)If $x=\frac{8ab}{a+b}$, determine the value of $(\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b})$

ans. $x=\frac{8ab}{a+b}$ **or,** $\frac{x}{4a}=\frac{2b}{a+b}$, **or,** $\frac{x+4a}{x-4a}=\frac{3b+a}{b-a}$

Again $x=\frac{2a\,x\,4b}{a+b}$ **or,** $\frac{x}{4b}=\frac{2a}{a+b}$, **or,** $\frac{x+4b}{x-4b}=\frac{3a+b}{a-b}$

$$\frac{x+4a}{x-4a}+\frac{x+4b}{x-4b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}=\frac{3b+a}{b-a}-\frac{3a+b}{b-a}=2$$

iv)calculate the amount on Rs 5000 at the rate of 8% compound interest per annum for 3 years.

Ans,Amount= $P(1+\frac{R}{100})^n$

$=5000(1+\frac{8}{100})^3 = 5000\ x\ (\frac{27}{25})^3$**= Rs 6298.56**

v)If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ **and** $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$ **calculate** $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

Ans.

$$a+b=\frac{\sqrt{5}+1}{\sqrt{5}-1}+\frac{\sqrt{5}-1}{\sqrt{5}+1}=\frac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)}$$

$$=\frac{5+1+2\sqrt{5}+5+1-2\sqrt{5}}{5-1}=\frac{12}{4}=3$$

$$\&\ a-b=\frac{\sqrt{5}+1}{\sqrt{5}-1}-\frac{\sqrt{5}-1}{\sqrt{5}+1}=\frac{(\sqrt{5}+1)^2-(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)}$$

$$=\frac{(5+1+2\sqrt{5})-(5+1-2\sqrt{5})}{5-1}=\frac{5+1+2\sqrt{5}-5-1+2\sqrt{5}}{4}=\frac{4\sqrt{5}}{4}=\sqrt{5}$$

$$ab=\frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\sqrt{5}-1}{\sqrt{5}+1}=1.$$

$$\frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{(a+b)^2-ab}{(a+b)^2-3ab}=\frac{3^2-1}{3^2-3x1}=\frac{9-1}{9-3}=\frac{8}{6}=\frac{4}{3}$$

vi)Prove that the opposite angles of a cyclic quadrilateral are supplementary.

Ans.

Let us prove with reason

Therom : 38. The opposite angles of a cyclic quadrilateral are supplementary.

Given : ABCD is a cyclic quadrilateral of a circle with centre O.

To prove : $\angle ABC + \angle ADC = 2$ right angles
and $\angle BAD + \angle BCD = 2$ right angles

Construction : A, O and C, O are joined

Proof : The reflex angle $\angle AOC$ at the centre and the angle $\angle ABC$ on the circle are forme with the circular are ADC.

$\therefore$ Reflex $\angle AOC = 2\angle ABC$

$\therefore \angle ABC = \frac{1}{2}$ reflex $\angle AOC$ .....(i)

Again, $\angle AOC$ is the angle at the centre and $\angle ADC$ is the angle on the circle formed with circular arc ABC.

$\therefore \angle AOC = 2\angle ADC$

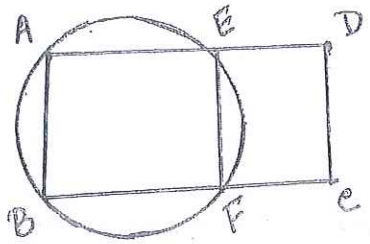
$\therefore \angle ADC = \frac{1}{2}\angle AOC$ .....(ii)

$\therefore$ From (i) and (ii), we get $\angle ABC + \angle ADC = \frac{1}{2}$ reflex $\angle AOC + \frac{1}{2}\angle AOC$

$= \frac{1}{2}$ (reflex $\angle AOC + \angle AOC$)

$= \frac{1}{2} \times 4$ right angles $= 2$ right angles

vii)ABCD is a parallelogram. A circle passing through the points A and B intersects the sides AD and BC at the points E and F respectively. Prove that E,F,C,D are concyclic.



ABCD is a parallelogram. Angle BAD + angle ADC = 2rt angles

Again ABEF is a cyclic quadrilateral

Angle BAE + angle BFE= 2rt angles

But angle BFE + angle CFE = 2 rt angles

Angle EFC = angle BAE , angle EFC= angle BAD

Angle EFC + angle ADC= 2 rt angles

Angles EFC + angle EDC=2 rt angles Therefore EFCD is a cyclic quadrilateral.

viii) Draw a right angled triangle having two sides 4cm and 8 cm length,containing right angle. Then draw the circumcircle and point out the circumcentre.

Ans. Step1: Draw the right angled Triangle with the given measure

Step2: Draw perpendicular bisectors on the perpendicular and base of the triangle.

Step3: Taking intersecting point of two perpendicular bisectors as centre now a circle will be drawn and the circle will be passing through the vertices of the triangle.

Circumcentre will be lying on exactly mid point of the hypotenuse of the triangle.

ix)Determine the value : $\frac{tan\,60°-tan30°}{1+tan60°tan30°} + \cos60°\cos30° + \sin60°\sin30°$

Ans. $\frac{tan\,60°-tan30°}{1+tan60°tan30°} + \cos60°\cos30° + \sin60°\sin30° = \frac{\sqrt{3}-1/\sqrt{3}}{1+\sqrt{3}x\,1/\sqrt{3}} + \frac{1}{2}x\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}x\frac{1}{2} = \frac{5}{2\sqrt{3}}$

x)If $\sin\alpha=\frac{a^2-b^2}{a^2+b^2}$, then show that $\cot\alpha=\frac{2ab}{a^2-b^2}$

Ans.$\cos^2\alpha=$ 1- $\sin^2\alpha=1-(\frac{a^2-b^2}{a^2+b^2})^2=\frac{(a^4+b^4+2a^2\ b^2\ )-(a^4+b^4-2a^2\ b^2\ )}{(a^2+b^2)^2}=\frac{4a^2\ b^2}{(a^2+b^2)^2}$

$\cos\alpha=\frac{2ab}{(a^2+b^2)}$ $\cot\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{2ab}{(a^2+b^2)}$ / $\frac{a^2-b^2}{a^2+b^2}=\frac{2ab}{a^2-b^2}$