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ST. LAWRENCE HIGH SCHOOL

A Jesuit Christian Minority Institution

Selection Test– 2018

Sub: Mathematics Model Answer **Class: X** **F. M. 90**

Date: 15/11/2018

1.Choose the correct answer.

i) Compound ratio of $ab:c^2$, $bc:a^2$ and $ca:b^2$ is b) 1:1

ii)A principal becomes twice of its amount in 20 years at a certain rate of simple interest. At the same rate of simple interest that principal becomes thrice of the amount in c) 40 years

iii)which smallest factor we should multiply with the denominator to rationalize the denominator of $7\div\sqrt{48}$ c)$\sqrt{3}$

iv)If each of radius and height of a cone is increased by twice of its length, then the volume of it will be d) 8 times

v)The circular value of each internal angle of a regular hexagon is b)$2\pi/3$

vi)The length of radius of a circle with centre o is 5 cm. P is a point at the distance of 13 cm from the point o. The length of two tangents are PQ and PR from the point P.The area of the quadrilateral PQOR is a)60 sq cm

2. Fill up the blanks.(any 5)

i) If numerical value of curved surface area of a solid sphere is three times of its volume , the length of the radius is 1 unit.

ii)If 35 is removed from the data 30, 34, 35 ,36 ,37,38,39 and 40 then median increases by 0.5

iii)A person goes 24 m west from a place and then he goes 10 m north . The distance of the person from the starting point is 26 m

iv) The value of x for 10:35::x:42 is 12

v) Value of $\sin^2 45^o - \text{cosec}^2 60^o + \sec^2 30^o =$ 1/2

vi) If x, 12, y,27 are in continued proportion, Then positive value of x is 8

3. Write True or False.(any 5)

i) If A+B=90° and tanA=$\frac{3}{4}$, then value of cotB is $\frac{3}{4}$. True

ii)Two figures are similar means they are congruent too. False

iii) If $x \propto y^2$ and y=4 when x=8, If x=32 then the value of y is 38. False

iv)If P+Q =$\sqrt{13}$ and P-Q=$\sqrt{5}$ then the value of PQ is 2. True

v) If the lengths of radii of two solid right circular cylinders are in the ratio 2:3 and their heights are in the ratio 5:3 , Then ratio of their lateral surface area is 10:11. False

vi) The roots of the equation x^2-x+2 =0 are not real. True

4. Answer the following questions. (any 10)

i) Find the values of a and b , if the roots of the equation ax^2+bx+35=0 are -5 and -7.

Ans. -b/a= -12 and 35/a=35 therefore a=1 and b=12

ii) If the length of each of two parallel chords are 16 cm.If the length of the radius of the circle is 10 cm. Find out distance between two chords.

Ans. Perpendicular drawn from the centre on the chord bisects the chord. If AB and CD two chords on opposites sides of the centre then half of the chord is 8 cm. Therefore perpendicular distance from thr centre to both the chords are $\sqrt{10^2 - 8^2}$= 6cm. Therefore distance between two chords is 12 cm.

iii)If the height of a right circular cylinder is 14 cm and lateral surface area is 264 sq cm. Write the volume of the cylinder.

Ans.2πrh=264 or r= 3

Volume= $\pi r^2 h$= 22/7 x 3x3 x 14= 396cu cm.

iv) Rationalise the denominator $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Ans. $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})\ \sqrt{5}-\sqrt{3}} = 4 + \sqrt{15}$

v)AB is the diameter of a circle with centre o. ABCD is a cyclic quadrilateral in which AB|| CD and if <BAC=25° then find out <DAC.

Ans Angle ACB=90^o , Angle ACD= 25^o Therefore Angle BCD=115^o. since angle BAD+ angle BCD=180^o , angle CAD=180^o-115^o -25^o= 40^o

vi)Find out total surface area of a cone whose length of the radius of the base is 1.5 cm and slant height is 2.5 cm.

Ans. Total surface area= 22/7 x 1.5 (1.5 +2.5) = 18.86 sq cm.

vii)Express 85.12^o into degrees, minutes and seconds.

Ans . 85^o+ (.12 x 60) = 85^o+ 7.2′ =85^o+ 7′ + (.2 x 60)=85^o 7′ 12 ″

viii) Prove that: $\frac{2tan30^o}{1-tan^2 30}=\sqrt{3}$

Ans. $\frac{2/\sqrt{3}}{1-1/3}=\sqrt{3}$

ix) Find out mode of the given data 102, 104, 117, 102, 118, 120, 104, 122, 102

Ans. 102 has the highest frequency. Mode=102

x) If x= acosθ and y=bsinθ, eliminate θ from these two relations.

Ans. $Sin^2\theta + cos^2\theta= (x/a)^2+ (y/b)^2=1$

xi) In triangle ABC if AB=(2a-1)cm, AC=$2\sqrt{2a}$ and BC= (2a+1) cm write the value of <BAC

Ans. $(2a-1)^2+(2\sqrt{2a})^2=(2a+1)^2$ or, $AB^2+ AC^2= BC^2$ Therefore ABC is a right triangle having A =90^o

xii)If α+β=90^o, show that $\frac{sec\alpha}{cos\alpha}$ - $cot^2\beta=1$

Ans. Cosec β/ sin β - $cot^2\beta= cosec^2\beta - cot^2\beta=1$

5. Answer any one question.

i)A weaver cooperative society takes a loan of Rs 15000 at the time of buying a power loom .After 5 years the society has to repay Rs 22125 for recovering the loan. What is simple rate of interest per annum?

Ans. Interest = Rs 22125 – Rs 15000= Rs 7125 ,Time =5 yrs

rate =(interest x 100)/(principal x time)=(7125 x 100)/ (15000 x 5)=9.5 %

ii)Two friends invested Rs 40000and Rs 50000respectively to start a business. They made a contract that they would divide 50% of the profit equally among themselves

and the remaining profit in the ratio of their capitals. Find out share of their profit when first friend's profit is Rs800 less than that of the second friend.

Ans let total profit be Rs x.

50% of x= Rs x/2 Each will get Rs x/4

Remaiming (x-x/2)=Rs x/2 will be divided in the ratio of 4:5

1st friend will get in total= Rs 2x/9 + Rs x/4 =17x/36

2nd friend will get in total = rs 5x/18+ Rs x/4=19x/36

According to the problem.

19x/36 – 17x/36=800 or, x=14400, 1st friend will get = Rs 6800

6.Answer any one question.

i)A superfast train runs having the speed 15km/hr more than that of an express train .Leaving same station the superfast train reached at a station of 180 km distance 1 hour before than the express train. Determine the speed of the superfast train.

Ans.Let the speed of the super fast train be x km/hr, that of express train be (x-15) km /hr

According to the problem

$180/(x-15) - 180/x = 1$ or, $180 [1/(x-15) - 1/x]= 1$ or, $x^2-15x=15 \times 180$

Or, $x^2-60x+ 45x -2700=0$ or, $(x-60)(x+45)=0$ therefore $x=60$

ii)Find the value of K in $x^2-2(5+2k)x+ 3(7+10k)=0$ when it has real and equal roots.

Ans.$[-2(5+2k)]^2- 4.1.3(7+10k)= 0$

Or, $4 (25 +20k+4k^2)- 12 (7+10k)=0$ or , $16k^2-40k +16=0$ OR, $(K-2)(2K-1)=0$

Therefore k= 2 or 1/2

7. Answer any one question.

i)If volume of a sphere varies directly with cube of length of its radius and surface area varies directly with the square of the length of the radius. Prove that the square of volume of sphere varies directly with cube of its surface area.

Ans,let volume of the sphere be V, Radius =R and surface area=S

$V=mR^3$, $s=nR^2$

$R^3=V/m$ or, $R^2= s/n$

$R=(v/m)^{1/3}$ or,$R^2= (v/m)^{2/3}$ or, $S/n=(v/m)^{2/3}$ or, $s= n\,(v/m)^{2/3}$ or, $s^3=n^3\,v^2/m^2$

or, $v^2 \alpha\, s^3$

ii)If $x=2+\sqrt{3}$, $y=2-\sqrt{3}$ calculate the value of a) $x^3-\frac{1}{x^3}$ b) $y^2+\frac{1}{y^2}$

ans. $X=2+\sqrt{3}$, $1/x= 2-\sqrt{3}$, a) $x^3-\frac{1}{x^3}= (x- 1/x)^3+ 3.x.1/x(x-1/x)=(2\sqrt{3})^3+3.1.2\sqrt{3}=30\sqrt{3}$

b) $y^2+\frac{1}{y^2}=(y+1/y)^2-2.y.1/y=4^2-2=14$

8.Answer any one question.

i)If $\frac{x}{y}=\frac{a+2}{a-2}$, show that $\frac{x^2-y^2}{x^2+y^2}=\frac{4a}{a^2+4}$

Ans. $\frac{x}{y}=\frac{a+2}{a-2}$ or, $\frac{x^2-y^2}{x^2+y^2}=\frac{(a+2)^2-(a+2)^2}{(a+2)^2+(a+2)^2}$ or, $\frac{x^2-y^2}{x^2+y^2}=8a/2(a^2+4)=4a/(a^2+4)$

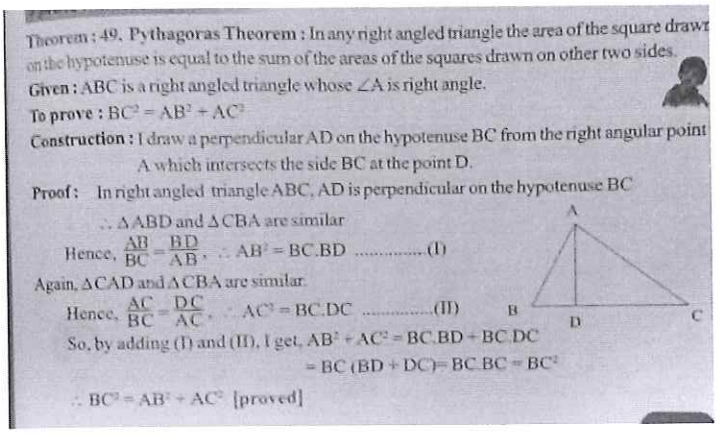
ii) $\frac{a+b}{b+c}=\frac{c+d}{d+a}$, prove that c=a or a+b+c+d=0

Ans $\frac{a+b+c+d}{d+c}=\frac{c+d+a+b}{d+a}$ or, $\frac{a+b+c+d}{d+c}-\frac{c+d+a+b}{d+a}=0$ or,$(a+b+c+d)\,[1/(c+d)\ -1/(a+d)]= 0$

$a+b+c+d=0$ or $c+d= a+d$ or,$c=a$

9.Answer any one question.

i)Prove that in any right angled triangle the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on other two sides.



Theorem : 49. Pythagoras Theorem : In any right angled triangle the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on other two sides.

Given : ABC is a right angled triangle whose ∠A is right angle.

To prove : $BC^2 = AB^2 + AC^2$

Construction : I draw a perpendicular AD on the hypotenuse BC from the right angular point A which intersects the side BC at the point D.

Proof : In right angled triangle ABC, AD is perpendicular on the hypotenuse BC

∴ ΔABD and ΔCBA are similar

Hence, $\frac{AB}{BC}=\frac{BD}{AB}$, ∴ $AB^2 = BC.BD$ .....(I)

Again, ΔCAD and ΔCBA are similar.

Hence, $\frac{AC}{BC}=\frac{DC}{AC}$, ∴ $AC^2 = BC.DC$ .....(II)

So, by adding (I) and (II), I get, $AB^2 + AC^2 = BC.BD + BC.DC$

$= BC\,(BD + DC) = BC.BC = BC^2$

∴ $BC^2 = AB^2 + AC^2$ [proved]

ii) Prove that the opposite angles of a cyclic quadrilateral are supplementary.

Let us prove with reason

Therom : 38. The opposite angles of a cyclic quadrilateral are supplementary.

Given : ABCD is a cyclic quadrilateral of a circle with centre O.

To prove : $\angle ABC + \angle ADC = 2$ right angles
and $\angle BAD + \angle BCD = 2$ right angles

Construction : A, O and C, O are joined

Proof : The reflex angle $\angle AOC$ at the centre and the angle $\angle ABC$ on the circle are formed with the circular arc ADC.

$\therefore$ Reflex $\angle AOC = 2\angle ABC$

$\therefore \angle ABC = \frac{1}{2}$ reflex $\angle AOC$ .....(i)

Again, $\angle AOC$ is the angle at the centre and $\angle ADC$ is the angle on the circle formed with the circular arc ABC.

$\therefore \angle AOC = 2\angle ADC$

$\therefore \angle ADC = \frac{1}{2}\angle AOC$ .....(ii)

$\therefore$ From (i) and (ii), we get $\angle ABC + \angle ADC = \frac{1}{2}$ reflex $\angle AOC + \frac{1}{2}\angle AOC$

$= \frac{1}{2}$ (reflex $\angle AOC + \angle AOC$)

$= \frac{1}{2} \times 4$ right angles $= 2$ right angles

Similarly by joining B, O and D, O; it can be proved that, $\angle BAD + \angle BCD = 2$ right angles (proved)

10. Answer any one question.

i) Mohit Drew two straight lines through any point x exterior to a circle to intersect the circles at points A ,B and C,D respectively. Prove that logically triangle XAC and triangle XBD are equiangular.

Ans. From an external point x ,two straigfht lines are drawn which cut the circle at A,B and C, D.

To prove- Triangle XAC and XBD are equiangular.

A,C and B,D are joined.

Proof-ABDC cyclic quadrilateral. $\angle ABD + \angle ACD = 180^o$ and

$\angle ACD + \angle ACX = 180^o$ therefore $\angle ABD = \angle ACX$. Again $\angle BDC + \angle BAC = 180^o$ and

$\angle BAC + \angle CAX = 180^o$ therefore $\angle BDC = \angle CAX$ and $\angle AXC = \angle BXD$ (common angle) therefore triangles XAC and XBD are equiangular.

ii)Prove that the sum of the area of the squares drawn on the sides of a rhombus is equal to the sum of the area of the squares drawn on two diagonals.

Ans. AC and BD are two diagonals af a rhombus ABCD.AC and BD are intersecting each other at o. to prove $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

ACis perpendicular on BD, AO=OC and BO=OD.

AOB, BOC,COD,AOD are right angled triangles.

$AB^2 = OA^2 + OB^2$, $BC^2 = OB^2 + OC^2$, $AD^2 = OD^2 + OA^2$, $CD^2 = OC^2 + OD^2$

$AB^2 + BC^2 + CD^2 + DA^2 = 2\ (OA^2 + OB^2 + OC^2 + OD^2) = 4(OA^2 + OB^2)$

Again $AC^2+BD^2=(2OA)^2+(2OB)^2=4(OA^2+OB^2)$

therefore $AB^2+BC^2+CD^2+DA^2=AC^2+BD^2$

11. Answer any one question.

i) Draw one triangle in which the length of two sides are 7.6 cm , 6 cm and the angle included by those two sides is 75^o. Draw incircle of the triangle and measure inradius.

Ans.step1- draw one triangleof the sides 7.6 cm and 6 cm.

Strep2- draw bisectors of two base angles of the tyriangle .

Step3- then from the intersecting point say I, of bisectors draw a perpendicular on the base.

Step4- say perpendicular touches the base say at D point,then keeping ID radius draw an incircle that touches all the sides of the triangle.

ii)Draw a circle with radius of 2.8 cm length. Take a point which is at a distance of 7.5 cm from the centre .From that point draw two tangents of the circle.

Ans.step1-a circle with radius 2.8 cm and centre o is drawn.

Step2-now OA=7.5 cm is cut from the ray OY from centre O.

Step3-Now bisect OA .Let the mid point be x

Step4-taking x centre and ox radius a circle is drawn. New circle intersects the previous circle at the points P and Q.

Step5- OP and OQ are joined and extended. OP and OQ are required tangents.

12. Answer any two questions.

i) If $x\sin60^o\cos^230^o=\frac{tan^245^o\ sec60^o}{cosec60^o}$ determine the value of x.

Ans, $x\sqrt{3}/2\ (\sqrt{3}/2)^2=\frac{2}{2/\sqrt{3}}$ or, $x=8/3$

ii) If $\frac{cosec\theta+sin\theta}{cosec\theta-sin\theta}=\frac{5}{3}$, then find out value of $sin\theta$.

Ans. $\frac{cosec\theta+sin\theta}{cosec\theta-sin\theta}=\frac{5}{3}$ or, $\frac{cosec\theta+sin\theta+cosec\theta-sin\theta}{cosec\theta+sin\theta-cosec\theta+sin\theta}=\frac{5+3}{5-3}$ or, $2\text{cosec}\theta/2\sin\theta=4$ or, $\sin2\theta=1/4$

Therefore sinθ=±1/2

iii) $\angle A + \angle B = 90^o$, show that $\text{cosec}^2 22^o \cot^2 68^o = \sin^2 22^o + \sin^2 68^o + \cot^2 68^o$

Ans. LHS-$\text{cosec}^2 22^o \cot^2 68^o = \sec^2 68^o \cot^2 68^o = \text{cosec}^2 68^o$

RHS- $\sin^2 22^o + \sin^2 68^o + \cot^2 68^o = \cos^2 68^o + \sin^2 68^o + \cot^2 68^o = \text{cosec}^2 68^o$

13. Answer any one question.

i) I am flying a kite having the length of thread 250metres, when the thread makes an angle 60^o with horizontal line and when the thread makes an angle of 45^o with the horizontal line, Calculate the height of the kite from me.

Ans. Let A be the position of the kite. AB= length of the thread=250 m.

1st case- $\sin 60^o = AC/AB$ or, $AC = 125\sqrt{3}$ m.

2nd case- $\text{Sin } 45^o = AD/AB$ or, $AD = 125\sqrt{2}$ m.

ii)A passenger of an aeroplane observes that Howrah station is at one side of the plane and saheedminar is just on the opposite side. The angle of depression of Howrah station and saheedminar from the passenger of the aeroplane are 60^o and 30^o respectively . If the aeroplane is at a height of $545\sqrt{3}$ m at that time, find out distance between Howrah station and saheedminar.

Ans.let height of the plane be $AD = 545\sqrt{3}$ m. B and C are the positions of Howrah station and saheedminar respectively.

$\tan 60^o = AD/BD$ or, BD=545 m.

$\tan 30^o = AD/DC$ or, DC=1635m. therefore BC= 545+1635=2180 m.

14. Answer any two questions.

i)If the height and slant height of a cone are 6cm and 10 cm respectively . Determine the total surface area and volume of the cone.

Ans.let the radius be r cm , $r^2 = l^2 - h^2 = 100 - 36 = 64$,r=8

total surface area= 22/7 x 8 (8+10)= 452.57 sq cm.

volume= 1/3 x 22/7 x 8x8 x 6= 402.28 cu cm.

ii)The length of outer and inner diameter of a hollow right circular cylinder are 16 cm and 12 cm respectively. Height of the cylinder is 36 cm. Calculate how many solid cylinders of 2cm radius and 6 cm length can be made by melting this cylinder.

Ans. Volume of hollow cylinder= π [8^2-6^2] x36 cu cm

Volume of x number of solid cylinders=x . π.1^2.6 cu cm.

According to the problem

x . π. 1^2.6 = π [8^2-6^2] x36

or, x=168

iii) Three spheres made of copper having the length of 3cm, 4 cm and 5 cm radii are melted and a large sphere is made . Calculate the length of radius of the large sphere.

Ans. .Volume of 3 small spheres= 4/3 π (3^3+4^3+5^3) cu cm= 4/3 π x 216 cu cm.

Let the radius of the big sphere be R , 4/3 πR^3=4/3 π x 216 or, R=6 cm

15.Answer any two questions.

i)Find out average age from the following data

Age(in yrs)	30-34	35-39	40-44	45-49	50-54	55-59
Number of persons	10	12	15	6	4	3

Ans.

Class boundary	Mid point(x)	Frequency(f)	fx
29.5-34.5	32	10	320
34.5-39.5	37	12	444
39.5-44. 5	42	15	630
44.5-49.5	47	6	282
49.5- 54.5	52	4	208
54.5-59. 5	57	3	171

Sum=2055 , mean= 2055/50 =41.1 yrs

ii) Find out median of the data from the following frequency distribution table

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	7	10	15	10	8	5

Ans.

Class boundary	frequency	Cumulative	

		frequency	
0-10	4	4	
10-20	7	11	
20-30	10	21	
30-40	15	36	
40-50	10	46	
50-60	8	54	
60-70	5	59	

Median= $l+\frac{\frac{n}{2}-c.f}{f}$ x h= 30+8.5/15 x10=30 + 5.666= 35.67

iii)Find out the mode from the following frequency distribution table of ages of examinees

Age(yrs)	**16-18**	**18-20**	**20-22**	**22-24**	**24-26**
Number of examinees	**45**	**75**	**38**	**22**	**20**

Ans. Mode= l+ $(f_1- f_0)/(2f_1 - f_o-f_2)$x h= 18+ $\frac{(75-45)}{(150-45-38)}$ x2= 18+.9= 18.9