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ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

First term examination 2019

Class 10

Subject : Mathematics

Model answer paper

date:16/04/2019

GroupA

I)   Choose the correct option.

1) C   2) c   3) b   4) b   5) b   6) d

II)   Fill in the blanks.

1. If the roots of $x^2+5x+k=0$ are -2 and -3 then the value of $k = \underline{6}$

2. The present price of a machine is P and the rate of depreciation is r% per annum. The price of the machine after n years will be $\underline{P(1-r/100)^n}$

3. O is the circumcentre of an equilateral triangle ABC Then measure of angle $AOB= \underline{120^o}$

4. The perpendicular distance of the greatest chord of a circle from the centre is <u>zero.</u>

5. Length , breadth and height of a rectangular parallelepiped are l, b and h. The length of the diagonal is
$$\sqrt{l^2 + b^2 + h^2}$$

6. If x be the volume and r be the radius of a cylinder, then the height of the cylinder is $\underline{x/\pi r^2}$

7. If $0^o \leq \theta \leq 90^o$ and $\cos\theta = \sec\theta$, then the value of $\theta = \underline{0^o}$

8. The value of $\tan 35^o \tan 55^o = \underline{1}.$

GROUP-B

Q.4) Short answer type questions:

i)   The compound interest of Rs 5000 in 2 years is Rs408. Find the rate of interest.

Ans. $5000(1+r/100)^2= 5408$ or, $(1+r/100)^2= =(26/25)^2$ Therefore r= 4%

ii)   If the interest in 5 years is <u>1/4</u>th the principle, what will be rate of simple interest.

Ans. SI= prt/100   or, p/4= p x rx 5/100 Therefore r= 5%

iii)   For what value of 'a' the equation $(a-2)x^2 + 2x + 3=0$ will not be a quadratic equation?

Ans. if a-2=0 or ,a=2 then the equation will not be a quadratic equatioin

iv)   Three times the sum of a number and its reciprocal is 10. Find the number.

Ans. let the number be x

$3(x+1/x) = 10$ or, $3x^2-10x =3=0$ or, x=3 or 1/3

v)   The length, breadth and height of a room are 12 m, 4 m and 3 m respectively. What is the length of the longest rod that can be placed inside the room.

Ans. Diagonal = $\sqrt{h^2 + l^2 + b^2}$ =13 m

vi)   If the height of a solid right circular cylinder is 6 cm and diameter 14 cm, find the area of the Whole surface.

Ans. h=6 cm, r= 7 cm   whole surface area = 2 x 22/7 x 7(7+6) = 572 sq cm.

vii) The diameter of a circle is 10 cm. If the distance of a chord from the centre is 4 cm, what is the length of the chord?

Ans. let length of the chord be 2x

$25= 16+ x^2$ or, x=3 cm Therefore length of the chord 6 cm

viii)   If $\sin 45^0 - x\cos 60^0 = 0$ then what is the value of x?

Ans, $1/\sqrt{2} -x/2=0$ or, x= $\sqrt{2}$

ix)   Express 3750 " into degrees, minitues and seconds.

Ans. 3750"= 62'30"= 1○2' 30"

Q.5) Short answer type questions (Attempt any 5):

i) Find an equation whose roots are reciprocal of the roots of $3x^2+8x+2=0$.

Ans. a +b = -8/3 and ab= 2/3

Sum of the roots $1/a+1/b= a+b/ab = -8/3 \div 2/3 = -4$, product of the roots $= 1/ab= 3/2$

Required equation $= x^2+4x +3/2=0$ or, $2x^2+8x +3=0$

ii) The hypotenuse of a right angled triangle is 13 cm and difference of other two sides is 7 cm. Find the two sides.

Ans. x-y=7   According to the problem , $x^2+y^2=169$ or, $(x-y)^2+2xy= 169$ or, $2xy= 120$

or, xy=60   therefore y=60/x

Or, x-60/x=7   or, (x-12)(x+5)=0   x=12 and y=5

iii) If $x=Sin^2 30^0+4cot^2 45^0-sec^2 60^0$ then find the value of x ?

Ans. $x= ¼ + 4-4 = \frac{1}{4}$

Iv) 0 is the circumcentre of the triangleABC. Prove that <OBC+<BAC=90⁰.

Ans. angle BOC= 2 x angle BAC

angle OBC = angle OCB   Again angle OBC + angle OCB + angle BOC=180⁰ or, 2 (OBC+BAC)= 180

or, angle OBC + angle BAC= 90⁰

v) The radius of a circle is 7cm. Determine the circular value of an angle subtented by an arc of 5.5cm length at the centre of that circle.

ANs.s=5.5 cm , r= 7 cm   θ= s/r= (5.5/7)ᶜ= (55/70)ᶜ= 5π/20=π/4

vi) The ratio of the angles of a triangle is 2:5:3. Calculate the circular value of the smallest and greatest angle of the triangle.

Ans. 2x +5x+3x=180 or, x=18   smallest angle = 36⁰ , greatest angle =90⁰

Group-C

v) a)A boatman goes to a place distant 45 km by boat and returns to the starting place. The time of return is 2 hrs more than the time to go to the place . If the speed of the stream is 3 km/hr .Find the speed of the boat.

Ans. let the speed of the boat be x km/hr

45/x-3 − 45/x+3 =2

Or,45x6/(x-30(x+3)= 2

Or, $x^2-9=135$ or, x=12 km/hr

b) Solve by Sridhara Acharyya's formula: ab x^2- (a^2-b^2)x-ab=0

Ans. x= $\frac{(a^2-b^2)\pm\sqrt{(a^2-b^2\ )+4a^2b^2}}{2ab}$ $\underline{\ } \frac{(a^2-b^2)\pm(a^2+b^2)}{2ab}$

when x=$\frac{(a^2-b^2)+(a^2+b^2)}{2ab}$ then x= a/b

when x=$\frac{(a^2-b^2)-(a^2+b^2)}{2ab}$ then x= - b/a

c) If α+β=90∘, prove that secα+sinβ /sinα = tanα + 2 tan β

Ans. LHS- $\frac{sec\alpha+sin\beta}{sin\alpha}$= 1/ cosα sinα + cosα /sinα =$\frac{1+cos^2\alpha}{sin\ \alpha\ cos\alpha}$

RHS- tanα +2 tanβ = sinα/cosα + 2 cosα/sinα =$\frac{sin^2\ \alpha+2cos^2\alpha}{sin\alpha\ cos\alpha}$ = $\frac{1+\ cos^2\alpha}{sin\alpha\ \ cos\alpha}$

Therefore LHS= RHS (proved)

VI)a) Prove that the angle subtended by an arc of a circle at the centre is double the angle subtended by the same arc on the circumference of the circle.

Ans. Refer to Theorem 34 , Page number -123(Ganit Prakash)

b) A car manufacturing company increases the manufacturing of a special car from 1,50,000 to 1,88,160 in 2 years . Find the rate of growth of production per year.

Ans. $188160 = 150000(1+\frac{r}{100})^2$

Or, $(1+\frac{r}{100})^2 = 188160/150000 = (56/50)^2$

Or, $1+r/100 = 56/50$ or, $r/100 = 56/50 -1$ or, $r = 12\%$

c) The diameter of cross section of a wire is reduced by 50%. If the volume remains constant , what percent of length of the wire should be increased?

Ans. old diameter= 2r unit , old radius= r unit , old volume= $\pi r^2 h_1$ cubic unit

     new diameter =r unit, new radius= r/2 unit , new volume = $\pi(r/2)^2 h_2$ cubic unit

$\pi r^2 h_1 = \pi(r/2)^2 h_2$ or, $4h_1 = h_2$

Change in length $3h_1$ units.   therefore percentage of change $3h_1/h_1 \times 100 = 300\%$

d) If one root of $x^2 + bx + 12 = 0$ and $x^2 + bx + q = 0$ be 2 find the value of q.

Ans. $4+2b = -12$ or, b= -8   again, $4-16+q=0$ Therefore q= 12

e)If $\sec\theta = a + 1/4a$, then prove that $\sec\theta + \tan\theta = 2a$ or 1/2a

Ans. $\tan^2\theta = (a+1/4a)^2 -1 = a^2 + (1/4a)^2 - 2 \times a \times 1/4a = (a-1/4a)^2$

$\tan\theta = \pm(a-1/4a)$

$\sec\theta + \tan\theta = a+1/4a + a - 1/4a = 2a$

or $\sec\theta + \tan\theta = a+ 1/4a - a + 1/4a = 1/2a$