FOR GOD AND COUNTRY

ST. LAWRENCE HIGH SCHOOL

FIRST TERM – 2018

Subject: MATHEMATICS **Class: IX** **F. M. 75**

Duration:02 Hours 30 Mins **Date:26.04.18**

Group-A

1. Choose the correct options: 1x14=14

i) Which of the following expressions is linear polynomial?

a) $x^2 + x$ b) $x + 4$ c) $x^2 - 3x + 2$ d) $x + 1/x$

ii) If $x = 5$ and $y = 3$ then the value of $(x + y)^{x/y}$ will be

a) 2 b) 4 c) 8 d) 32

iii) The number $\sqrt{7}$

a) lies between 1 and 2 b) lies between 2 and 3 c) lies between 3 and 4 d) lies between 6 and 7

iv) If in the parallelogram ABCD, $\angle A : \angle B = 2:3$ then measure of $\angle D$ is

a) 36° b) 54° c) 72° d) 108°

v) The distance between the points (9,0) and (0,-12) is

a) 10 units b) 12 units c) 11 units d) 15 units

vi)Which of the following statements is wrong?

a) Sum of two rational numbers is always whole number

b) The quotient of two rational numbers maybe negative integer

c) 0 is rational number and whole number

d) All integers are rational numbers.

vii) If $3^x = 9^y$ Then value of x:y is

a) 1 : 1 b) 1 : 2 c) 2 : 1 d) 4 : 1

viii) The simultaneous equations $8x + 7y = 56$ and $4x + 4y = 28$

a) have only one common solution b) have infinite number of common solutions

c) have no common solution d) None of these

ix) If the denominator of a proper fraction is 3 greater than the numerator (x) then the fraction will be

a) $^{x}/_{3}$ b) $^{x+3}/_{3}$ c) $^{x}/_{x+3}$ d) $^{x}/_{x-3}$

x) The polynomial $x^2 - x - 12$ is

a) divisible by $x + 2$ b) divisible by $x + 3$ c) divisible by $x + 4$ d) divisible by $x - 2$

xi) Perimeter of the parallelogram ABCD is 36 cm. If AB = 9.5 cm then length of the side AD is

a) 8 cm. b) 8.5 cm c) 9 cm. d) 7.5 cm

xii) $\overline{AB}$ is a diameter of a circle. If A(3, -3), B(x, -7) and $\overline{AB}$=5 units then the values of x are :

a) 5 or 1 b) 2 or 3 c) 0 or 6 d) -6 or 0

xiii) If the distance between the points (a, 0) and (0, a) is $4\sqrt{2}$ then the value of a is

a) 2 b) 4 c) 8 d) $2\sqrt{2}$

xiv) Which of the following points is nearest to the origin

a) (2, 3) b) (-3, 1) c) (0, 4) d) (3, 4)

Group-B

2 . Answer the following. (8 x 2 = 16)

(i) Write 2 rational number between ½ and ⅙ (by d-method).

(ii) Given an example where difference of two irrationals is a rational.

(iii) Solve: $2^{x+2} + 2^{x-1} = 9$.

(iv) Factorise: $x^3 - 7x - 6$.

(v) Calculate the distance between two points $(\frac{-3}{2}, 0)$ and (0, -2).

(vi) If p (x) = x + 4, write the value of p (x) + p (-x).

(vii) Establish the relation between a, b and c if $a^3 + b^3 + c^3 - 3abc = 0$ and $a + b + c \neq 0$.

(viii) Applying Remainder Theorem write the remainder when $x^3 - 6x^2 + 9x - 8$ is divided by $x - 3$.

Group-C

3. Answer any nine questions : 5x9=45

a) Simplify: $-20\sqrt{2} + 10(1+2\sqrt{2})$

b) Factorize : $a^3 - 12a - 16$.

c) With the help of graph solve the equations $3x + 2y = 18$ and $3y - 2x = 1$.

d) Solve (By the method of elimination):

$3x - {}^{2}/_{y} = 5$; $x + {}^{4}/_{y} = 4$

e) 10 years ago the age of father was 7 times the age of son. After 2 years, twice the age of father will be 5 times the age of son. What are the present ages of father and son?

f) If the polynomial $x^4 + 2x^3 - 3x^2 + ax - b$ is divided by $(x-1)$ and $(x+1)$, the remainders are 5 and -13 respectively. Find the values of a and b.

g) If $a^x = b^y = c^z$ and $b^2 = ac$ then prove that, $1/x + 1/z = 2/y$.

h) Show that, the points A (3, 3), B (8, -2), C (-2, -2) are the vertices of a right angled isosceles triangle. Find the length of the hypotenuse of the triangle ABC.

i) In the quadrilateral ABCD, AD = BC and $\angle BAD = \angle ABC$. Prove that, the quadrilateral is an isosceles trapezium.

j) Prove that diagonals of a parallelogram bisect each other.

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27.4.18.

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Subject: Mathematics **Class: IX** **F. M. 75**

MODEL ANSWERS

Group-A

1. Choose the correct options:

i) b) x+4

ii) d) 32

iii) b) lies between 2 and 3

iv) d) 108°

v) d) 15 units

vi) a) sum of two rational numbers is always whole number.

vii) c) 2:1

viii) a) have only one common solution

ix) c) x/x+3

x) b) divisible by x+3

xi) b) 8.5 cm

xii) c) 0 or 6

xiii) b) 4

xiv) a) (2, 3)

Group-B

2. Answer the following.

(i) Write 2 rational number between ⅐ and ⅙ (by d-method).

Sol.: Let $a = \frac{1}{7}$ and $b = \frac{1}{6}$

$$d = \frac{b-a}{n+1} = \frac{\frac{1}{6}-1/7}{2+1} = \frac{1/42}{3} = \frac{1}{42} \times \frac{1}{3} = \frac{1}{126}$$

$$1^{st} \text{ no. } \frac{1}{7} + \frac{1}{126} = \frac{18+1}{126} = \frac{19}{126}$$

$$2^{nd} \text{ no. } \frac{1}{7} + \frac{2}{126} = \frac{1}{7} + \frac{1}{63} = \frac{9+1}{63} = \frac{10}{63}$$

(ii) Given an example where difference of two irrationals is a rational.

Sol.: $(\sqrt{5} + 2) - (\sqrt{5} - 2) = 4$

(iii) Solve: $2^{x+2} + 2^{x-1} = 9$.

Sol.: $2^{x+2} + 2^{x-1} = 9$

$$\text{Or } 4 \times 2^x + \frac{2^x}{2} = 9$$

$$\text{Or } \frac{2^x (8 + 1)}{2} = 9$$

Or $2^x = 2$ or $x = 1$

(i v) Factorise: $x^3 - 7x - 6$.

Sol.: $f(-1) = 0$

$\therefore x^3 - 7x - 6$

$= x^2 (x + 1) - x (x + 1) - 6 (x + 1)$

$= (x + 1) (x^2 - x - 6)$

$= (x + 1) (x - 3) (x + 2)$.

(v) Calculate the distance between two points $(\frac{-3}{2}, 0)$ and $(0, -2)$.

Sol.: Let $a = (\frac{-3}{2}, 0)$ $B = (o, 2)$

Length of AB = required distance

$$= \sqrt{(-\frac{3}{2} - 0)^2 + (0 + 2)^2}$$

$$= \sqrt{\frac{9}{4} + 4}$$

$$= \frac{\sqrt{25}}{4} = \frac{5}{2} = 2\frac{1}{2} \text{ unit}$$

(vi) If $p(x) = x + 4$, write the value of $p(x) + p(-x)$.

Sol.: $P(x) = x + 4$

$P(-x) = -x + 4$

$\therefore P(x) + P(-x) = x + 4 + 4 = 8$

(vii) Establish the relation between a, b and c if $a^3 + b^3 + c^3 - 3abc = o$ and $a + b + c \neq o$.

Sol.: $a + b + c = 0$

$\rightarrow a^2 + b^2 + c^2 - ab = bc - ca = 0$

Or $2a^2 + 2b^2 + 2c^2 - 2ab = 2bc - 2ca = 0$

Or $(a - b)^2 + (b - c)^2 + (c - a)^2$

Sum of three squares or positive numbers is zero

$\therefore$ each is zero

$\therefore a = b = c$.

(viii) Applying Remainder Theorem write the remainder when $x^3 - 6x^2 + 9x - 8$ is divided by $x - 3$.

Sol.: $f(3) = 3^3 - 6 \times 3^2 + 9 \times 3 - 8 = 27 - 54 + 27 - 8 = -8$

Group -C

3.Answer any nine questions:

a) The given expression

$= -20\sqrt{2} + 10(1+2\sqrt{2})$

$= -20\sqrt{2} + 10 + (10x2)\sqrt{2}$

$= 20\sqrt{2} + 10 + 20\sqrt{2} = 10.$

b) The given expression

$= a^3 + 8 - 12a - 24 = a^3 + 2^3 - 12(a+2)$

$= (a+2)(a^2 - a \cdot 2 + 2^2) - 12(a+2)$

$= (a+2)(a^2 - 2a + 4 - 12)$

$= (a+2)(a^2 - 2a - 8)$

$= (a+2)(a^2 - 4a + 2a - 8)$

$= (a+2)\{a(a-4) + 2(a-4)\}$

$= (a+2)(a-4)(a+2)$

d) We get, $3x - 2/y = 5$ _____(1)

$x + 4/y = 4$ _____(2)

Multiplying equation (1) by (2) we get, $6x - 4/y = 10$ _____(3)

By (2)+(3) we get,

$x + 6x = 4 + 10$

or, $7x = 14$

or, $x = 14/7 = 2$

Putting $x = 2$ in (2) we get,

$2 + 4/y = 4$

or, $4/y = 2$

or, $2y = 4$

or, $y = 4/2 = 2$.

e) Let the present age of father and son be x years and y years respectively.

10 years ago, the age of father and son was (x-10) years and (y-10) years respectively.

After 2 years, the age of father and son will be (x+2) years and (y+2) years respectively.

From the first condition,

$x - 10 = 7(y - 10)$

or, $x - 7y = -16$ -----(1)

From the second condition,

$2(x + 2) = 5(y + 2)$

or, $2x - 5y = 6$ -----(2)

Solving equations (1) and (2) we get,

$x = 38, y = 14$

So, the age of father and son at present are 38 years and 14 years respectively.

f) Let, $f(x) = x^4 + 2x^3 - 3x^2 + ax - b$

If f(x) is divided by (x-1) and (x+1), the remainders will be f(1) and f(-1) respectively.

Now, $f(1) = 1^4 + 2 . 1^3 - 3 . 1^2 + a . 1 - b$

$= 1 + 2 - 3 + a - b$

According to the question, $a-b = 5$ ----(1)

Again, $f(-1) = (-1)^4 + 2(-1)^3 - 3(-1)^2 + a(-1) - b$

$= - 1 - 2 - 3 - a - b = -(a+b) -4$

According to the question, $f(-1) = -13$

or, $- (a+b) - 4 = -13$

or, $a + b + 4 = 13$

or, $a + b = 13 - 4 = 9$ ____(2)

By (1) + (2) we get, $2a = 5+9$

or, $a = 14/2 = 7$

From (2) we get, $b = 9 - a = 9 - 7 = 2$

$a = 7, b = 2.$

g) Let, $a^x = b^y = c^z = k$

$(a^x)^{1/x} = k^{1/x}$

or, $a = k^{1/x}$

again, $b^y = k$

or, $b = k^{1/y}$

$c^2 = k$

or, $c = k^{1/z}$

$b^2 = ac$

Therefore, $(k^{1/4})^2 = k^{1/x} . k^{1/z}$

or, $k^{2/y} = k^{1/x + 1/z}$

or, $1/x + 1/z = 2/y$. (Proved)

h) $AB=\sqrt{(3-8)^2 + (3+2)^2} = 5\sqrt{2}$ units

$BC=\sqrt{(8+2)^2 + (-2+2)^2} = 10$ units

$AC= \sqrt{(3+2)^2 + (3+2)^2} = 5\sqrt{2}$ units

Since, AB=AC therefore the triangle is isosceles.

Again, $AB^2 + AC^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2 = 100$

And $BC^2 = 100$

Since, $AB^2 + AC^2 = BC^2$ Therefore, it is a right angle isosceles triangle.

i) In the quadrilateral ABCD, AD = BC and <BAD = <ABC.

It is required to prove that, ABCD is an isosceles trapezium.

Construction : From D and C, DE and CF are drawn perpendiculars on AB.

Proof : In AED and BFC , <AED = <BFC (each is a right angle)

<EAD = < FBC (by hypothesis), AD = BC (by hypothesis).

Therefore,

Therefore, DE = CF

Again, DE and CF are perpendiculars on the same straight line AB,

Therefore, DE ||CF

Therefore, In the quadrilateral EFCD, DE = CF and DE || CF.

Therefore, EFCD is a parallelogram.

Therefore, DC || EF i.e., DC ||AB.

The quadrilateral ABCD is an isosceles trapezium. (Proved)

j) The diagonals AC and BD of the parallelogram ABCD intersect each other at the point O.

It is required to prove that, AO = OC and BO = OD.

Proof : In AOB and COD, < OAB = <OCD (AB||CD and AC is their transversal

<BAC = alternate <ACD i.e., < OAB = <OCD)

<AOB = vertically opposite <COD

and AB = DC (they are opposite sides of a parallelogram)

Therefore,

Therefore, AO = OC (corresponding sides of congruent triangles)

BO = OD (corresponding sides of congruent triangles). Proved.