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ST. LAWRENCE HIGH SCHOOL

FOR GOD AND COUNTRY

ANNUAL EXAMINATION-2018

MATHEMATICS

Sub: Mathematics | **Class: 9** | **F. M. 75**

Duration: 2½ Hrs. | **Date:** 13 - 11 - 18

[Relevant rough work must be done in the margin of the page containing the answers]

GROUP-A

1. Choose the correct alternatives : **1x5=5**

i) If $(a^m)^n = \frac{a^m}{a^n}$ then

a) $n=\frac{m}{m+1}$ b) $n=\frac{m}{m-1}$ c) $n=\frac{m+1}{m}$ d) $n=\frac{m-1}{m}$

Ans. a

ii) If $a+b+c=0$ then the value of $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$ is

a) 1 b) abc c) 2 d) 3

Ans. d

iii) The area of the trapezium is 132 sq.cm. The length of one parallel side of the trapezium is 23 cm and its height is 6 cm. The length of the other parallel side is

a) 27 cm b) 31 cm c) 21 cm d) 20 cm

Ans. c

iv) If $\log_{10} x - \log_{10}\sqrt{x} = 1$; then the value of x is

a) 10 b) 100 c) $\frac{1}{10}$ d) $\sqrt{10}$

Ans. b

v) If the cost price of 10 pens is equal to the selling price of 8 pens then the percentage of profit or loss is

a) 20 profit b) 20 loss c) 25 profit d) 25 loss

Ans. c

2. Fill in the blanks : **1x5=5**

i) The value of $\log_6 \log_{\sqrt{2}} 8$ is ___1___

ii) The distance of the point $(-5,-7)$ from the y-axis is ___7___ units.

iii) If $(a-1)$ is a factor of a^3-2a^2+1, then the other factor(s) is/are ___a^2-a-1___

iv) The height of an equilateral triangle of side 4 cm is ___$2\sqrt{3}$___ cm

v) If the vertices of a triangle are $(-1,0),(0,0) and (0,1)$ then its area is ___$\frac{1}{2}$___ sq.unit.

3. **State whether the following statements are TRUE/FALSE :** **1x4=4**

i) If the area of a square inscribed in a circle is 196 sq.cm. then the area of the circle is 308 sq.cm TRUE

ii) The value of k for which the points (2,–1), (k, –1), (1,–1) lie on the same straight line is all real values of k. TRUE

iii) In a classified data the percentage of the frequency of a class is 14%. If the total frequency is 50; then the frequency of the class is 14. FALSE

iv) The point of concurrence of the three medians of a triangle is orthocenter. FALSE

GROUP : B

Answer all the questions : **2x8=16**

4. Simplify : $9^{-3}\times\frac{16^{1/4}}{6^{-2}}\times\left(\frac{1}{27}\right)^{-\frac{4}{3}}$

Ans. $9^{-3}\times\frac{16^{1/4}}{6^{-2}}\times\left(\frac{1}{27}\right)^{-4/3}$

$= 3^{-6}\times\frac{2}{2^{-2}.3^{-2}}\times\left(\frac{1}{3}\right)^{3\times\left(-\frac{4}{3}\right)}$

$= 3^{-6}\times 2.2^{2}.3^{2}.3^{4}$

$= 3^{-6+2+4}.2^{3} = 8$ (Ans)

5. By using factor theorem, justify whether $(x-2)$ is a factor of $4x^4+4x^3-19x^2-16x+12$

Ans. $f(x)=4x^4+4x^3-19x^2-16x+12$

$\therefore f(2)=4\times2^4+4\times2^3-19\times2^2-16\times2+12$

= 64 + 32 – 76 – 32 + 12

= 108 – 108 = 0

$\therefore (x-2)$ is a factor of $f(x)$.

6. ABCD is a parallelogram and ABCE is a quadrilateral. If the diagonal AC bisects the quadrilateral then prove that AC||DE

Ans. ABCD is a parallelogram and ABCE is a quadrilateral.
The diagonal AC bisects the quadrilateral. D and E are joined.
It is required to prove that, $AAC \parallel DE$.
Proof : $\because$ the diagonal AC bisects the quadrilateral ABCE
$\therefore \Delta ABC = \Delta ACE$
Again, AC is a diagonal of the parallelogram ABCD
$\therefore \Delta ABC = \Delta ACD$
$\therefore$ we get, $\Delta ACD = \Delta ACE$
Therefore, areas of ΔACD and ΔACE are equal
and they are on the same side of the base AC.
$\therefore AC \parallel DE$ (proved)

7. Factorise : x^3-4x+3

Ans. The given expression = $x^3-1-4x+4$

= $(x-1)(x^2+x+1)-4(x-1)$

= $(x-1)(x^2+x+1-4)$

= $(x-1)(x^2+x-3)$

8. If $x=\log_{2a}a,\ y=\log_{3a}2a, z=\log_{4a}3a$; then prove that $xyz+1=2yz$.

Ans. $LHS=xyz+1$

= $\log_{2a}a.\log_{3a}2a.\log_{4a}3a+1$

= $\log_{4a}a+\log_{4a}4a$

= $\log_{4a}(4a^2)=2\log_{4a}2a$

= $2\log_{3a}2a.\log_{4a}3a=2yz$ = RHS

9. The area of a rectangle is 60 sq.cm and its perimeter is 34 cm. Find the length of the diagonal.

Ans. Let, length of the rectangle = xcm; breadth = y cm

$\therefore xy=60;\ 2(x+y)=34 \Rightarrow x+y=17$

$\therefore x^2+y^2=17^2-2\times 60=169$

$\therefore$ Length of the diagonal = $\sqrt{x^2+y^2}=13cm$ (Ans)

10. If the co-ordinates of the three consecutive vertices of a parallelogram are (7,3), (9,6) and (10,12) then find the co-ordinates of the fourth vertex.

Ans. Let in the parallelogram ABCD, A(7,3), B(9,6), C(10,12) and the fourth vertex is D(x,y).

$\because$ the diagonals of a parallelogram bisect each other

$\therefore$ the mid points of the diagonals AC and BD are the same point.

Now, mid point of the diagonal AC is $\left(\frac{7+10}{2},\frac{3+12}{2}\right) or, \left(\frac{17}{2},\frac{15}{2}\right)$ and the mid point of the diagonal BD is $\left(\frac{9+x}{2},\frac{6+y}{2}\right)$.

According to the condition, $\frac{9+x}{2}=\frac{17}{2}$ and $\frac{6+y}{2}=\frac{15}{2}$

$\therefore 9+x=17\ or, x=17-9=8$ and 6+y = 15 or, y = 15 – 6 = 9

$\therefore\ x=8, y=9$

$\therefore$ co-ordinates of the fourth vertex are (8,9).

11. A man sold an article at a gain of 5%. Had he sold it for Rs. 240 more, he could have gained 8%. Find the cost price of the article.

Ans. Let, the cost price of the article = Rs. 100

$\therefore$ the selling price of the article at a profit of 5% = Rs. (100+5) = Rs. 105

$\therefore$ the selling price of the article at a profit of 8% = Rs. (100+8) = Rs. 108

$\therefore$ had the article been sold at (Rs. 108 – Rs. 105) i.e. Rs. 3 more the profit would have been 8%.

Now, if the selling price is Rs. 3 more, the cost price of the article = Rs. 100

if the selling price is Rs. 1 more, the cost price = Rs. $\frac{100}{3}$

" " " " Rs. 240 " " " = Rs. $\frac{100 \times 240}{3}$ = Rs. 8,000

$\therefore$ the required cost price of the article is Rs. 8,000.

GROUP : C

Answer any nine questions : **5x9=45**

12. If 12 articles are sold per rupee there is a loss of 4%. How many articles should be sold per rupee to make a profit of 44%.

Ans. If 12 articles are sold in 1 rupee then selling price of 1 article is Rs. $\frac{1}{12}$

$\because$ when sold at Rs. $\frac{1}{12}$ there is 4% loss

$\therefore$ selling price is Rs. 96 when cost price is Rs. 100

selling price is Rs. $\frac{1}{12}$ when cost price is Rs. $\frac{100}{96} \times \frac{1}{12} = Rs. \frac{25}{3 \times 96}$

$\therefore$ cost price of 1 article = Rs. $\frac{25}{3 \times 96}$

Now, to make a profit of 44%

if cost price is Rs. 100 then selling price is Rs. 144

if cost price is Rs. $\frac{25}{3 \times 96}$ then selling price is Rs. $\frac{144}{100} \times \frac{25}{3 \times 96} = Rs. \frac{1}{8}$

$\therefore$ if 1 article is sold at Rs. $\frac{1}{8}$ then there will be 44% profit

$\therefore$ in Rs. $\frac{1}{8}$, 1 article should be sold

in Rs. 1,8 articles should be sold

$\therefore$ if 8 articles are sold per rupee there will be 44% profit.

13. Solve by cross – multiplication method.

$$\left.\begin{aligned} \frac{6}{x} + \frac{2}{y} = 5 \\ \frac{8}{x} - \frac{3}{y} = 1 \end{aligned}\right\}$$

Ans. Let, $\frac{1}{x} = u, \frac{1}{y} = v$

$\therefore$ The given two equations are,

$6u + 2v - 5.1 = 0$

$8u - 3v - 1.1 = 0$

By cross multiplication we get,

$$\frac{u}{2(-1)-(-3)(-5)} = \frac{v}{(-5)8-(-1).6} = \frac{1}{6(-3)-8 \times 2}$$

or, $\frac{u}{-2-15}=\frac{v}{-40+6}=\frac{1}{-18-16} or, \frac{u}{-17}=\frac{v}{-34}=\frac{1}{-34}$

or, $u=\frac{-17}{-34}=\frac{1}{2}, v=\frac{-34}{-34}=1$

$\because u=\frac{1}{2} \; or, \frac{1}{x}=\frac{1}{2} \; or, x=2$

$\because v=\frac{1}{y} \; or, \frac{1}{y}=1 \; or, y=1$

$\therefore$ the required solution is $x=2, y=1$

14. If $a^x=bc; b^y=ca; c^z=ab$; show that

$$\frac{x}{x+1}+\frac{y}{y+1}+\frac{z}{z+1}=2$$

Ans. $a^x=bc \; or, a^x.a=abc \; or, a^{x+1}=(abc)^1 \; or, a=(abc)^{\frac{1}{x+1}} \; or, a^x=(abc)^{\frac{x}{x+1}}$

Again, $b^y=ca \; or, b^y.b=cab \; or, b^{y+1}=(abc)^1 \; or, b=(abc)^{\frac{1}{y+1}}$

or, $b^y=(abc)^{\frac{y}{y+1}} \; and \; c^z=ab \; or, c^z.c=abc \; or, c^{x+1}=(abc)^1$

or, $c=(abc)^{\frac{1}{z+1}} \; or, c^z=(abc)^{\frac{z}{z+1}}$

we get, $(abc)^{\frac{x}{x+1}}.(abc)^{\frac{y}{y+1}}.(abc)^{\frac{z}{z+1}}=a^x.b^y.c^z$

or, $(abc)^{\frac{x}{x+1}+\frac{y}{y+1}+\frac{z}{z+1}}=(bc)(ca)(ab)$

or, $(abc)^{\frac{x}{x+1}+\frac{y}{y+1}+\frac{z}{z+1}}=(abc)^2$

$\therefore \frac{x}{x+1}+\frac{y}{y+1}+\frac{z}{z+1}=2$ (proved)

15. Prove that the three medians of a triangle are concurrent.

Ans. Let, the two medians BE and CF of ΔABC intersect each other at the point G. A and G are joined and extended, then it intersects BC at the point D. It is required to prove that, AD is a median. Then it will be proved that, the medians are concurrent.

Construction : AD is extended upto H in such a way that AG = GH. B and H, C and H are joined.

Proof : In ΔABH, F and G are the mid points of AB and AH.

$\therefore FG \parallel BH \; i.e., GC \parallel BH$

Again, in ΔAHC, G and E are the mid points of AH and AC

$\therefore GE \parallel HC \; i.e., BG \parallel HC$.

$\therefore$ opposite sides of the quadrilateral BHCG are parallel to each other.

$\therefore$ BHCG is a parallelogram.

$\because$ the diagonals of a parallelogram bisect each other

$\therefore$ D is the mid point of BC, i.e., AD is a median.

$\therefore$ the three medians of a triangle are concurrent. (Proved).

16. The ratio of the area of an equilateral triangle and the area of a square is $\sqrt{3}:2$. If the length of the diagonal of the square is 60 cm; find the perimeter of the triangle.

Ans. Let, the length of the side of the equilateral triangle be a cm and the length of the side of the square be x cm.

According to the equation, $x\sqrt{2} = 60 \; or, \; x = \frac{60}{\sqrt{2}} = \frac{60\sqrt{2}}{2} = 30\sqrt{2}$

$$\because \frac{area \; of \; the \; equilateral \; triangle}{area \; of \; the \; square} = \frac{\sqrt{3}}{2} \; \therefore \frac{\frac{\sqrt{3}}{4}a^2}{x^2} = \frac{\sqrt{3}}{2}$$

or, $2 \times \frac{\sqrt{3}}{4}a^2 = \sqrt{3}x^2 \; or, \; a^2 = \frac{\sqrt{3}x^2.4}{2\sqrt{3}} = 2x^2$

$\therefore a = \sqrt{2}x = \sqrt{2}.30\sqrt{2} = 60$

$\therefore$ the required perimeter = $3a \; cm = 3 \times 60cm = 180cm$.

17. Find the condition that the three points $(a,b),(c,d) \; and \; (a-c,b-d)$ will be collinear.

Ans. The points (a,b), (c,d) and (a–c,b–d) will be collinear if the area of the triangle formed by the three points is zero.

i.e., if $\frac{1}{2}|a(d-b+d)+c(b-d-b)+(a-c)(b-d)| = 0$

or, $\frac{1}{2}|a(2d-b)+c(-d)+(a-c)(b-d)| = 0$

or, $2ad - ab - cd + ab - ad - bc + cd = 0$

or, $bd - bc = 0 \; or, ad = bc$

$\therefore$ the required condition for collinearity is ad = bc.

18. Draw a triangle of sides 5 cm, 8 cm and 11 cm. Draw a rectangle equal in area to this triangle. (Only traces of construction are required).

Ans. (Do yourself)

19. Solve for x :

$$\log_5\left(5^{\frac{1}{x}} + 125\right) = \log_5 6 + 1 + \frac{1}{2x}$$

Ans. $\log_5\left(5^{1/x} + 125\right) - \log_5 6 = 1 + \frac{1}{2x}$

$$\Rightarrow \log_5\left(\frac{5^{1/x} + 125}{6}\right) = 1 + \frac{1}{2x} \Rightarrow 5^{\frac{1}{x}} + 125 = 6.5^{1+\frac{1}{2x}}$$

$$\Rightarrow 5^{\frac{1}{x}} - 30.5^{\frac{1}{2x}} + 125 = 0$$

$$\Rightarrow \left(5^{\frac{1}{2x}} - 5\right)\left(5^{\frac{1}{2x}} - 25\right) = 0$$

$$\left.\begin{array}{l} 5^{\frac{1}{2x}} = 5 \Rightarrow x = \frac{1}{2} \\ 5^{\frac{1}{2x}} = 25 \Rightarrow x = \frac{1}{4} \end{array}\right\} \text{(Ans)}$$

20. If the polynomial $x^4+2x^3-3x^2+ax-b$ is divided by $(x-1)\,and\,(x+1)$, the remainders are 5 and –13 respectively. Evaluate a and b.

Ans. Let, $f(x)=x^4+2x^3-3x^2+ax-b$

If $f(x)$ is divided by $(x-1)\,and\,(x+1)$, the remainders will be $f(1)$ and $f(-1)$ respectively.

Now, $f(1)=1^4+2.1^3-3.1^2+a.1-b$

$= 1+2-3+a-b=a-b$

According to the question, $a-b=5....(1)$

Again, $f(=1)=(-1)^4+2(-1)^3-3(-1)^2+a(-1)-b$

$= 1-2-3-a-b=(a+b)-4$

According to the question, $f(-1)=-13$

or, $-(a+b)-4=-13\ or, a+b+4=13$

or, $a+b=13-4=9....(2)$

By $(1)+(2)$ we get, $2a=5+9\ or,\ a=\frac{14}{2}=7$

from (2) we get, $b=9-a=9-7=2$

$\therefore a=7, b=2$

21. Prove that, area of the square ABCD is greater than the area of the rhombus ABEF.

Ans. ABCD is a square and ABEF is rhombus.

It is required to prove that, square ABCD > rhombus ABEF.

Construction : FP is drawn perpendicular from F on AB

Proof : AF is the hypotenuse of the right angled triangle APF.

$\therefore AF > FP\ or, AB.AF > AB.FP$

or, $AB.AB > AB.FP$ [$\because ABEF$ is a rhombus $\therefore$ AF = AB]

$\therefore$ square ABCD > rhombus ABEF (Proved).
