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FOR GOD AND COUNTRY

ST. LAWRENCE HIGH SCHOOL

Annual Examination – 2019

Sub: Mathematics | **Class: 9** | **F. M. 75**

Duration: 2 hrs 30 min | **Model Answers** | **Date: 20.11.19**

GROUP-A

Q.1) Choose the correct options: **1x6=6**

i) If the straight line 3x+4y =5 and 4mx-3y=2 are perpendicular, then m= (a)1
ii) If the equation 3x+4y=5 and 3x+ky=6 have no solution, then k= (c) 4
iii) If $\log_2 3$ = a, then $\log_8 27$ is: (d) a
iv) If in a number, digit in the unit place be y and digit in the tenth place is x, then the number will be: (c) 10x+y
v) Two successive discounts of 20% and 10% is equivalent to a discount of (c) 28%
vi) If the areas of a square and a rectangle of equal perimeter be S and R, then (c) S>R

Q.2) Fill in the blanks: **1x4=4**

i) The point (-4,-3) lies in the __3rd__ quadrant.
ii) If the frequency of a class 70-105 is 7, then the frequency density is __0.2__.
iii) If A = (6,0), B = (-6, 0) and C = (6,6), then area of triangle ABC is __36 sq. units__.
iv) If a/b + b/a = 1, then $a^3 + b^3$ is __0__.

Q.3) State whether the statement is True or False: **1x4=4**

i) The population of a village is variable. True
ii) The ratio of SP and CP is 8:9, then the loss is 20%. False
iii) The area of a square is half the square of the diagonal. True
iv) If the denominator of a proper fraction is 3 greater than the numerator (x), then the fraction will be x/3. False

GROUP-B

Q.4) Short answer type questions: **2x9=18**

i) The area of a rectangle is 60 sq cm and its perimeter is 34 cm. Find the length of the diagonal.

Let the length and breadth of rectangle be 'l' cm and 'b' cm respectively.
Area = lb = 60 cm -(i)
Perimeter = 2(l+b) = 34 cm -(ii)
Putting l as (17-b) in equation (i),
b(17-b) = 60
or, (b-5) (b-12) = 0
b=5 or b=12
Considering b=5,
∴ l = 60/b = 12
Length of diagonal = $\sqrt{l^2 + b^2}$ = 13 cm

ii) If the area of an equilateral triangle is 9√3 sq m, then find its median.

Let the length of side of the triangle be a m.

Area = $\frac{\sqrt{3}}{4}a^2 = 9\sqrt{3}$ or, a = 6

We know median of an equilateral triangle divides the opposite side into equal parts and is a perpendicular bisector.

∴ BTP, $\frac{\sqrt{3}}{4}a^2 = \frac{1}{2} \cdot a \cdot x$ (x is the median) or, $x = \frac{\sqrt{3}}{2}a = 3\sqrt{3}$ m

iii) A man gets CP of 120 mangoes by selling 110 mangoes, what is the profit %?

BTP, CP of 120 mangoes = SP of 110 mangoes. Let CP of 120 mangoes = SP of 110 mangoes = x

∴CP of 1 mango = $\frac{x}{120}$ and SP of 1 mango = $\frac{x}{110}$

∴Profit made by selling 1 mango = $\frac{x}{110} - \frac{x}{120} = \frac{x}{1320}$

Profit % = $\frac{Profit}{CP} \times 100$

$= 9\frac{1}{11}\%$

iv) Calculate the perimeter of a circle whose diameter is 14 cm.

Diameter of circle = 14 cm. ∴ Radius (r) = 7 cm and hence Perimeter of a circle = $2\pi r$ = 44 cm

v) Find the value of $\log_4\log_4\log_4 256$.

$\log_4\log_4\log_4 256 = \log_4\log_4\log_4 4^4 = 0$

vi) If x/y=5/16 and x+y=21 then find the value of x-y?

$\frac{x}{y} = \frac{5}{16}$ -(i)

x + y = 21 or, x = 21 – y -(ii). Putting x as (21 – y) in equation (i),we get y = 16 and solving x = 5

∴ x – y = 5 – 16 = -11

vii) If $x^2+y^2-2x+4y=-5$, then what are the values of x and y?

$x^2 + y^2 - 2x + 4y = -5$, or, $x^2 + y^2 - 2x + 4y + 5 = 0$, or, $(x-1)^2 + (y+2)^2 = 0$

We know if the sum of two squares is 0, it means they are individually 0.

∴ x-1 = 0 or, x = 1 and y = -2

viii) If the ratio of CP and SP is 25:26, then what is the % of profit?

CP : SP :: 25 : 26. Let CP and SP be 25x and 26x respectively.

∴ Profit = 26x – 25x = x and Profit% = $\frac{Profit}{CP} \times 100$, = 4%

ix) Find the area of a triangular field whose sides are 13m, 14m and 15m.

Sides of field are 13 m, 14 m and 15 m. Let a = 13 m , b = 14 m and c = 15 m

∴ Semi perimeter = $\frac{a+b+c}{2}$ = 21 m

We know area = $\sqrt{s(s-a)(s-b)(s-c)}$ = 84 m^2

Q.5) Short answer type questions (Attempt any 5): 3x5=15

i) Solve: 2/x+5/y=1 and 3/x+2/y=19/20. (by method of comparision).

$\frac{2}{x}+\frac{5}{y}=1$ -(i) , $\frac{3}{x}+\frac{2}{y}=\frac{19}{20}$ -(ii)

$\frac{2}{x}+\frac{5}{y}=1$ or, x = $\frac{xy-2y}{5}$ -(iii)

$\frac{3}{x}+\frac{2}{y}=\frac{19}{20}$ or, x = $\frac{19xy-60y}{40}$ -(iv)

Comparing equation (iii) and (iv), $\frac{xy-2y}{5} = \frac{19xy-60y}{40}$

or, x = 4

Putting x as 4 in equation (i), we get y = 10

ii) Factorise $a^3 - 2a^2 + 1$.

$f(a) = a^3 - 2a^2 + 1$

$f(1) = 1^3 - 2(1)^2 + 1$

= 0

∴ f(1) = 0

or, a = 1

∴ (a – 1) is a factor of the expression $a^3 - 2a^2 + 1$

$a^2(a-1) - a(a-1) - (a-1)$

$= (a-1)(a^2 - a - 1)$

iii) The medians BE and CF of triangle ABC intersects each other at G. Prove that triangle BCG = quadrilateral AFGE.

Here, medians bisect the triangle into two equal parts.

∴ ΔBEC = ΔBEA -(i)

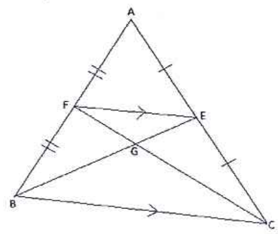
and ΔBCF = ΔACF

From the mid point theorem, FE // BC, ΔBCF = ΔBEC

ΔBCF - ΔBGC = ΔBEC - ΔBGC

∴ ΔCGE = ΔBGF -(ii)

∴ (i) – (ii) gives ΔBGC = quad AFGE



iv) Find the condition that the three points (a,b), (c,d), and (a-c, b-d) will be collinear.

The points are (a, b), (c, d), and (a – c, b – d).

For the pints to be collinear, the area of the triangle formed by them has to be 0.

∴ 0 = | a{d – (b – d)} + c(b – d – b) + (a – c) (b – d) |

or, ad – bc = 0

v) Find the measurement of square tiles of maximum size that may be used to cover a pavement of area 385mx 60m and also find the number of tiles required.

Area of pavement = 23100 m^2

Measurement of tiles is the HCF of 385 and 60.

The dimension of square tiles is 5m x 5m,

No. of tiles required = $\frac{Area\ of\ pavement}{Area\ of\ each\ tile}$

= 924 square tiles

vi) If the area of a circle is 616 sq cm then what is the length of its largest chord?

Area of a circle = 616 cm^2

Let the radius be r.

BTP, $\pi r^2 = 616$

or, r = 14 cm

We know that the largest chord of a circle is the diameter.

∴ Length of diameter = 2r = 28 cm

Group – C

(V) Answer the following questions:- **(ANY TWO)** (4 x 2 = 8)

1. If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$, then show that $x^a \cdot y^b \cdot z^c = 1$

   Let, $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k\ (k \neq 0)$

   Now, $\log(x^a \cdot y^b \cdot z^c) = \log x^a + \log y^b + \log z^c$

   $= k \times 0 = 0 = \log 1$

   So, $x^a \cdot y^b \cdot z^c = 1$(Proved)

2. A trader sells a television at 10% profit. If the cost price is less by 10% and selling price is less by ₹ 180, then the trader would make a 20 % profit. Calculate the cost price of the television.

   Let the cost price of the television be ₹ 100x.

   So, the SP is ₹ $100x \times \frac{110}{100}$ = ₹ 110x Hence, present SP is ₹ $90x \times \frac{120}{100} = 108x$

   According to the condition, 110x – 180 = 108x

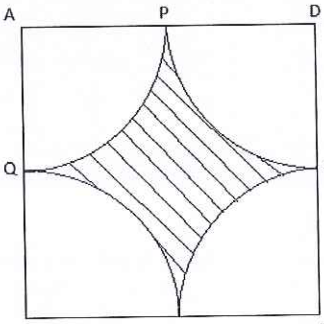
   ∴ The cost price of the television is ₹ 9000.

3. At each corner of a square garden there is a flower bed and the remaining part in the middle is ploughed for vegetables. If each flower bed is in the form of a sector of circular field with length of radius 3.5 m, then calculate the perimeter of middle part of the field of vegetables and area of the garden and how much land of area is ploughed for flower and for vegetable.

   Let ABCD be the square garden. A, B, C and D are the centres of circles with length of radius 3.5 m.

   APQ is a flower bed with a sector of circle of centre A.

   Length of arc $\widehat{PQ} = \frac{90}{360} \times$ perimeter of circle



∴ Perimeter of making vegetable bed = $4 \times \frac{11}{2}$ m = 22 m

Area of garden = $(AD)^2 = (2 \times 3.5)^2$ sq. m = 49 sq. m

The area of sector APQ = $\frac{90}{360} \times \frac{22}{7} \times (3.5)^2$ sq. m

∴ Ploughing for flower = $4 \times \frac{77}{8}$ sq. m = 38.5 sq. m

∴ The area of land for vegetable is (49 – 38.5) sq. m = 10.5 sq. m

(VI) Answer the following questions:- **(ANY FOUR)** (5 x 4 = 20)

1. If $a^{2-x} \cdot b^{5x} = a^{x+3} \cdot b^{3x}$, then show that $x \log \frac{b}{a} = \log \sqrt{a}$

   Cross multiplying the given pats and taking log on both sides, the result follows.

2. Prove that the three medians of a triangle are concurrent.

   Refer to Pg – 292 – Theorem 30 of the board book.

3. The medians BE and CF of Δ ABC intersect each other at G. P and Q are the midpoints of BG and CG respectively. Join P, F and Q , E. Prove that

   (i) PQEF is a parallelogram.

   (ii) Point G divides BE and CF in the ratio 2 : 1.

   Refer to Pg – 294 – Application 5 of the board book.

4. Factorise :- (x - 1) (x - 2) (x + 3) (x + 4) + 6

   (x - 1) (x - 2) (x + 3) (x + 4) + 6

   $= (x^2 - x + 3x - 3)(x^2 + 2x - 8) + 6$

   [We suppose,[$x^2 + 2x = a$], we get (a – 6) (a – 5), or $(x^2 + 2x - 6)(x^2 + 2x - 5)$

5. The present ages of A and B are in the ratio 9 : 4.Seven years hence, the ratio of their ages will be 5 : 3. Find their present ages.

   Let the ages of A and B be x and y respectively. $\therefore \frac{x}{y} = \frac{9}{4}$ -(i)

   BTP, $\frac{x+7}{y+7} = \frac{5}{3}$, or or, 3x = 5y + 14 -(ii)

   Putting x as $\frac{9y}{4}$ in equation (ii),we get y=8

   Putting y as 8 in equation (i),we get x=18

   ∴ The ages of A and B are 18yrs and 8yrs respectively.