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FOR GOD AND COUNTRY

St. Lawrence High School

A JESUIT CHRISTIAN MINORITY INSTITUTION

3rd Term Examination – 2019

Model Answer

Sub: Algebra and Geometry **Class: 8** **F.M: 80**

Duration: 2 Hour 30 minutes **Date: 27. 11. 2019**

Group – A

1. **Choose the correct option for the following questions.(Answer all the questions)** **1x5=5**

i) If $64^x = \frac{1}{256^y}$, then $3x + 4y$ equals to

a) 2 b) 4 c) 8 **d) 0**

ii) The value of $\frac{a+b}{\sqrt{(a-b)^2+4ab}}$ is

a) $(a+b)$ b) $(a-b)$ **c) 1** d) None of these.

Iii) If $-m > -2$, then which one is not a probable value of $'m'$?

a) -2 b) -1.01 c) 0 **d) 2.01**

iv) If $4x - 5 = 16 + 12x$, then x is ,

a) A fraction b) an integer **c)a rational number** d) a whole number

v) The sum of all the internal angles of a regular polygon of number of sides $(n-1)$ is

a) $(2n-4)\times 90^0$ b) $(2n-3)\times 90^0$ **c) $(2n-6)\times 90^0$** d) $(2n-2)\times 90^0$

2. **Fill in the blanks. (Answer all the questions)** **1x5=5**

i) The value of $(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1}$ is <u>44.</u>

ii) If, $x + \frac{1}{x} = 3$, then the value of $x - \frac{1}{x} = \underline{\sqrt{5}}$.

iii) Factorizing $\left(5x^2 - \frac{1}{5}\right)$ we get $\underline{5\left(x+\frac{1}{5}\right)\left(x-\frac{1}{5}\right)}$.

iv) Two times a number divided by 3 is 18. The number is <u>27</u>.

v) Each external angle of a regular polygon measures 18^0. The number of sides is <u>20</u>

3. **Write 'True' or 'False'. (Don't write 'T' or 'F'). (Answer all the questions)1x5=5**

i) The value of $(1^0 - 2^0 + 12^0)^{-1}$ is – 9. – **False.**

ii) $(-x+y)(-x-y) = (x+y)(x-y)$.- **True.**

iii) $a^2 + 6a - 9$ is a perfect square expression. . – **False.**

iv) Solution of $\frac{a}{0.25} = \frac{4}{5}$ will be greater than 1. – **False.**

v) The external angle formed by producing a side of any quadrilateral inside a circle, is always equal to the opposite interior angle. – **False.**

Group B

1. **Answer the following questions** **2 x 5 = 10**

1.1. Point H (a, b) is reflected in the x axis to H1(-7, 8). Write down the values of a and b. State which quadrant it lies in.

Ans: a= -7, b= -8, Quadrant= 3rd

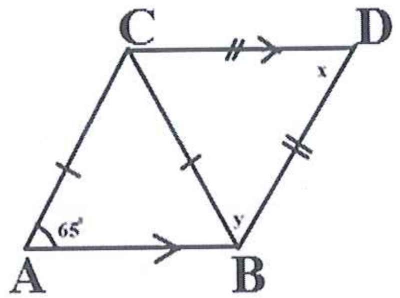
1.2. Solve for x: 3(x+1) + 4x =24

Ans: 3(x+1) + 4x =24

Or, 3x + 3 + 4x = 24

Or, 7x + 3 = 24
Or, 7x = 24-3
Or, 7x = 21
Or, x= 21/7 =**3 Ans**

1.3.Find the value of x and y:



Ans: Let the figure be name ABCD.
Angle CAB= 65°
Angle CBA = 65°
Since AC=BC, triangle ABC is isosceles.
Base angles of an isosceles triangles are equal.
Angle ACB = 180°-2x 65°= 50°
Sum of three angles of a triangle = 180°
Angle DCA = 180o- angle CAB =180°-65°=115°
AB is parallel to CD. Co interior angles are supplementary.
Now angle DCA = angle ACB + angle BCD
Or, 115° = 50° + angle BCD
Or, angle BCD = 115°-50°= 65°
Now, y= Angle DBC= 65°
Since BD= CD, triangle BCD is isosceles.
Base angles of an isosceles triangle are equal.
So, x= angle BDC = 180°-2x 65°= 50°
Sum of three angles of a triangle = 180°
Thus, x= 50°, y= 65° Ans

1.4.From the sum of x + 3y and -3x –y subtract x-y

Ans: (x+3y) + (-3x –y) – (x-y)
= x +3y -3x –y –x +y
=(x-3x-x) + (3y –y +y)
= **-3x+3y Ans**

1.5.If x= 2^k and y = 2^{k+3}, find the value of $\frac{x}{y}$

Ans: $\frac{x}{y} = \frac{2^k}{2^{k+3}} = 2^{k-k-3} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ **Ans**

2. Answer the following questions (any 5) **3 x 5 = 15**

2.1. Find the value of a-b, if a+b =3, ab=2

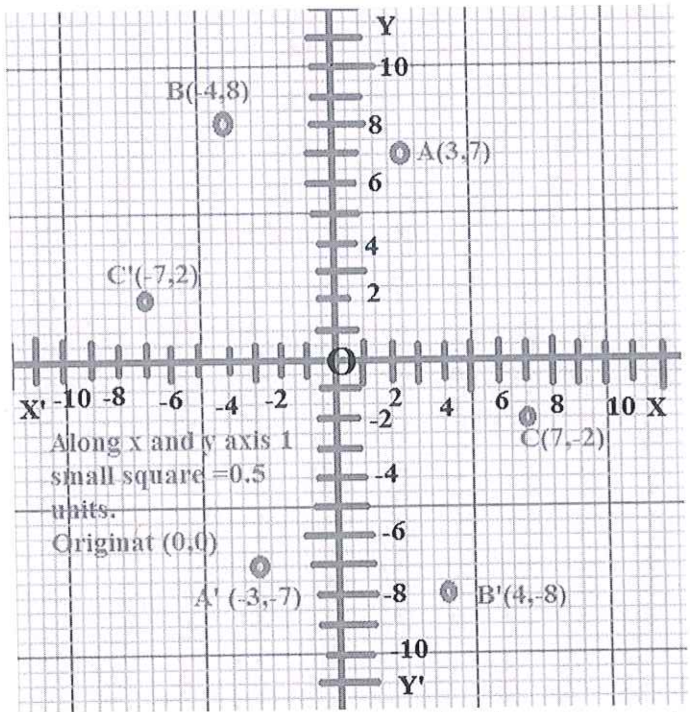
Ans. $(a-b)^2 = (a+b)^2 - 4ab = 3^2 - 4 \times 2 = 9-8 = 1$
So, a-b = √1 = **±1Ans**

2.2.Factorise $x^4-y^4+x^2-y^2$.

Ans: $x^4-y^4+x^2-y^2 = (x^2-y^2)(x^2+y^2) + x^2-y^2 = (x^2-y^2)(x^2+y^2+1)$=**$(x-y)(x+y)(x^2+y^2+1)$**
Using the identity $a^2-b^2= (a+b)(a-b)$

2.3.Write the coordinates of the following points when reflected in the origin.

(i) A(3,7), (ii) B(-4,8), (iii) C(7,-2)



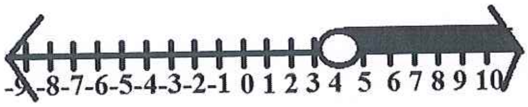
2.4.Solve the inequality in a system of real numbers and graph the solution set on a number line: **0> -4-p**

Ans: 0> -4-p

Or, 0< 4+p

Or, 4+p > 0

Or, **p>4**



Solution p={x:x>4 and x∈R}

2.5.The sum of three consecutive odd natural numbers is 87. What are the three numbers?

Ans: Let the first of the three numbers be x

Second number = x+2

Third number = x+4

According to the problem,

X + (x+2) + (x+4) = 87

Or, 3x + 6 = 87

Or, 3x = 87-6= 81

Or, x = 81/3 = 27

X+2 = 29,

X+4 = 31

Therefore the numbers are **27, 29, 31 Ans**

2.6.Solve for z: 3(z+1) +4(z+0.3) = 20z +0.95

Ans: 3(z+1) +4(z+0.3) = 20z +0.95

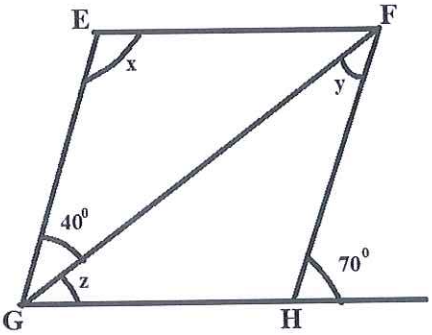
Or, 3z+ 3 + 4z + 1.2 = 20z + 0.95

Or, 3 z + 4z -20 z = 0.95 -1.2- 3

Or, -13 z = -3.25

Or z = 3.25/13 = **0.25 or ¼ Ans**

2.7.EFGH is a parallelogram. Find x, y and z. Also state the property you use to find them.



Ans: Angle FHG = 180°-70° (Linear Pair)
x= 110° (Opposite angles of a parallelogram are equal)
x+40°+ z = 180° (Co interior angles)
Or, z = 30°
And y = 40° (Alternate angle)
Thus **x= 110°, y = 40°, z = 30° Ans**

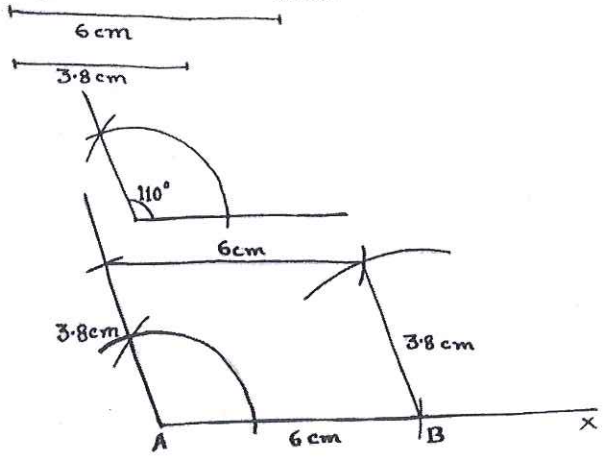
Group C

3. **Answer the following questions(any 8)** **5 x 8 = 40**
   i. Two cars A and B leave Mumbai at the same time, travelling in opposite directions. If the speed of car A is 8 Km/hr more than car B and they are 300 Km apart at the end of 6 hours, then calculate their speeds.

Ans: Let the speed of one car be x Km/hr
Speed of the other car = x+ 8 Km/hr
Distance covered in 6 hours = 6x + 6(x+8)
According to the problem,
6x + 6(x+8) = 300
Or, 12 x +48 = 300
Or, 12x = 200-48
Or, 12 x =252
Or, x = 252/12 = 21
Speed of one car = 21 Km/hr. Speed of the other car =(21 + 8) Km/hr = 29 Km hr
Speed of car A = 29 Km/hr, Speed of car B = 21 Km/hr Ans

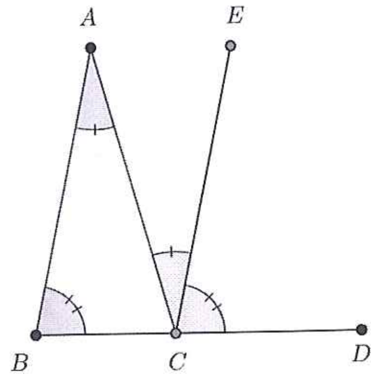
   ii. Construct a parallelogram ABCD with AB = 6 cm, BC = 3.8 cm and ∠A = 110°

Ans:



iii. If one side of a triangle is produced then prove that the external angle so formed is the sum of two opposite internal angles of a triangle.

Ans. Given: Let $\triangle ABC$ be a triangle.
Let BC be extended to a point D.
RTP: The external angle is the sum of two opposite internal angles of a triangle.
Construction: Construct CE through the point C parallel to the straight line AB.



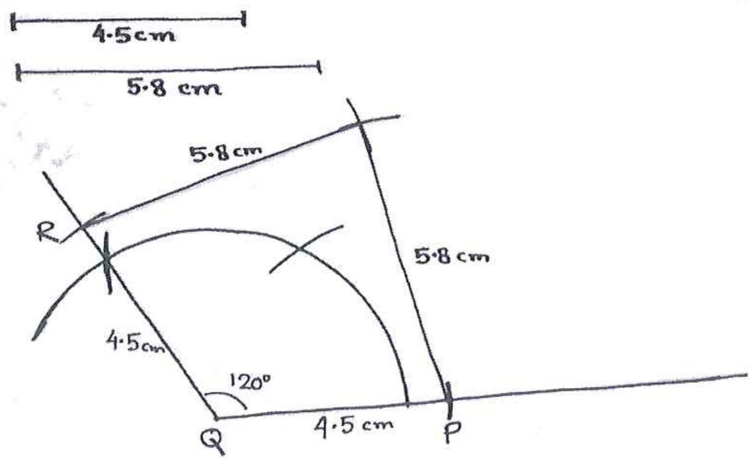
Proof:

Statement	Reason
We have that AB∥CE and AC is a transversal that cuts them. $\angle BAC = \angle ACE$	From Parallelism implies Equal Alternate Interior Angles
Similarly, we have that AB∥CE and BD is a transversal that cuts them. $\angle ECD = \angle ABC$	From Parallelism implies Equal Corresponding Angles
$\angle ACD = \angle ABC + \angle BAC$	By Euclid's Second Common Notion

Hence Proved

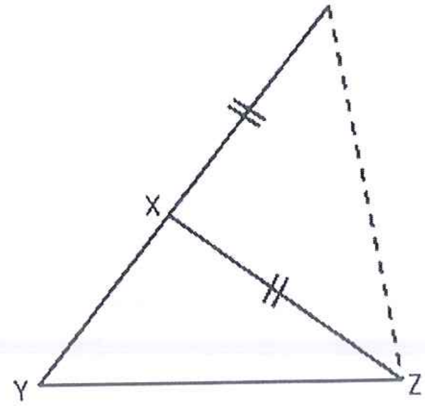
iv. Construct a quadrilateral PQRS in which PQ = QR = 4.5 cm, PS = RS = 5.8 cm and $\angle PQR = 120^\circ$.

Ans:



v. Prove that the sum of two sides of a triangle is greater than the third side.

Ans. **Given:** XYZ is a triangle.
Required To Prove: (XY + XZ) > YZ, (YZ + XZ) > XY and (XY + YZ) > XZ
Construction: Produce YX to P such that XP = XZ. Join P and Z.



Statement	**Reason**
1. ∠XZP = ∠XPZ.	1. XP = XZ.
2. ∠YZP > ∠XZP.	2. ∠YZP = ∠YZX + ∠XZP.
3. Therefore, ∠YZP > ∠XPZ.	3. From 1 and 2.
4. ∠YZP > ∠YPZ.	4. From 3.
5. In ΔYZP, YP > YZ.	5. Greater angle has greater side opposite to it.
6. (YX + XP) > YZ.	6. YP = YX + XP
7. (YX + XZ) > YZ. (Proved)	7. XP = XZ

Similarly, it can be shown that (YZ + XZ) >XY and (XY + YZ) > XZ.

Hence Proved

vi. Find the value of x such that $\left(\frac{7}{4}\right)^{-3} \times \left(\frac{7}{4}\right)^{-5} = \left(\frac{7}{4}\right)^{3x-2}$

Ans:

$$\left(\frac{7}{4}\right)^{-3} \times \left(\frac{7}{4}\right)^{-5} = \left(\frac{7}{4}\right)^{3x-2}$$

$$Or, \quad \left(\frac{7}{4}\right)^{-3-5} = \left(\frac{7}{4}\right)^{3x-2}$$

$$Or, \quad \left(\frac{7}{4}\right)^{-8} = \left(\frac{7}{4}\right)^{3x-2}$$

When bases are same powers can be equated.

$Or, -8 = 3x - 2$

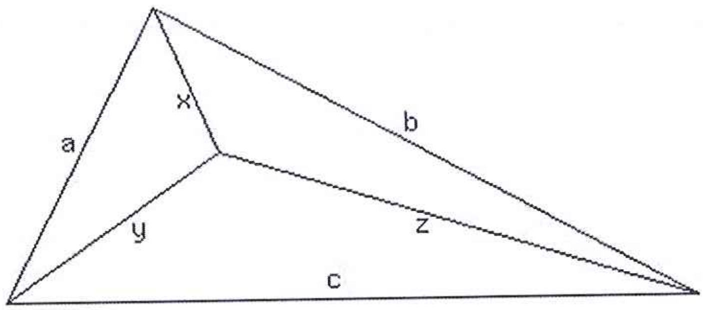
$Or, 3x = -8+2$

$Or, 3x = -6$

$Or, x = -6/3 =$ **-2 Ans**

vii. Prove that the sum of the distances of a point within a triangle from the vertices of the triangle is greater than the semi-perimeter.

Ans: Let the sides of the large triangle be a, b, and c.
Let the distances to the vertices from an interior point be x, y, and z.
Half the perimeter of the large triangle is $\frac{a+b+c}{2}$



RTP: $x + y + z > \frac{a+b+c}{2}$

Proof:

Statement	Reason
x + y > a x + z > b y + z > c	Using the triangular inequality on each of the three smaller triangles that make up the large triangle
x + y > a x + z > b y + z > c -------------------- 2x + 2y + 2z > a+b+c	Adding those three inequalities term by term

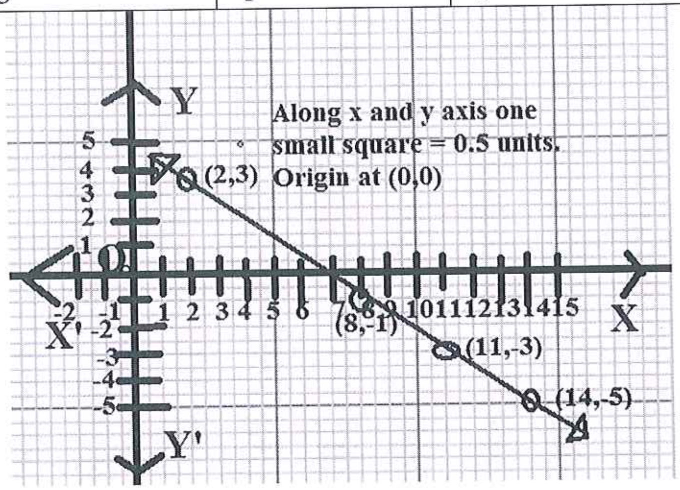
$x + y + z > \frac{a+b+c}{2}$	Dividing through by 2

Hence Proved

viii. Draw the graph of the line 2x + 3y = 13

Ans:

X	2	8	11	14
y	3	-1	-3	-5



ix. The angle between the bisectors of the vertical angle of a triangle and the perpendicular drawn from the vertex of the triangle on the base of the triangle is equal to half the difference of the angles at the base.

Ans: Given: ΔABC with vertical angle bisector AE, perpendicular from vertex to the base AD

Required to prove: $\angle DAE = \frac{1}{2}(\angle ABC - \angle ACB)$

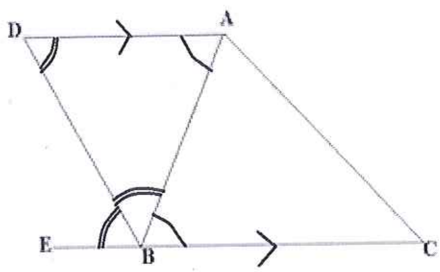
Proof:

Statement	Reason
∠BAC + ∠ABC + ∠ACB = 180°………(i)	Now in ΔABC, We know in a triangle the sum of all three interior angles is equal to 180°.
∠BAE = ∠CAE ⇒ ∠BAC = 2∠BAE	Given AE is angle bisector of ∠BAC
2∠BAE + ∠ABC + ∠ACB = 180°………(ii)	Substituting the above value in equation (i) we get
∠BAE = ∠BAD + ∠DAE	from figure,
2(∠BAD + ∠DAE) + ∠ABC + ∠ACB = 180° ⇒ 2∠BAD + 2∠DAE + ∠ABC + ∠ACB = 180°………(iii)	Substituting this value in equation (ii), we get
In right - angled ΔBAD, ∠ABD + ∠BAD = 90° ⇒ ∠ABC + ∠BAD = 90° ⇒ ∠BAD = 90° - ∠ABC…….(iv)	Given AD is perpendicular to BC, so ΔBAD and ΔDAE is right - angled triangle,
2(90° - ∠ABC) + 2∠DAE + ∠ABC + ∠ACB = 180° ⇒ 180° - 2∠ABC + 2∠DAE + ∠ABC + ∠ACB = 180°	Substituting equation (iv) in equation (iii), we get

$\Rightarrow 180° - \angle ABC + 2\angle DAE + \angle ACB = 180°$ $\Rightarrow 2\angle DAE = 180° - 180° + \angle ABC - \angle ACB$ $\Rightarrow 2\angle DAE = \angle ABC - \angle ACB$ $\Rightarrow \angle DAE = \frac{1}{2}(\angle ABC - \angle ACB)$ …..(v)	

Hence Proved

x. In triangle ABC, the external bisector of angle ABC and the parallel line of BC through A intersect each other at D. Prove that $\angle ADB = 90^{o} - \angle ABC/2$.



Ans.Given: In triangle ABC, the external bisector of $\angle$ ABC and the parallel line of BC through A intersect each other at D.

Required to prove: $\angle ADB = 90^{o} - \angle ABC/2$.

Proof:

Statement	Reason
$\angle ABC = \angle BAD$	Alternate angles
Ext $\angle ABC = 180^{o} - \angle ABC$	Linear pair
In triangle ADB,	Sum of three angles of a triangle = 180o
$\angle ADB + \angle BAD + \angle ABD = 180^{o}$	
$\angle ABD = ½$ ext $\angle ABC = ½ (180^{o} - \angle ABC)$	BD was bisector
$½ (180^{o} - \angle ABC) + \angle ABC + \angle ADB = 180^{o}$	Replacing the values
Solving, $\angle ADB = 90^{o} - \angle ABC/2$	

Hence Proved.