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# ST. LAWRENCE HIGH SCHOOL

**A JESUIT CHRISTIAN MINORITY INSTITUTION**

**Pre – Test Examination**

**Sub: Mathematics Solution** **Class: X** **FM: 75**

**Duration: 2 hrs 30mins** **Date:06.08.19**

## Group- A

( I ) Choose the correct option:- ( 1 X 6 = 6 )

1. In a business the ratio of capitals of Ramesh and Piyush is 5 : 6. The profit in the business is₹220, then the part of the profit of Ramesh is
   a) ₹ 900 **b) ₹ 100** c) ₹ 1200 d) ₹ 1500
2. If $a^2 \alpha bc$ and $b^2 \alpha$ ca, then
   a) c α a b) c constant c) abc constant **d) a α b**
3. If ( 3a + 5b ) : ( 3a - 5b ) = 13 : 8, then a : b is equal to
   a) 7 : 3 **b) 7 : 1** c) 5 : 1 d) 5 : 3
4. ABCD is a cyclic trapezium of which AD // BC. If ∠ABC = 75°, then ∠BCD is
   a) 105° **b) 75°** c) 45° d)90°
5. IF the area of lateral surface of a cone is twice the area of its base, then the ratio of its height and diameter is
   a) 2 : 1 b) $\sqrt{2}$: 1 c) $\sqrt{3}$: 1 **d) $\sqrt{3}$: 2**
6. If the length of the shadow of a 15m height palm tree is $15\sqrt{3}$ m, then theangle of elevation of the sun is
   a) 45° **b) 30°** c) 60° d) 90°

( II ) Fill in the blanks :- ( 1 X 4 = 4 )

1) The number which is to be subtracted from $\sqrt{72}$ to make the difference $\sqrt{32}$ is $2\sqrt{2}$
2) Two perpendicular tangents are drawn to a circle from an external point. If the radius of the circle be a unit, then the length of the tangent is a unit
3) Ratio of whose surface area of a solid sphere and a solid hemisphere of same radius is 4:3
4) The value of tan35° tan55° = 1

( III ) Write True or False :- ( 1 X 4 = 4 )

1) Compound interest for the second year on ₹ 8000 at 10% per annum is ₹ 880. **True**
2) The circumcentre of a triangle always lies inside the triangle. **False**
3) The % increase in the volume of a solid hemisphere if its radius is doubled, is 300%. **False**
4) If x varies directly as y and y varies inversely as z, then x varies directly as z. **False**

## Group – B

( IV ) Answer the following questions :- (2 X 9 = 18 )

1) If the interest in 5 years is $1/4^{th}$ the principal, what will be the rate of simple interest per annum?**Ans. 5%**
2) Two friends start a business with the capitals ₹ 16000 and ₹ 24000. They make a profit of ₹ 3375 in a year. How much each friend will get as a part of the profit?

**Ans. Ratio 2:3. First friend's profit = Rs. 1350. Second friend's profit = Rs. 2025.**

3) If y be the mean proportional of x and z, then prove that xy + yz is the mean proportional of $(x^2 + y^2)$ and $(y^2 + z^2)$

$\frac{x}{y} = \frac{y}{z} \Rightarrow \frac{xy}{y^2} = \frac{yz}{z^2} \Rightarrow \frac{xy}{yz} = \frac{y^2}{z^2} \Rightarrow \frac{xy+yz}{yz} = \frac{y^2+z^2}{z^2} \Rightarrow \frac{xy+yz}{y^2+z^2} = \frac{y}{z}$

Similarly,
$\frac{x}{y} = \frac{y}{z} \Rightarrow \frac{x^2}{xy} = \frac{y^2}{yz} \Rightarrow \frac{x^2}{y^2} = \frac{xy}{yz} \Rightarrow \frac{x^2+y^2}{xy+yz} = \frac{y}{z}$. So, we have $\frac{x^2+y^2}{xy+yz} = \frac{xy+yz}{y^2+z^2}$.

4) If m = $\sqrt{\frac{n}{n+b}}$, then express $n$ in terms of $m$ and $b$.

**Ans.** $n = \frac{bm^2}{1-m^2}$.

5) If $a^3 + b^3 \alpha a^3 - b^3$, then show that a α b.

$\frac{a^3+b^3}{a^3-b^3} = k \rightarrow \frac{a^3+b^3+a^3-b^3}{a^3+b^3-a^3+b^3} = \frac{k+1}{k-1} \Rightarrow \frac{a^3}{b^3} = \frac{k+1}{k-1} \Rightarrow \frac{a}{b} = m \Rightarrow a\ \alpha\ b.$

6) In a cyclic quadrilateral the ratio of three consecutive angles are 1 : 2 : 3, find the values of first and third angle.

**Ans.** $x + 3x = 180°, x = 45°, 3x = 135°.$

7) Two circles with radius 17cm and 8cm touch each other. Find the distance between the centres when they touch internally.

**Ans. (17-8)cm = 9cm**

8) What portion of a ditch 48m long, 16.5m wide and 4m deep can be filled with stones and earth available during excavation of a tunnel of 4m diameter and 56m length?

**Ans.** $\frac{2}{9}$. $(\pi \times 4 \times 56) \div (48 \times 16.5 \times 4)$

9) By eliminating θ from x = a sin θ and y = b tanθ, find a relation between x and y.

**Ans.** $\frac{y}{b} = \frac{x}{\sqrt{a^2-x^2}}, \frac{a^2}{x^2} - \frac{b^2}{y^2} = 1.$

( V ) Answer the following questions:- ( Any Five ) ( 3 X 5 = 15 )

1) Solve: $x^2$ - 2x - a ( a + 2 ) = 0

**Ans.** $(x - a - 2)(x + a) = 0, x = (a + 2), -a.$

2) The present population of a town is 2,50,000. It is increasing at the rate of 12% every year. What will be the population after 2 years?

**Ans.** $2,50,000\left(1 + \frac{12}{100}\right)^2 = 3,13,600.$

3) The diameter of a solid copper sphere is 14cm. By melting this sphere how many solid spheres of diameter 7cm can be formed?

**Ans. Volume of the solid copper sphere= 4/3xπx7x7x7** $cm^3$
**Volume of solid spheres of diameter 7cm= 4/3xπx3.5x3.5x3.5**$cm^3$
**Number of solid spheres formed= volume of the solid copper sphere ÷ volume of the solid sphere=8**

4) O is the circumcentre of Δ ABC. Prove that $\angle OBC + \angle BAC = 90°$.

Ans. Given O is the circumcenter of triangle ABC.
R.T.P. - ∠OBC + ∠BAC = 90°

Proof, OB = OC. ∠OBC = ∠OCB. ∠BOC = 2∠BAC.
∠BOC+∠OCB = 180°-∠OBC
2∠BAC+2∠OBC = 180°
∠BAC+∠OBC = 90° (Hence proved.)

5) Prove that tan 9° tan27° tan60° tan81° = $\sqrt{3}$

**Ans. Tan(90-81) tan81 tan(90-63) tan63 tan60 =cot81 tan81 cot63 tan63 $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$**

6) If x = $Sin^2 30° + 4\ Cot^2\ 45° - Sec^2\ 60°$, then find the value of x.

**Ans. X = $\frac{1}{2} x \frac{1}{2} + 4 \times 1 - 2 \times 2 = \frac{1}{4}$**

## Group – C

( VI ) Answer the following questions:- ( Any 2 ) ( 4 X 2 = 8 )

1) The distance through which a heavy body falls form rest under gravity varies as the square of the time it falls. The body falls a distance of 125m in 5 secs. How far does the body falls in 10 sec?

**Ans.** $d = kt^2 \rightarrow k = 5, d = 500\ m.$

2) A and B start a joint business with capital ₹ 45000 and ₹ 75000 respectively. After 6 months, A invests ₹ 60000 more but B withdraws ₹ 15000 for personal reason. If at the end of the year the total profit is ₹ 28500, then how much each will get as a part of profit?

**Ans. A's profit = Rs. 15000, B's profit = Rs. 13500.**

3) Construct a right angled triangle whose sides including the right angle are of length 7cm and 9cm. Draw a circumcircle of this triangle.

**First draw a straight line AB measuring 9 cm. Then construct 90° at A. Draw an arc at A measuring 7 cm and name it C. Join BC. Draw two perpendicular bisectors on AB and BC to intersect at O. With O as centre and OC as radius, draw a circle which will touch A and B. Thus a circumcircle is constructed.**

( VII ) Answer the following questions:- ( Any 4 ) ( 4 X 5 = 20)

1) $\frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-\sqrt{18}} - \frac{\sqrt{18}}{3-2\sqrt{3}}$

**Ans.** $4\sqrt{3} + 2\sqrt{6} - 4\sqrt{3} - 3\sqrt{2} + 3\sqrt{2} + 2\sqrt{6} = 4\sqrt{6}.$

2) At the starting of the year two friends Sujit and Chandan start a business with a capital of ₹ 6000 and ₹ 5000. After some months one of their friendsPradip became a partner of the business by investing ₹ 6000 as capital. After one year the total profit of the business is ₹15000 and Pradip gets ₹ 4000 as profit. After how many months Pradip joined the business.

**Ans. Pradip invested for $(12 - x)$ months.**

**Therefore ratio of the profit of Sujit, Chandan, and Pradip = 12:10 :$(12 - x)$.**

So, Pradip's profit = $15{,}000 \times \frac{12-x}{(12+10+12-x)}$.

**$x = 4$ months.**

3) As seen from a point on the roof of a four storeyed building 18m high, the angle of elevation of the top of a monument and the angle of depression of its bottom are 60° and 30° Find the height of the monument.

**Ans. (54+18)m = 72m**

4) There is some water in a cylindrical pot. Two iron spherical balls are dropped in water. The two balls are completely immersed in water. For this water is raised by 6cm. If the diameter of one ball is twice the diameter of the other and the diameter of the cylindrical pot is 8cm, find the radius of each ball.

**Ans. Let the diameter of one ball be $d$ cm and the diameter of another ball be $2d$ cm. Therefore radius of the first ball is $\frac{d}{2}$ cm and the radius of the second ball is $d$ cm. Volume of water displaced is $\pi * 4^2 * 6 = 96\pi$ cm. Therefore $= \frac{4}{3}\pi d^3 + \frac{4}{3}\pi\frac{d^3}{8}$. So $d = 4$ cm. So the radii of the balls are 2 cm and 4 cm.**

5) Prove that if two tangents are drawn to a circle from a point outside it, then the line segment joining the points of contact and the exterior point are equal and they subtend equal angles at the centre.

Ans. Given – 2 tangents PA and PB are drawn at points A and B on a circle from a point P outside the circle.

RTP – i) PA=PB

ii) $\angle POB = \angle POA$

Construction – Join O,P; O,A; O,B.

Proof – OA=OB

_____ OP is the common side

∠OAP=∠OBP

ΔOAP ≅ ΔOBP (RHS congruency)

Therefore,

PA = PB (c.p.c.t)

∠POA = ∠POB (hence proved).