## ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

**STUDY MATERIAL - 3 (PART - II)**

**Subject : PHYSICS**

Date : 8.6.20

CLASS : XII

Chapter : Electrostatics

Topic : Capacitor

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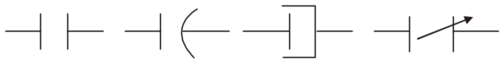
**D : Short Notes**

1. **Capacitors :** A capacitor is a device to store charge or electrostatic energy.
**Capacitance :** It is the capacity of the capacitor to store charge. In a capacitor $Q \propto V$ or $Q = CV$; $C$ is called the capacitance.

$$C = (M^{-1}L^{-2}T^{4}A^{2})$$

The following Fig. (a) or (b) represent circuit symbol for a simple capacitor. The Fig. (c) represents electrolytic and Fig (d) represents variable capcitor (tuner or trimmer).



a b c d

From the point of view of the shape of capacitors, they are of three types : spherical, parallel plate and cylindrical.

Unit of capacitance is Farad $1F = \frac{1C}{1V}$.

$1F$ is a very big unit. Therefore smaller units like $\mu F$, $nF$ or $\mu\mu F$ (also called $pF$) are used very commonly.

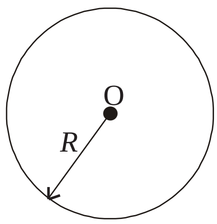
**Spherical Capacitors :** These are of two types :

(a) **Isolated spherical capacitors** (b) **Concentric spherical capacitors**

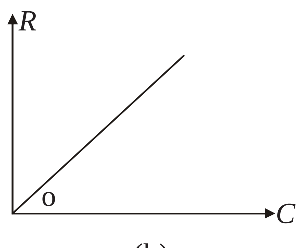
(a) Isolated spherical capacitor consists of a single sphere. Its capacitance $C = 4\pi\varepsilon_0 R$ i.e. $C \propto R$ where $R$ is radius of the sphere. See the Fig.

(b) Two spherical shells (or inner one may be solid) form a concentric spherical capacitor as shown in the following figure. Note that normally outer sphere is grounded.

$$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$




(a)
Isolated capacitor




(b)
Capacitance vs radius

If a dielectric of strength $K$ is introduced between $R_1$ and $R_2$

$$C = 4\pi\varepsilon_0 K \frac{R_1 R_2}{R_2 - R_1}$$

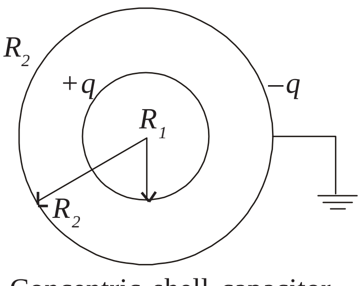



Fig. Concentric shell capacitor

2. **Parallel Plate Capacitor :**

If two plates each of area $A$ are separated by a distance $d$ in vacuum as shown in the Figure.

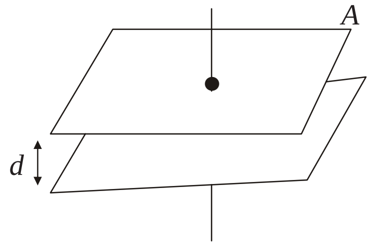



Fig. Parallel plate capacitor

then $C = \frac{\varepsilon_0 A}{d}$

$C = \frac{k\varepsilon_0 A}{d}$ if a dielectric of strength $k$ is completely filled in the gap.

$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{k}\right)}$ if the dielectric slab has thickness $t(t<d)$

3. Force between the plates of a capacitor is attractive and its magnitude is $F = \frac{Q^2}{2A\varepsilon_0}$

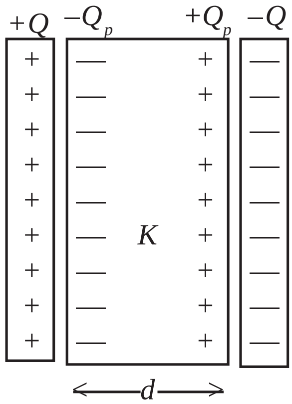



Fig. Polarization illustration

4. **Energy stored in a capacitor**

$$U = \frac{1}{2}CV^2 = \frac{QV}{2} = \frac{Q^2}{2C}$$

If the charge is uniformly distributed throughout the volume then energy stored $U = \frac{1}{2}\int V \rho dv$ where $dv$ is volume element and $V$ is potential difference. Volume density of electric field energy

$u = \frac{E\sigma}{2} = \frac{\varepsilon_0 E^2}{2}$ in free space

volume density of electric field energy in a medium $u_{med} = \frac{\varepsilon_0 K E^2}{2} = \frac{\varepsilon_0 \varepsilon_r E^2}{2}$.

5. **Capacitor in series :** In series magnitude of the charge on each plate is equal but voltage across each capacitor is different.

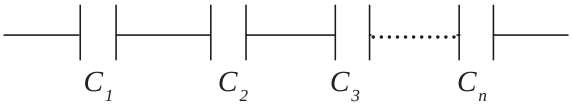



**Fig.** Capacitors in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots\ldots\ldots \frac{1}{Cn}$$

If $n$ equal capacitors are in series then $C_{eq} = \frac{C}{n}$

6. **Capacitors in Parallel**

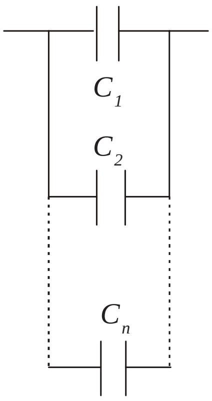



Fig : Capacitors in Parallel

If $C_1$, $C_2$, $C_3$, ........ $C_n$ are connected in parallel as shown in the above figure then

$C_{eq} = C_1, C_2, C_3, \ldots\ldots\ldots +C_n$

Note that in parallel, charge on each capacitor is different while potential drop or voltage across each capacitor is equal. If $n$ equal capacitors are in paraooel then $C_{eq} = nC$.

There are four methods to simplify capacitance networks

(a) Series parallel method

(b) Wheatstone bridge method

(c) Charge distribution method

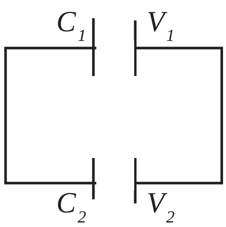
(d) Star/delta method.

7. **Charge Distribution Method :** It can be applied in principle anywhere in ture with Kirchhoff's law but in symmetrical circuits it makes the problem very simple. In symmetrical circuits charge entering a branch — charge leaving the branch (identical) or mirror image branch.

If two capacitors $C_1$ and $C_2$ charged to $V_1$ and $V_2$ are joined together then common potential is

$$V_{common} = \frac{V_1C_1+V_2C_2}{C_1+C_2} = \frac{Q_1+Q_2}{C_1+C_2}$$

Charge on capacitors after joining $\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$




Common potential

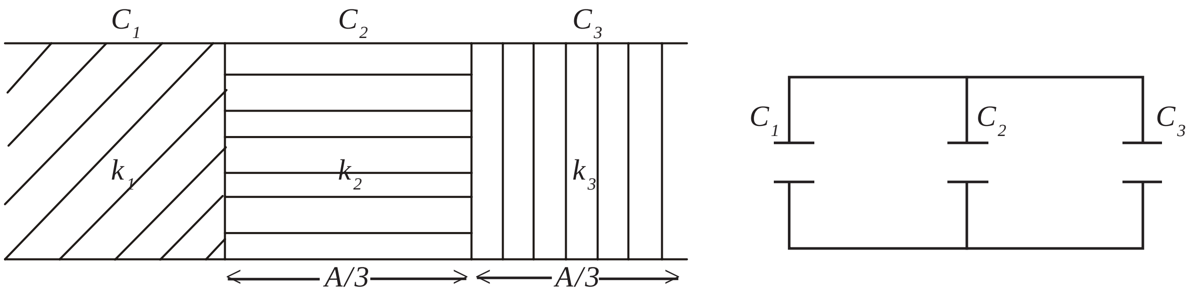
Loss in energy when two capacitors $C_1$ and $C_2$ charged to $V_1$ and $V_2$ are joined together as shown in the figure is

$$\Delta E = \frac{C_1 C_2}{(C_1 + C_2)}(V_1 - V_2)$$

If dielectrics are added in the manner shown in the following fig. (a) then net capacitance is in a parallel combination of $C_1$, $C_2$ and $C_3$ as illustrated in the fig. (b)

$$C_1 = \frac{\varepsilon_0 k_1 A/3}{d}, \qquad C_2 = \frac{\varepsilon_0 k_2 A/3}{d}$$

$$C_3 = \frac{\varepsilon_0 k_3 A/3}{d} \text{ and } \qquad C = C_1 + C_2 + C_3$$




(a) (b)

Fig. Effect of dielectrics on capacitor

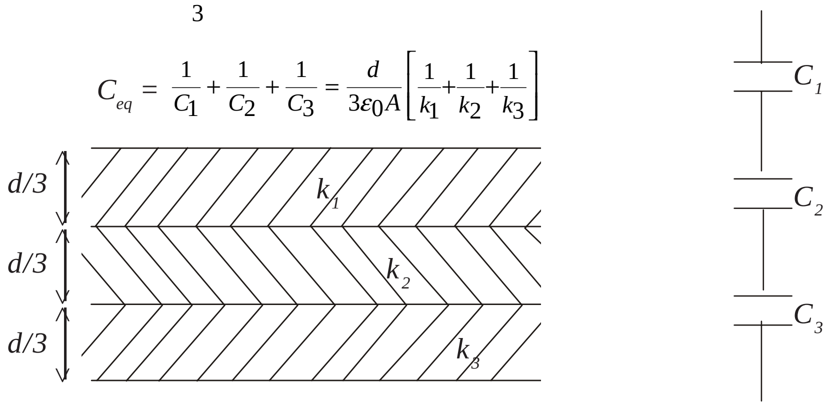
If dielectrics are arranged as shown in Fig. (a) then $C_{eq}$ IS series combination of $C_1$, $C_2$ and $C_3$ as illustrated in Fig. (b).

$$C_1 = \frac{\varepsilon_0 K_1 A}{\frac{d}{3}}$$

$$C_2 = \frac{\varepsilon_0 K_2 A}{\frac{d}{3}}$$

$$C_3 = \frac{\varepsilon_0 K_3 A}{\frac{d}{3}}$$

$$C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right]$$




(a) (b)

Fig. Effect of dielectrics in a capacitor

**E. Exercise Problems :**

1. A parallel plate capacitor with plate area 100 $cm^2$ and separation between the plate 5 mm is connected across a 24 $V$ battery. The force of attraction between the plates is of the order of —

Ans. : $$F = \frac{Q^2}{2A\varepsilon_0} = \frac{(CV)^2}{2A\varepsilon_0} = \frac{\left(\frac{A\varepsilon_0}{d}\right)^2 V^2}{2A\varepsilon_0}$$

$$= \frac{A\varepsilon_0 V^2}{2d^2} = \frac{10^{-2}\times 8.85\times 10^{-12}\times 24^2}{2\times 25\times 10^{-6}} = 1.08 \times 10^{-6}\text{N}.$$

2. A capacitor 10 $\mu$F charged to 50 $V$ is joined to another uncharged 50 $\mu C$ capacitor. Find the loss in energy.

**Ans :** Energy loss = $\frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2 = \frac{10\times 50\times 10^{-12}}{2(10+50)\times 10^{-6}}(50-0)^2 = 1.04\times 10^{-4}\,J$

3. Two spheres of radius 5 cm and 10 cm, both charged to 120 $\mu C$, are joined by a metal wire and then metal wire is removed. What is the charge on each after removal of the wire?

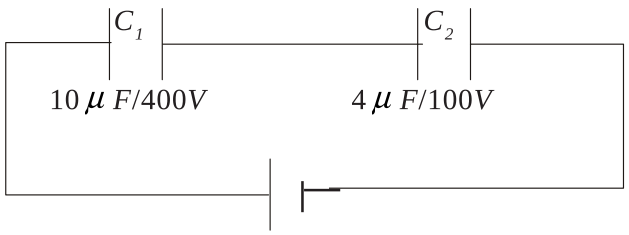
**Ans :** $Q_1' = \frac{(Q_1+Q_2)r_1}{r_1+r_2} = \frac{240\times 5}{15} = 80\mu C$

$Q_2' = 240 - 80 - = 160\,\mu C$

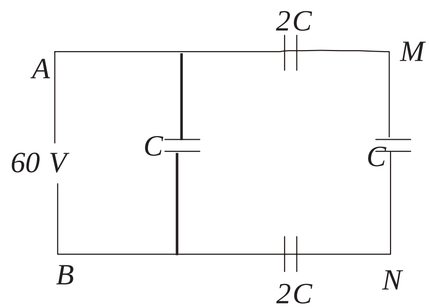
4. A 10 $\mu F$/400 $V$ and a 4 $\mu F$/100 $V$ capacitors are connected in series. Find the maximum potential which can be applied.

**Ans :** The maximum charge which can be applied is 400 $\mu C$ (maximum rating of 4 $\mu F$/100 $V$) capacitor.

Thus, potential which can be applied is 100 + 40 = 140 $V$.




5. In the circuit shown, a potential difference of 60 $V$ is applied across $AB$. The potential difference between the points $M$ and $N$ is




**Ans :** Let $Q$ amount of charge flow through the $MN$ branch

$V = 60\ V = \frac{Q}{2C} + \frac{Q}{C} + \frac{Q}{2C} = 2\frac{Q}{C}$ or $Q = 30C\ V$.

Potential difference between $M$ and $N$ = $\frac{Q}{C} = \frac{30C}{C}V$

= 30 $V$

*Ambarnath Banerjee*