ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-6
SUBJECT – STATISTICS

Pre-test

Chapter: RANDOM VARIABLE **Class: XII**

Topic: Random Variables **Date:18.05.2020**

RANDOM VARIABLES

PART 1

Random variable:

A variable which has a pre assigned set of observations or realizations but the actual outcome is not known prior to the experiment (a random variable is always denoted by a capital letter).

A random variable are of two types namely, viz, discrete random variable and continuous random variable.

If the set of observations be discrete, i.e., countable (finite or infinite) is known as discrete random variable and if the same be continuous i.e., uncountable whether finite or infinite are known as continuous random variable.

Let us consider the following example.

X: Face value of a die when rolled once

The set of observations is x = { 1, 2, 3, 4, 5, 6 }

Now P(X = 1) = $\frac{1}{6}$

P(X = 2) = $\frac{1}{6}$

P(X = 3) = $\frac{1}{6}$

P(X = 4) = $\frac{1}{6}$

P(X = 5) = $\frac{1}{6}$

P(X = 6) = $\frac{1}{6}$

So from the above example we can say P($X = x$) = $\frac{1}{6}$ for all x=1(1)6

We define $P(X = x) = f(x)$ which is known as probability distribution of the random variable X.

The probability distribution of a random variable is defined as the probability of a function of the observations along with the known parameters.

The probability distribution of a discrete random variable is known as probability mass function (pmf) and that of continuous random variable is known as probability density function (pdf).

Condition for pmf & pdf

i. For both pmf and pdf f(x) ≥ 0 for all x.
ii. For pmf $\sum_x f(x) = 1$and for pdf $\int f(x)dx = 1$

Note : The above conditions are basically in probability the axiom I and

axiom II.

Expectation of a random variable X.

Expectation of a random variable X is the population mean which is formulated as

$E(X) = \sum_x x.f(x)$ for discrete random variable and $E(X) = \int x\, f(x)dx$ for continuous random variable.

The question is that why do we say expectation as population mean ?

Consider the form of expectation of X and since $f(x)$

Is a probability $f(x) = \frac{n(x)}{n}$.

So $E(X) = \sum_x x.\frac{n(x).}{n}$ x: observation in sample space

$n(x)$: frequency of x in sample space

n: total frequency in sample space

The arithmetic mean $\bar{x} = \sum_i x_i \frac{f_i}{N}$

x_i: observation in sample

f_i: frequency of x in sample

N: total frequency in sample

So instead of sample if we talk about sample space, i.e., population we get the form of $E(X)$. Hence it is known as population mean.

Some important results in Expectation.

- If X=c , a constant , E(X) = c.

Pf: E(X) = $\sum_x x.f(x) = \sum_x c.f(x) = c \sum_x f(x) = cX1 = c$

- If Y=cX, c being a constant, then E(Y) = cE(X)

Pf: E(Y)= $\sum_y y.f(y) = \sum_x c.x.f(x) = c \sum_x xf(x) = cE(X)$

- If Y=a +bX, c being a constant, then E(Y) = a + bE(X)

Pf: E(Y)= $\sum_y yf(y) = \sum_x (a + bx) f(x) = a \sum_x f(x) + b \sum_x xf(x)$

$= a + bE(X)$

- Variance of X, $\sigma_X^2 = E(X - E(X))^2$

$= E(\, X^2 + \left(E(X)\right)^2 - 2X.E(X))$

$= E(X^2) + \left(E(X)\right)^2 - 2E(X).E(X)$

$= E(X^2) - \left(E(X)\right)^2$

- If Y= c , c being a constant, then V(Y) = 0

Pf: $\sigma_Y^2 = E(\, Y - E(Y))^2 = E(c - c)^2 = 0.$

- If Y= a +bX , a and b being a constants, then V(Y) = a +bV(Y)

Pf: $\sigma_Y^2 = E(\, Y - E(Y))^2 = E((a + bX - a - bE(X))^2$

$= E(b\ (X - E(X))^2$

$= b^2 E(x - E(X))^2$

$= b^2 \sigma_X^2$

$\Rightarrow\ \sigma_Y = |b|\sigma_X$

Prepared by

Sanjay Bhattacharya