



ST. LAWRENCE HIGH SCHOOL
1st Term Exam – 2019



Sub: Algebra and Geometry
 Duration: 2hrs 30 mins

Class: 8

F. M. : 80
 Date: 13.04.19

Group – A

1. Choose the correct option for the following questions. (Answer all the questions) 5x1=5

i) The value of $(5 - 1)^{-1}$ is

d) $\frac{1}{4}$

ii) What should be multiplied with $\frac{a^2x}{by^2}$ to get $\frac{ax}{by}$?

d) $\frac{y}{a}$

iii) Two sides of a triangle are 7cm and 5cm. what can be the highest measure of the 3rd side among the given options?

d) 11.9cm

iv) What should be added with $(x^2 + 2)$ to make it a perfect square number?

b) $\frac{1}{x^2}$

v) Two angles of a triangle are 60° and 65° . Which option of the following options is correct?

d) Side opposite to 65° angle is the largest side.

2. Fill in the blanks. (Answer all the questions)

5x1=5

i) $\frac{(-2)^{-3}}{(4)^{-2}} = \frac{(4)^2}{(-2)^3} = \frac{16}{-8} = -2$

ii) For a right angled triangle, the side opposite to **90° angle or right angle** is the largest side.

iii) $(a^2b^{-1})^2 \times (a^{-4}b^2) = a^4b^{-2} \times a^{-4}b^2 = a^0b^0 = 1$.

iv) $a^2 - 16b^2 = (a + 4b)^2 - \underline{(8ab + 32b^2)}$.

v) In an obtuse angle triangle one angle is **greater or $>$** than 90° .

3. Write 'True' or 'False'. (Don't write 'T' or 'F'). (Answer all the questions)

5x1=5

i) If two sides of a triangle are equal then angles opposite to those sides will also be equal. -

True

ii) Both the diagonals of any quadrilateral are always greater than its all sides. - **False**

iii) $xy \times (x - y)$ is greater by x^2y than $-xy^2$. - **True**

iv) $\left(\frac{a^3}{b^{-1}}\right)^2$ is equal to $\left(\frac{a^2}{b^{-1}}\right)^3$. - **False**

v) $3x^2 + 4x + 1 + x^2$ is not a perfect square number. - **False**

Group - B

(For this group explanation and diagrams are needed where applicable)

6. Very Short Answer Type Questions. (Answer all the questions)

5x2=10

- i) **Without actual multiplication, find the square of 99.99**

$$\begin{aligned}\text{Ans: } (99.99)^2 &= (100 - 0.01)^2 \\ &= (100)^2 - 2 \times 100 \times 0.01 + (0.01)^2 \\ &= 10000 - 2 + 0.0001 \\ &= 9998 + 0.0001 \\ &= 9998.0001\end{aligned}$$

- ii) **Simplify : $(6x^2 - 1) - (3x^2 - 6x + 2)$.**

$$\begin{aligned}\text{Ans: } (6x^2 - 1) - (3x^2 - 6x + 2) \\ &= 6x^2 - 1 - 3x^2 + 6x - 2 \\ &= 3x^2 + 6x - 3\end{aligned}$$

- iii) **Simplify : $\frac{14a^2b+7ab}{-7ab}$.**

$$\begin{aligned}\text{Ans: } \frac{14a^2b+7ab}{-7ab} \\ &= \frac{14a^2b}{-7ab} + \frac{7ab}{-7ab} \\ &= -2a - 1\end{aligned}$$

- iv) **Simplify: $(xy^{-1})^2 \times (xy)^{-2}$.**

$$\begin{aligned}\text{Ans: } (xy^{-1})^2 \times (xy)^{-2} \\ &= x^2y^{-2} \times x^{-2}y^{-2} \\ &= x^0y^{-4} \\ &= y^{-4} \quad \text{or } = \frac{1}{y^4}\end{aligned}$$

- v) **Determine the value of x , if $16x = (62)^2 - (26)^2$.**

$$\begin{aligned}\text{Ans: } 16x &= (62)^2 - (26)^2 \\ \text{Or, } 16x &= (62 + 26)(62 - 26) \\ \text{Or, } 16x &= 88 \times 36 \\ \text{Or, } x &= \frac{88 \times 36}{16} \\ \text{Or, } x &= 198\end{aligned}$$

7. Short answer type question.(Any five)

5x3=15

- i) **Divide: $-\frac{4}{5}a^2b^2c^4$ by $-\frac{6}{15}abc^2$.**

$$\begin{aligned}\text{Ans: } -\frac{4}{5}a^2b^2c^4 \div -\frac{6}{15}abc^2 \\ &= \frac{-4a^2b^2c^4}{5} \div \frac{-6abc^2}{15} \\ &= \frac{-4a^2b^2c^4}{5} \times \frac{15}{-6abc^2} \\ &= 2abc^2\end{aligned}$$

- ii) **Simplify: $(2x^2 - 3x - 1) \times (x - 5)$.**

$$\begin{aligned}\text{Ans: } (2x^2 - 3x - 1) \times (x - 5) \\ &= 2x^2 \cdot x - 3x \cdot x - 1 \cdot x - 2x^2 \cdot 5 + 3x \cdot 5 + (1 \times 5) \\ &= 2x^3 - 3x^2 - x - 10x^2 + 15x + 5\end{aligned}$$

$$= 2x^3 - 13x^2 + 14x + 5$$

iii) **Simplify:** $\{(2)^a\}^x \times \{(2)^{-x}\}^a$.

$$\begin{aligned} \text{Ans: } & \{(2)^a\}^x \times \{(2)^{-x}\}^a \\ & = (2)^{ax} \times (2)^{-xa} \\ & = (2)^{ax-ax} \\ & = 2^0 \\ & = 1 \end{aligned}$$

iv) **Simplify:** $\left(\frac{1}{2^{-1}} + \frac{1}{3^{-1}}\right)^{-1}$.

$$\begin{aligned} \text{Ans: } & \left(\frac{1}{2^{-1}} + \frac{1}{3^{-1}}\right)^{-1} \\ & = (2^1 + 3^1)^{-1} \\ & = (5)^{-1} \\ & = \frac{1}{5} \end{aligned}$$

v) **Find the value of** $4x^2 + \frac{1}{x^2}$ **when** $2x + \frac{1}{x} = 2$.

$$\text{Ans: Given, } 2x + \frac{1}{x} = 2$$

Squaring both the sides of the above equation, we get

$$\left(2x + \frac{1}{x}\right)^2 = 2^2$$

$$\text{Or, } 4x^2 + 2 \cdot 2x \cdot \frac{1}{x} + \frac{1}{x^2} = 4$$

$$\text{Or, } 4x^2 + 4 + \frac{1}{x^2} = 4$$

$$\text{Or, } 4x^2 + \frac{1}{x^2} = 4 - 4$$

$$\text{Or, } 4x^2 + \frac{1}{x^2} = 0$$

vi) **In triangle ABC, $\angle ABC$ is a right angle. $AB=3\text{cm}$ and $AC=5\text{cm}$. Find out the measure of BC .**

Ans: According to the sum, the triangle will be as shown in the figure.

$$\text{Now, by Pythagoras theorem, } (AB)^2 + (BC)^2 = (AC)^2$$

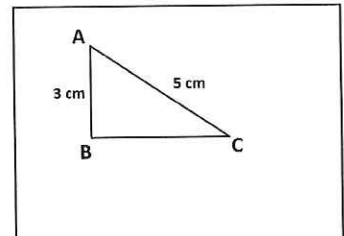
$$\text{Or, } 9 + (BC)^2 = 25$$

$$\text{Or, } (BC)^2 = 25 - 9$$

$$\text{Or, } (BC)^2 = 16$$

$$\text{Or, } BC = 4$$

Hence the measure of BC side will be 4 cm.



vii) **In triangle PQR, it is given that $PQ=QR$ and $\angle PQR = 40^\circ$. Determine $\angle QPR$.**

Ans: According to the sum, the triangle will be as shown in the figure.

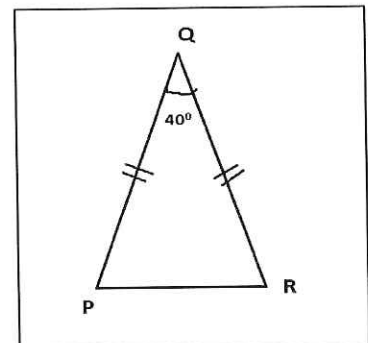
As it is an isosceles triangle, where $PQ = QR$, hence the angles opposite to those will also be equal. i.e $\angle QPR = \angle QRP = x$ (let)

Then $x + x + 40^\circ = 180^\circ$ [as the sum of three angles of a triangle is 180°]

$$\text{Or, } 2x = 180^\circ - 40^\circ$$

$$\text{Or, } x = 70^\circ$$

Hence , $\angle QPR = x = 70^\circ$



Group C

I. Answer the following questions (any 8)

5 x 8 = 40

1. If $\left(x - \frac{1}{x}\right) = 4$ find the value of $\left(x^4 + \frac{1}{x^4}\right)$

Ans: $\left(x - \frac{1}{x}\right) = 4$

Squaring on both sides of the equation, we have $\left(x - \frac{1}{x}\right)^2 = 16$

Or, $\left(x^2 - 2 + \frac{1}{x^2}\right) = 16$

Or, $\left(x^2 + \frac{1}{x^2}\right) = 16 + 2$

Or, $\left(x^2 + \frac{1}{x^2}\right) = 18$

Squaring on both sides of the equation

$\left(x^2 + \frac{1}{x^2}\right)^2 = 18^2$

Or, $\left(x^4 + 2 + \frac{1}{x^4}\right) = 324$

Or $\left(x^4 + \frac{1}{x^4}\right) = 324 - 2 = 322$. **Ans**

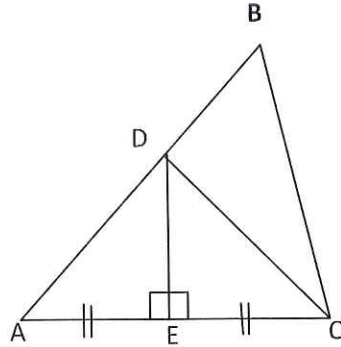
2. What must be subtracted from $x^4 + 6x^3 + 13x^2 + 13x + 8$ so that the resulting polynomial is exactly divisible by $x^2 + 3x + 2$?

Ans: Dividing $x^4 + 6x^3 + 13x^2 + 13x + 8$ by $x^2 + 3x + 2$ we have

$$\begin{array}{r}
 \overline{) x^4 + 6x^3 + 13x^2 + 13x + 8} \\
 \underline{x^4 + 3x^3 + 2x^2} \\
 3x^3 + 11x^2 + 13x + 8 \\
 \underline{3x^3 + 9x^2 + 6x} \\
 2x^2 + 7x + 8 \\
 \underline{2x^2 + 6x + 4} \\
 x + 4
 \end{array}$$

Ans: The remainder is x+4 and must be subtracted from the quotient to make it exactly divisible by the divisor.

3. In a triangle ABC, the perpendicular bisector of AC meets AB at D. Prove that $AB = BD + DC$



Ans: In Triangles ADE and DEC:

Given $AE = EC$, $\angle DEA = \angle DEC = 90^\circ$, DE is common

Therefore the triangles are congruent by SAS congruency.

$AD = DC$ (cpct)

$AB = AD + BD = DC + BD$.

Hence proved.

4. If $4^x + 4^x + 4^x + 4^x + 4^x + 4^x + 4^x + 4^x = \frac{1}{512}$ then what is the value of $-\frac{3}{x}$?

Ans: We have $8 \cdot 4^x = \frac{1}{512}$

Or, $2^3 \cdot 2^{2x} = 2^{-9}$

Or, $2^{(3+2x)} = 2^{-9}$

When bases are same the powers can be equated. Therefore,

We have, $3 + 2x = -9$

Or $2x = -9 - 3$

Or $2x = -12$

Or $x = -6$

Therefore $-\frac{3}{x} = \frac{-3}{-6} = \frac{1}{2}$

5. Multiply $x^2 + xy - y^2$ by $x^2 - xy + y^2$

Ans: $(x^2 + xy - y^2)(x^2 - xy + y^2)$

$= x^2 \cdot (x^2 - xy + y^2) + xy \cdot (x^2 - xy + y^2) - y^2 \cdot (x^2 - xy + y^2)$

$= x^4 - x^3y + x^2y^2 + x^3y - x^2y^2 + xy^3 - x^2y^2 + xy^3 - y^4$

$= x^4 - x^2y^2 + 2xy^3 - y^4$ (Ans)

6. Can 6 metres, 8 metres and 10 metres form the sides of a right angled triangular field?

Ans: Yes

We know by Pythagoras theorem that for a right angled triangle $\text{base}^2 + \text{perpendicular}^2 = \text{hypotenuse}^2$ where hypotenuse is the largest side of the triangle.

In the given question $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

hence the given sides can form a right angled triangular field.

7. The angles of a quadrilateral are in the ratio 2 : 3 : 5 : 8. Find the measure of each of the four angles.

Ans: We know that the sum of four angles of a quadrilateral = 360°

Given the ratio of the angles is 2:3:5:8.

Let the common factor be x .

$$\text{Then } 2x + 3x + 5x + 8x = 360^\circ$$

$$18x = 360^\circ$$

$$\text{Or } x = 20^\circ$$

Therefore the angles are $2 \times 20^\circ$, $3 \times 20^\circ$, $5 \times 20^\circ$ and $8 \times 20^\circ$

Ans: 40° , 60° , 100° , 160°

8. Find the product of $(a-b-c)$ and $(a-b+c)$

Ans: $(a-b-c)(a-b+c)$

$$= \{(a-b)-c\} \{(a-b)+c\}$$

Let $(a-b)$ be m

Therefore the expression becomes

$$(m-c)(m+c)$$

Using the identity $(x+y)(x-y) = x^2 - y^2$, we have

$$m^2 - c^2$$

$$= (a-b)^2 - c^2$$

Using identity $(x-y)^2 = x^2 - 2xy + y^2$, the expression becomes

$$a^2 - 2ab + b^2 - c^2 \text{ (Ans)}$$

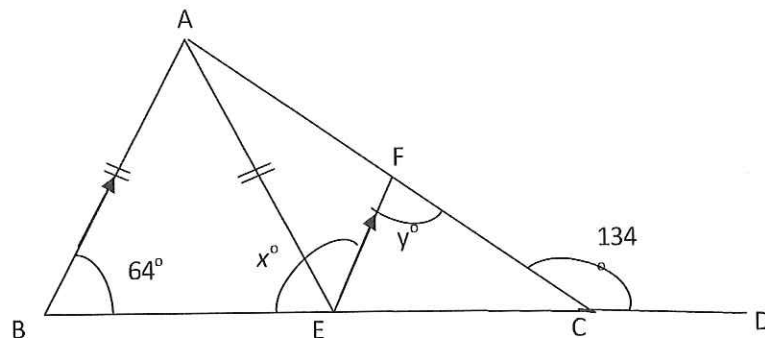
9. When the sun is directly overhead a 4 m rod is held in an inclined position so that its shadow is 3 m long. How much higher is one end of the rod than the other?

Ans: 4 cm is the inclined end of the rod. We can assume it as hypotenuse, 3 m is the base. We require to find the perpendicular height of a right angled triangle. We may use the Pythagoras theorem as follows:

$$\text{height} = \sqrt{(\text{hypotenuse}^2 - \text{base}^2)}$$

$$\text{Height} = \sqrt{(4^2 - 3^2)} = \sqrt{(16 - 9)} = \sqrt{7} = \underline{2.65 \text{ m (Ans)}}$$

10. Find the value of x and y in the adjoining diagram where similar markings show equal sides and similar arrowheads show parallel lines.



Ans: In triangle AB is parallel to EF and BE is transversal.
Therefore $x^\circ + 64^\circ = 180^\circ$ (Co-interior angles)

$$x^\circ = 180^\circ - 64^\circ = 116^\circ$$

Now, $\angle FEC$ is corresponding angle to $\angle ABE$
So, $\angle FEC = 64^\circ$

In triangle EFC, $\angle FCD$ is the external angle and $\angle FEC$ and $\angle EFC$ are opposite to it.
Since external angle = sum of internal opposite angles of a triangle

$$\text{We have, } y^\circ + 64^\circ = 134^\circ$$

$$\text{Or, } y^\circ = 134^\circ - 64^\circ = 70^\circ$$

The required angles are $x^\circ = 116^\circ$ and $y^\circ = 70^\circ$