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ST.LAWRENCE HIGH SCHOOL

FOR GOD AND COUNTRY

Model answer

CLASS 10

SUBJECT: Mathematics F.M: 90

DURATION: 3hrs 15mins DATE: 11.11.2019

Group-A

1. CHOOSE THE CORRECT OPTION TO EACH OF THE FOLLOWING QUESTIONS : - (1x6=6)

(i) If a principal becomes twice of its amount in 10 years, the rate of simple interest per annum is –

(b) 10%

(ii) The value of k for which the roots of the equation $2x^2+kx+k-3=0$ are reciprocal to each other is –

(c)5

(iii) The perpendicular distance from the centre of a circle of radius 13cm upon a chord of length 10cm is : **(c) 12cm**

(iv) If tan $\alpha=\frac{1}{x}$ and sin $\alpha=\frac{1}{y}$ then the value of $x^2 - y^2$ is –

(b) -1

(v) The ratio of radii of two solid cylinders is 5:3 and the ratio of their heights is 3:5. Then the ratio of their volumes is –

(c) 5:3

(vi) The frequency of three numbers 12 , 15, 20 are (x+2) , x , (x-1) respectively. If the mean of the distribution is 14.5, then the value of x is ?

(b)3

2. FILL IN THE BLANKS (ANY FIVE): - (1x5=5)

(i). The median of the numbers 9, 13, 7, 11, 17, 15, 19 is _____**13**_____ .

(ii). If the height and the slant height of a cone are 12cm and 13cm respectively, then the volume of the cone is **100π** cc.

(iii). $\cos 20° \operatorname{cosec} 70° + \sin 20° \sec 70°$ = **2** .

(iv). In a cyclic quadrilateral ABCD, the side BC is the diameter of the circle. If $\angle ABC = 62°$ then the value of $\angle ADB =$ **28°** .

(v). 4a=5b and 8b=9c . Find a:b:c = **45 : 36 : 32**

(vi). In a partnership business the total profit of Suman and Soumya is Rs.20000. If the capital of Suman is Rs.6000 and profit is Rs.1200, then the capital of Soumya is **Rs. 94000/-**

3. IN EACH CASE, STATE WHETHER THE STATEMENT IS TRUE OR FALSE (ANY FIVE) :- (1x5=5)

(i). A cyclic parallelogram must be a rectangle.**TRUE**

(ii). If $\propto yz$ and $y \propto zx$, then $x \propto y$. **TRUE**

(iii). Compound interest for the second year on Rs.8000 at 10% per annum is Rs.888 .**FALSE**

(iv). If $0° \leq \theta \leq 90°$ then, the minimum value of $5 - \sin\theta$ is 5.**FALSE**

(v). Circumference of the circular base of a cylinder is 5π cm. and height is 10cm. Then the area of the curved surface is 25π sq. cm.**FALSE**

(vi). The mode of the scores 8, 10, 7, 6, 10, 11, 6, 11, 10, is 11. **FALSE**

Group-B

4.

(i) The present value of a machine is rs.25600. If the value decreases at a rate of $\frac{25}{2}$% every year, what will be the price after 3 years ?

Ans:-The price of the machine after three years will be

Rs. $25600\{1 - \frac{\frac{25}{2}}{100}\}^3$ = Rs. 17150/-

(ii) A sum of money is invested in compound interest. The interest of two consecutive years are Rs.225 and Rs.240. Find the rate of interest ?

Ans:-The interest of Rs. 225 in one year is Rs. (240-225) = Rs. 15. Hence,

The interest of Rs. 100 in one year is Rs. $(\frac{15}{225} \times 100)$ = Rs. $6\frac{2}{3}$

Rate of interest = $6\frac{2}{3}$ %

(iii) Solve $\frac{3}{x} + \frac{x}{3} = 4\frac{1}{4}$

Ans:-Or, $4x^2 - 51x + 36 = 0$ $Or, (x-12)(4x-3) = 0$ $Or,\ x = 12, \frac{3}{4}$

(iv) Find the positive value of x and y so that x, 12, y, 27 are in continued proportion.

Ans:- $\frac{x}{12} = \frac{12}{y} = \frac{y}{27}$; Solving it we shall get x=8 , y=18.

(v) AB is a chord of a circle with centre O. From O, OP is perpendicular to AB and OP produced to meet the circle at C. If AB=6 cm PC=1 cm, find the radius of the circle ?

Ans:-

$\because OP \perp AB$

$\therefore P$ is the mid point of AB.

$\therefore AP = \frac{1}{2} AB$

$= \frac{1}{2} \times 6 = 3$ cm ($\because$ Given $AB = 6$ cm)

Let the radius of the circle be x cm.

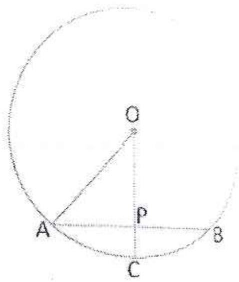
$\therefore OA = OC = x$ cm

$\because$ Given $PC = 1$ cm

$\therefore OP = OC - PC = (x - 1)$ cm

From right-angled triangle OAP,

$OA^2 = OP^2 + AP^2$



$$x^2 = (x-1)^2 + 3^2 \quad Or, x = 5.$$

(vi) ABEC is a cyclic quadrilateral, AB is a diameter of the circle. AE and BC intersect at D. If $\angle CBE = 25°, \angle BCE = 20°$, then find the value of $\angle ADC$?

Ans:-

$\because AB$ is a diameter of the circle,

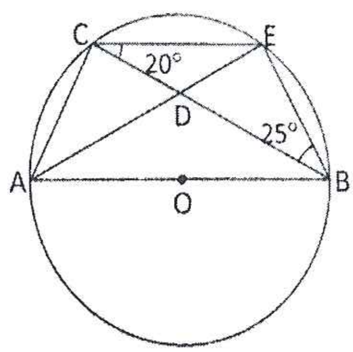
$\therefore \angle AEB = 90°$ ($\because$ angle on the semicircle) ;

Now, Ext. $\angle ADB = \angle DEB + \angle DBE$

$= 90° + 25°$ ($\because \angle CBE = 25°$) $= 115°$

$\therefore \angle ADC = 180° - \angle ADB$

$= 180° - 115° =$ **65°**



(vii) **AB is a chord of a circle with centre O. The tangent at B intersects produced AO at T. If $\angle BAT = 25°$, then find (a) $\angle ABT$ (b) $\angle BTA$**

Ans:-

$\because$ OB is the radius through the point of contact $\therefore$ $OB \perp BT$.

$\therefore \angle OBT = 90°$

In ΔAOB, $OA = OB$ (radius of same circle)

$\therefore \angle ABO = \angle OAB$

Now, $\angle BAT = 25°$ (given)

$\therefore \angle OAB = 25°, \angle OBA = 25°$.

Ext. $\angle BOT = \angle OAB + \angle OBA$

$= 25° + 25° = 50°$.

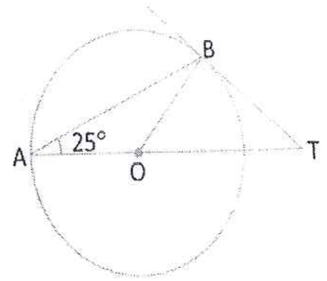
From ΔBOT, $\angle BOT = 50°, \angle OBT = 90°$

$\therefore \angle OTB = 180° - (50° + 90°) = 40°$

i.e., $\angle ATB = 40°$

(*i*) $\angle ABT = \angle ABO + \angle OBT = 25° + 90° = 115°$

(*ii*) $\angle BTA = 40°$.



(viii) **In ΔXYZ, $\angle Y = 90°$. If XY= $2\sqrt{6}$ and XZ-YZ=2 then, what is the value of $\sec X + \tan X$?**

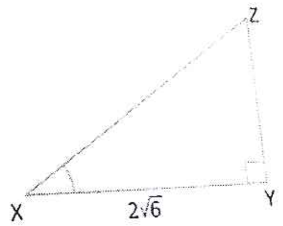
Ans:-

$\sec X + \tan X$

$$= \frac{(\sec X + \tan X)(\sec X - \tan X)}{\sec X - \tan X}$$

$$= \frac{\sec^2 X - \tan^2 X}{\sec X - \tan X} = \frac{1}{\sec X - \tan X}$$

$[\because \sec^2 X - \tan^2 X = 1]$



$$= \frac{1}{\frac{XZ}{XY} - \frac{YZ}{XY}} = \frac{1}{\frac{XZ - YZ}{XY}} = \frac{1}{\frac{2}{2\sqrt{6}}} = \sqrt{6}.$$

$[\because$ Given $XZ - YZ = 2$ and $XY = 2\sqrt{6}\,]$

(ix) **Prove that :** $\frac{\sin 52°+\cos 38°}{\sin 38°+\cos 52°} = \cot 38°$

Ans:-L.H.S.= $\frac{\sin 52°+\cos 38°}{\sin 38°+\cos 52°} = \frac{\sin 52°+\cos(90°-52°)}{\sin(90°-52°)+\cos 52°} = \frac{2\sin 52°}{2\cos 52°} = \tan 52° = \cot 38°$

$= R.H.S.$

(x) **The volume and lateral surface area of a right circular cone are numerically equal. If the height and the radius of the cone be *h* unit and *r* unit respectively then what is the value of $\frac{1}{h^2}+\frac{1}{r^2}$?**

Ans:-Let , the slant height be l unit.

Hence, $l^2 = h^2 + r^2$.

By the given condition $\frac{1}{3}\pi r^2 h = \pi r l$ Or, $l^2 = \frac{1}{9}r^2h^2$

Hence, $\frac{1}{h^2}+\frac{1}{r^2} = \frac{1}{9}$

(xi) **The base of a right circular cylinder and cone is same and the ratio of their volumes is 3:2. Prove that the height of the cone is twice the height of the cylinder.**

Ans:-Let, h_1 and h_2 be the heights of cylinder and cone respectively.

$\frac{\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2} = \frac{3}{2}$ Or. $h_2 = 2h_1$

(xii) **Find the median of the following sets of numbers :**

(a) 7,9,11,13,15,16,17,19,21.

(b) 111, 12, 98, 62,75,30,48,94.

Ans:-

(a) N=9 ; Median= $\frac{N+1}{2} th\ observation = 5th\ observation = 15$

(b) N=8 ; Median= $\frac{1}{2}[\frac{N}{2} th\ observation + \left(\frac{N}{2}+1\right) thobservation]$ =

$\frac{1}{2}[4th + 5th\ observation] = \frac{1}{2}[62+75] = 68.5$

Group-C

5. **(i) At the starting of the year two friends Rima and Raima start a business with a capital of Rs.6000 and Rs.5000 respectively. After some months one of their friends Rakhi became a partner of the business by investing Rs.6000 as capital. After one year the total profit of business is Rs.15000 and Rakhi gets Rs.4000 as profit. After how many months Rakhi joined the business ?**

Ans:-Let, Rakhi joined in the business after x months of start. Hence, ratio of profit of Rima , Raima and Rakhi

= $(6000 \times 12) : (5000 \times 12) : \{6000 \times (12 - x)\}$

$= 12 : 10 : (12 - x)$

According to the problem , $15000 \times \frac{(12-x)}{(12+10+12-x)} = 4000$

Hence, $x = 4$.

(ii) A certain sum of money becomes Rs.6600 in one year and Rs.7986 in three years by compound interest. Find the rate of interest and the sum .

Ans:-Let, the amount of money be Rs. P and the rate of interest be $r\%$ per annum.

Now, by the problem $P\left(1+\frac{r}{100}\right)^1 = 6600 \ldots\ldots\ldots\ldots\ldots\ldots.(i)$

And $P\left(1+\frac{r}{100}\right)^3 = 7986 \ldots\ldots\ldots\ldots\ldots\ldots.(ii)$

Solving $(i)\&(ii)$, $r = 10$ & $P = 6000$

6. **(i) Simplify :** $(\sqrt{5}+\sqrt{3})(\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} - \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}})$

Ans:-$(\sqrt{5}+\sqrt{3})\left(\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} - \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}\right)$

$$= (\sqrt{5}+\sqrt{3})\left\{\frac{3\sqrt{3}(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} - \frac{\sqrt{5}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}\right\}$$

$$= (\sqrt{5}+\sqrt{3})\{\sqrt{3}(\sqrt{5}-\sqrt{2}) - \sqrt{5}(\sqrt{3}-\sqrt{2})\}$$

$$= 2\sqrt{2}$$

(ii) If $(a^2+b^2) \propto ab$ then prove that, $(a+b) \propto (a-b)$

Ans:-$\frac{a^2+b^2}{ab} = k\ (Where, k\ is\ a\ constant)$

or , $\frac{a^2+b^2}{2ab} = \frac{k}{2}$

now, by componendo and dividendo, we get

$$\frac{(a+b)^2}{(a-b)^2} = \frac{k+1}{k-1} = m^2(say)$$

Hence, $(a+b) = \pm m(a-b)$ $i.e. (a+b) \propto (a-b)$

7. **(i) Solve :** $\frac{1}{(x-a-b)} = \frac{1}{x} - \frac{1}{a} - \frac{1}{b}$

Ans:-$\frac{1}{(x-a-b)} = \frac{1}{x} - \frac{1}{a} - \frac{1}{b}$

$$or, \frac{1}{(x-a-b)} - \frac{1}{x} = \frac{1}{a} - \frac{1}{b}$$

$$or, x^2 - ax - bx - ab = 0,$$

$$or, (x-a)(x-b) = 0 \quad or, \quad x = a, b$$

(ii) A boatman goes to a place distant 45km by boat and returns to the starting place. The time of return is 2 hours more than the time to go to the place. If the speed of the stream is 3km/hr, find the speed of the boat.

Ans:-Let , the speed of the boat in still water be x km/hr.

By the problem , $\frac{45}{(x-3)} - \frac{45}{(x+3)} = 2 \quad or, x = 12$

Hence, the speed of the boat in still 12 km/hr.

8. (i) If a,b,c are in continued proportion, show that,

$$\boldsymbol{abc(a+b+c)^3 = (ab+bc+ca)^3}$$

Ans:- $\frac{a}{b} = \frac{b}{c} = k(say)$ Hence, $b = ck$, $a = bk = ck \times k = ck^2$

By, putting the values of a and b in L.H.S and R.H.S , we shall get-

L.H.S = $c^6k^3(k^2+k+1)^3$=R.H.S

(ii) If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$ **then prove that,** $\frac{a}{y+z-x} = \frac{c}{x+y-z} = \frac{b}{z+x-y}$

Ans:-$\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b} = k(say)$, $\therefore x = k(b+c), y = k(c+a),$

$$z = k(a+b)$$

$$y+z-x = 2ka, \quad z+x-y = 2kb, \quad x+y-z = 2kc.$$

$$\frac{a}{y+z-x} = \frac{c}{x+y-z} = \frac{b}{z+x-y} = \frac{1}{2k}$$

Hence, $\frac{a}{y+z-x} = \frac{c}{x+y-z} = \frac{b}{z+x-y}$

9. (i) Prove that in a cyclic trapezium the oblique sides are equal.

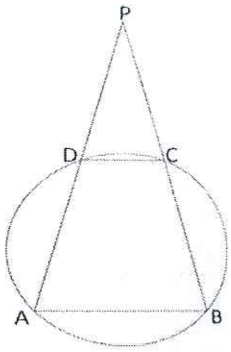
Ans:- **P.T.O.**

Suppose, $ABCD$ is a cyclic trapezium whose oblique sides are AD and BC.

To Prove : $AD = BC$.

Construction : AD and BC are produced. Suppose, they meet at P.

Proof : $\because$ $DC \parallel AB$



$\therefore$ $\angle BAD + \angle ADC = 2$ right angle ... (1)

$\because$ $ABCD$ is a cyclic quadrilateral

$\therefore$ $\angle ABC + \angle ADC = 2$ right angle ... (2)

From (1) and (2) we get,

$\angle BAD + \angle ADC = \angle ABC + \angle ADC$

i.e., $\angle BAD = \angle ABC$ i.e., $\angle PAB = \angle PBA$

$\therefore$ In ΔPAB, $PB = PA$.

Again, $\because$ $DC \parallel AB$ and PA and PB are two secant

$\therefore$ $\angle PDC = \angle PAB$ and $\angle PCD = \angle PBA$

$\because$ $\angle PAB = \angle PBA$ $\quad\therefore$ $\angle PDC = \angle PCD$

$\therefore$ In ΔPDC, $PC = PD$.

we get, $PA - PD = PB - PC$ ($\because$ $PA = PB$, $PD = PC$)

i.e., $AD = BC$. **(Proved)**

❖ **Hints on alternative proof :**

Join B and D and A and C. $\Delta ABD \cong \Delta ABC$ [$\because$ $\angle BAD = \angle ABC$ (proved earlier), $\angle ADB = \angle ACB$ (angle on same segment) and AB is common]

$\because$ $\Delta ABD \cong \Delta ABC$ $\quad\therefore$ $AD = BC$.

(ii) ***In $\triangle ABC$, $AB = AC$ and E is a point on extended BC.***

The circumcircle of $\triangle ABC$ meets AE at D. prove that, $\angle ACD = \angle AEC$.

In ΔABC, $AB = AC$ and E is a point on BC produced. The circumcircle of ΔABC meets AE at D. Join C and D.

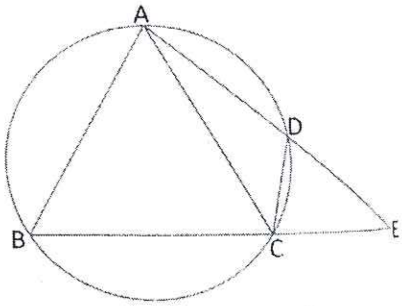
To Prove : $\angle ACD = \angle AEC$.

Proof : In ΔABC, $AB = AC$; $\therefore$ $\angle ACB = \angle ABC$.

Now, $ABCD$ is a cyclic quadrilateral $\therefore$ $\angle ABC + \angle ADC = 2$ right angle

i.e., $\angle ACB + \angle ADC = 2$ right angle [$\because$ $\angle ABC = \angle ACB$]

But, $\angle ACB + \angle ACE = 2$ right angle



$\therefore$ $\angle ACB + \angle ACE = \angle ACB + \angle ADC$

$\therefore$ $\angle ACE = \angle ADC$

Now, $\angle ACE = \angle ACD + \angle DCE$

In ΔDCE, Ext. $\angle ADC$

$= \angle DCE + \angle DEC$

$\therefore$ $\angle ACD + \angle DCE = \angle DCE + \angle DEC$ $\quad$ [$\because$ $\angle ADC = \angle ACE$]

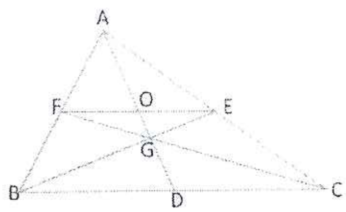
$\therefore$ $\angle ACD = \angle DEC = \angle AEC$. **(Proved)**

10. (i) In triangle ABC the medians BE and CF meet at G and the straight line segment FE meets AG at O. Prove that , AO=3OG.

Ans:- In triangle ABC, G is the centroid. AG produced to meet BC at D. Hence, BD=DC.

IN triangle ABC , mid points of AB and AC are F and E respectively. ∴ FE || BC.

$\therefore FO \parallel BD \therefore \frac{AO}{OD} = \frac{AF}{FB} = 1$



$\therefore AO = OD$

Again, G is the centroid of ΔABC

$\therefore \frac{AG}{GD} = \frac{2}{1}$ or, $AG = 2GD$.

$\therefore AO + OG = 2(OD - OG)$

or, $AO + OG = 2(AO - OG)$ [$\because OD = OA$]

$\therefore 2AO - AO = OG + 2OG \therefore AO = 3OG$. **(Proved)**

(ii) In triangle ABC , P is a point on the median AD. Extended BP and CP intersect AC and AB at Q and R respectively. Prove that RQ is parallel to BC.

Ans:-

Solution : Let P be a point on the median AD of the ΔABC. Extended BP and CP intersect AC and AB at the points Q and R respectively.

To prove : RQ || BC.

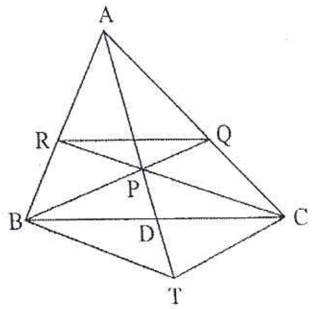
Construction : Let us extend PD to T in such a way that PD = DT. Let us join B, T and C, T.

Proof : In the quadrilateral BPCT, BC and PT are two diagonals. Also, BD = CD. [∵ AD is the median] and PD = DT [by construction]

i..e, the diagonals of the quadrilateral BPCT bisects each other.

∴ BPCT is a parallelogram.

∴ BP || TC and TB || CP.



Now, in ΔABT, RP || BT [∵ TB || CP]

∴ by Thales' theorem, $\frac{AR}{BR} = \frac{AP}{TP}$ ....(1)

Again, in ΔACT, QP || CT [∵ BP || TC]

∴ by Thales' theorem, $\frac{AQ}{CQ} = \frac{AP}{TP}$ ....(2)

Then from (1) and (2) we get, $\frac{AR}{BR} = \frac{AQ}{CQ}$.

∴ in ΔABC, $\frac{AR}{BR} = \frac{AQ}{CQ}$

∴ by the converse of Thales' theorem, RQ || BC.

Hence RQ || BC. **(Proved)**

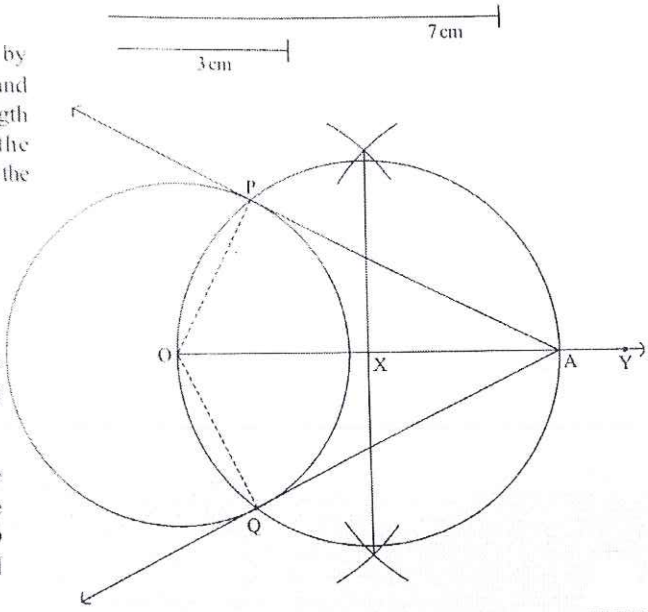
11. **(i) Draw a circle with a radius of 3cm length. Then draw two tangents of the circle from an external point at a distance of 7cm away from the centre. (Only traces of construction are required).**

Ans:-

(i) I have drawn a circle with centre O whose radius is 3 cm. and I cut off the line-segment OA from the ray OY at a distance of 7 cm. from the centre O.

(ii) I bisect the line-segment OA. Let the mid-point of OA be X.

(iii) I draw a circle by taking X as centre and the radius of XO length which intersects the circle with centre O at the points P and Q.



(iv) By joining A, P and A, Q I extended them.

∴ $\overrightarrow{AP}$ and $\overrightarrow{AQ}$ are two tangents of the circle with centre O from the external point A.

(ii) Find the value of $\sqrt{21}$ by gemetrical method. (Only traces of construction are required).

Ans:-$\sqrt{21} = \sqrt{7 \times 3}$; Now, taking two line segments of 7 unit and 3 unit , we can draw the mean proportional of 7 & 3 i.e. $\sqrt{21}$

12. **(i)If $\sin\alpha = m\sin\beta$ *and* $\tan\alpha = n\tan\beta$ then, prove that, $(\cos\alpha)^2 = \frac{m^2-1}{n^2-1}$.** **Ans:-**Since the part which we have to prove does not contain β, so from the two given relations we have to eliminate β.

$$\sin\alpha = m\sin\beta \quad or, \operatorname{cosec}\beta = \frac{m}{\sin\alpha} \qquad and\ \tan\alpha = n\tan\beta \quad or, \quad \cot\beta = \frac{n}{\tan\alpha}$$

Now, using the relation $(\operatorname{cosec}\beta)^2 - (\cot\beta)^2 = 1$ *we shall get,* $(\cos\alpha)^2 = \frac{m^2-1}{n^2-1}$

(ii). If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$ *then, prove that* $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Ans:-$\text{Sin}\,\theta + \cos\theta = \sqrt{2}\cos\theta$

$$Or, \quad (\sqrt{2}-1)\cos\theta = \sin\theta \quad Or, \quad \frac{(\sqrt{2}+1)\sin\theta}{(\sqrt{2}+1)(\sqrt{2}-1)} = \cos\theta$$

Or, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

(iii). $If\ (\cos\alpha)^2 - (\sin\alpha)^2 = (\tan\beta)^2$ then prove that,

$$(\cos\beta)^2 - (\sin\beta)^2 = (\tan\alpha)^2$$

Ans:-$(\cos\alpha)^2 - (\sin\alpha)^2 = (\tan\beta)^2$

Or, $(\cos\alpha)^2 - 1 + (\cos\alpha)^2 = (\tan\beta)^2$

Or, $2(\cos\alpha)^2 = (\tan\beta)^2 + 1 = (\sec\beta)^2$

Or, $2(\cos\alpha)^2(\cos\beta)^2 = 1$

Or, $2(\cos\beta)^2 = (\sec\alpha)^2 = (\tan\alpha)^2 + 1$

Or, $(\cos\beta)^2 - 1 + (\cos\beta)^2 = (\tan\alpha)^2$

Or, $(\cos\beta)^2 - (\sin\beta)^2 = (\tan\alpha)^2$

13. (i) The angles of depression of the top and bottom of a lamp post as seen from the roof of a building with height 12m are $45°$ *and* $60°$. Find the height of the lamp post. ($\sqrt{3} = 1.732$)

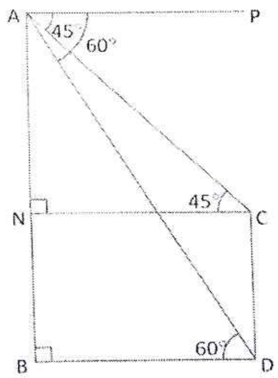
Ans:-**P.T.O.**

Let the height of house and the lamp-post be AB and CD. Here, $AB = 12$ metre. From C draw CN perpendicular to AB. Then $CN = BD$.

Now, the angle of depression of the top of the lamp-post from A is $\angle PAC = 45°$ and the angle of depression of the foot $= \angle PAD = 60°$.

$\therefore \angle ACN =$ alternate $\angle PAC = 45°$ and $\angle ADB =$ alternate $\angle PAD = 60°$.

Now, in $\triangle ABD$, $\angle ABD = 90°$ and $\angle ADB = 60°$



$\therefore \frac{BD}{AB} = \cot 60°$ or, $\frac{BD}{AB} = \frac{1}{\sqrt{3}}$

or, $BD = \frac{AB}{\sqrt{3}} = \frac{12}{\sqrt{3}}$ metre $= 4\sqrt{3}$ metre.

Again, in triangle ACN, $\angle ANC = 90°$ and $\angle ACN = 45°$

$\therefore \frac{AN}{CN} = \tan 45°$ or, $\frac{AN}{CN} = 1$

$\therefore AN = CN = 4\sqrt{3}$ metre

$[\because CN = BD = 4\sqrt{3}$ metre$]$

$\therefore$ Height of the lamp-post $= CD = NB = AB - AN$

$= (12 - 4\sqrt{3})$ metre

$= (12 - 4 \times 1{\cdot}732)$ metre

$= (12 - 6{\cdot}928)$ metre $=$ **5·072 metre.**

(ii) The angle of elevation of the top of a chimney situated at some distance from the roof of a house with 12m height is 30° and the angle of depression of its foot is 60°. Find the height of the chimney.

Let the height of the house be AC and the height of the chimney be BD.

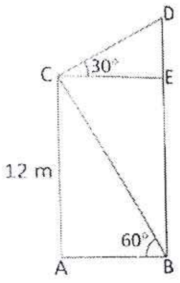
Draw $CE \perp BD$.

Now, angle of elevation of D from C is $30°$ $\therefore \angle DCE = 30°$.

Again, the angle of depression of B from C is $60°$ $\therefore \angle CBA = 60°$.

Since $AC = 12$ metre.

From right-angled triangle CDE we get,



$\frac{DE}{CE} = \tan 30° = \frac{1}{\sqrt{3}}$ $\therefore DE = \frac{1}{\sqrt{3}} CE$

From right-angled triangle ABC we get,

$\frac{AB}{AC} = \cot 60° = \frac{1}{\sqrt{3}}$

$\therefore AB = \frac{1}{\sqrt{3}} \cdot AC = \frac{1}{\sqrt{3}} \cdot 12$ m $(\because AC = 12$ m$)$

$\therefore CE = \frac{12}{\sqrt{3}}$ m $(\because CE = AB)$

$\therefore DE = \frac{1}{\sqrt{3}} \cdot CE = \frac{1}{\sqrt{3}} \times \frac{12}{\sqrt{3}}$ m $= \frac{12}{3}$ m $= 4$ m.

$\therefore$ **Height of the chimney** $= BD = BE + DE$

$= AC + DE = 12$ m $+ 4$ m $=$ **16 metre.**

14. (i)Monica has a piece of canvas whose area is 551 sq.m. She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 sq.m , find the volume of the tent that can be made with it.

Ans:- Since the area of the canvas = 551 sq.m. and area of the canvas lost in wastage is 1 sq.m, therefore the area of canvas available for making the tent is (551 – 1) sq.m. = 550 sq.m.

Now, the surface area of the tent = 550 sq.m and the base radius of the conical tent = 7 mNote that a tent has only a curved surface (the floor of a tent is not covered by canvas!!). Therefore, curved surface area of tent = 550 sq.m.

That is, πrl = 550

l = 25

$$l^2 = r^2 + h^2\ , now, putting\ the\ values\ of\ l\ \&\ r\ , we\ get\ h = 24$$

Hence, volume is $\frac{1}{3}\pi r^2 h = 1232\ m^3$

(ii) The ratio of the length, breadth and height of a solid cuboid is 4 : 3 : 2 and the area of a whole surface is 468 sq.cm. Find the volume of the cuboid.

Ans:- Let, the length, breadth and height of a solid cuboid is 4x, 3x, 2x.

Hence,$2(12x^2 + 6x^2 + 8x^2) = 468$ Or, x^2 = 9 Or, x = 3.

∴ length = 12cm , breadth = 9cm , height = 6cm.

Hence volume = (length × breadth × height) cc = 648 cc.

(iii) The curved surface of a sphere is 554 sq.cm. Find the diameter and volume of the sphere.

Ans:- Let, radius be r cm. By the problem , $4\pi r^2 = 554$ Or , $r = \sqrt{\frac{1939}{44}}$

∴ Diameter = $\sqrt{\frac{1939}{11}}$, Volume = $\frac{4}{3}\pi r^3 = \frac{277}{3}\sqrt{\frac{1939}{11}}$ cc

15.

(i) Find the mean from the following frequency distribution table :

Class interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	10	16	20	30	13	11

(ii) Find the median from the following frequency distribution table :

Class interval	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30	31 - 35
Frequency	2	3	6	7	5	4	3

(iii) Find the mode from the following frequency distribution table :

Class interval	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40
Frequency	2	6	10	16	22	11	8	5

Ans:-

(i) Mean = 40.3 (ii) Median = 18.36 (approx.) (iii)Mode = 21.76 (approx.)