ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-31

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Applications of derivatives **Class: XII**

Topic: Tangent & Normals **Date: 10.08.2020**

➢ Tangents and Normals : -

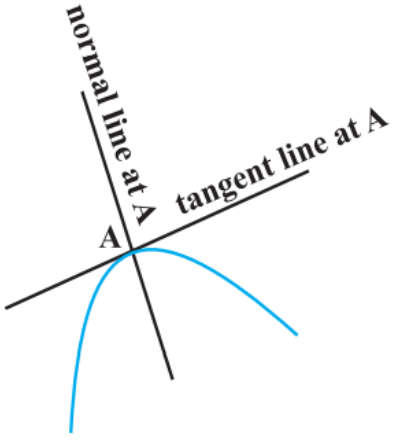
In this section, we shall use differentiation to find the equation of the tangent line and the normal line to a curve at a given point.

Recall that the equation of a straight line passing through a given point (x_0, y_0) having finite slope m is given by

$$y - y_0 = m\,(x - x_0)$$

Note that the slope of the tangent to the curve $y = f(x)$ at the point (x_0, y_0) is given by $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$ $(= f'(x_0))$. So the equation of the tangent at (x_0, y_0) to the curve $y = f(x)$ is given by

$$y - y_0 = f'(x_0)(x - x_0)$$



Also, since the normal is perpendicular to the tangent, the slope of the normal to the curve $y = f(x)$ at (x_0, y_0) is $\frac{-1}{f'(x_0)}$, if $f'(x_0) \neq 0$. Therefore, the equation of the normal to the curve $y = f(x)$ at (x_0, y_0) is given by

$$y - y_0 = \frac{-1}{f'(x_0)}(x - x_0)$$

i.e. $$(y - y_0) f'(x_0) + (x - x_0) = 0$$

Example Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at (1, 1).

Solution Differentiating $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ with respect to x, we get

$$\frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$$

or $$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Therefore, the slope of the tangent at (1, 1) is $\left.\frac{dy}{dx}\right]_{(1,1)} = -1$.

So the equation of the tangent at (1, 1) is

$$y - 1 = -1\,(x - 1) \quad \text{or} \quad y + x - 2 = 0$$

Also, the slope of the normal at (1, 1) is given by

$$\frac{-1}{\text{slope of the tangent at } (1,1)} = 1$$

Therefore, the equation of the normal at (1, 1) is

$$y - 1 = 1\,(x - 1) \quad \text{or} \quad y - x = 0$$

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