ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-29

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Integration **Class: XII**

Topic: Indefinite integrals **Date: 20.07.2020**

Solved Examples (Part 3)

31. The value of $\int e^x \frac{(x^3+x+1)}{(1+x^2)^{3/2}} dx$ is equal to

(A) $\frac{xe^x}{(1+x^2)^{1/2}}+C$ **(B)** $\frac{x^2e^x}{(1+x^2)^{1/2}}+C$

(C) $\frac{e^x}{(1+x^2)^{1/2}}+C$ **(D)** None of these

32. $\int \frac{\cos 2x}{\cos x} dx$ is equal to

(A) $2\sin x+\log|(\sec x-\tan x)|+c$

(B) $2\sin x-\log|(\sec x-\tan x)|+c$

(C) $2\sin x+\log|(\sec x+\tan x)|+c$

(D) $2\sin x-\log|(\sec x+\tan x)|+c$

33. The value of $\int \frac{\sin x}{\cos^{3/2} x} dx$ is equal to

(A) $2\sqrt{\sin x}+c$ **(B)** $2\sqrt{\cos x}+c$

(C) $2\sqrt{\sec x}+c$ **(D)** $2\sqrt{\operatorname{cosec} x}+c$

34. The value of $\int e^x \left[\frac{x+2}{x+4}\right]^2 dx$ is equal to

(A) $\frac{e^x x}{x+4}+c$ **(B)** $\frac{e^x}{x+4}+c$

(C) $\frac{e^x}{(x+4)^2}+c$ **(D)** $\frac{e^x x^2}{x+4}+c$

35. The value of $\int\sqrt{\dfrac{x-a}{b-x}}\,dx$ is equal to

(A) $(a+b)\left[\dfrac{\sin 2\theta}{2}+\theta\right]+c$, where $a\sin^2\theta+b\cos^2\theta=x$

(B) $(a-b)\left[\dfrac{\sin 2\theta}{2}+\theta\right]+c$, where $a\sin^2\theta+b\cos^2\theta=x$

(C) $(a-b)\left[\dfrac{\sin 2\theta}{2}+\theta\right]+c$, where $a\sin^2\theta-b\cos^2\theta=x$

(D) $(a+b)\left[\dfrac{\sin 2\theta}{2}+\theta\right]+c$, where $a\sin^2\theta-b\cos^2\theta=x$

36. If $\int\dfrac{bx\cos 4x-a\sin 4x}{x^2}dx=\dfrac{a\sin 4x}{x}+c$, then a and b may be

(A) $a=2, b=2$ **(B)** $a=1, b=4$

(C) $a=-1, b=4$ **(D)** $a=\dfrac{1}{4},\ b=2$

37. If $\int\dfrac{dx}{x\sqrt{1-x^3}}=a\log\left|\dfrac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|+c$, then

(A) $a=\dfrac{1}{2}$ **(B)** $a=\dfrac{2}{3}$

(C) $a=\dfrac{1}{3}$ **(D)** $a=-\dfrac{2}{3}$

38. If $\int x\log_e(1+1/x)dx=P(x)\ln\left(1+\dfrac{1}{x}\right)+\dfrac{1}{2}x-\dfrac{1}{2}\ln(1+x)+c$,

then

(A) $p(x)=\dfrac{x^2}{2}$ **(B)** $p(x)=-1$

(C) $p(x)=1$ **(D)** None of these

39. $\int\dfrac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ is equal to

(A) $2\cos\sqrt{x}+c$ **(B)** $\sqrt{\dfrac{\cos x}{x}}+c$

(C) $\sin\sqrt{x}+c$ **(D)** $2\sin\sqrt{x}+c$

40. $\int\dfrac{1+\tan x}{x+\log\sec x}dx=$

(A) $\log(x+\log\sec x)+c$ **(B)** $-\log(x+\log\sec x)+c$

(C) $\log(x-\log\sec x)+c$ **(D)** None of these

41. If $\int\dfrac{(x^2-1)}{(x^4+3x^2+1)\tan^{-1}\left(\dfrac{x^2+1}{x}\right)}dx=k\log\left|\tan^{-1}\dfrac{x^2+1}{x}\right|+c$,

then k is equal to

(A) 1 **(B)** 2

(C) 3 **(D)** 5

42. The value of $\int\dfrac{\ln x}{(1+\ln x)^2}dx$ is equal to

(A) $\dfrac{x}{1-\ln x}+c$ **(B)** $\dfrac{x\ln x}{1+\ln x}+c$

(C) $\dfrac{x}{1+\ln x}+c$ **(D)** $\dfrac{\ln x}{x+x\ln x}+c$

43. The value of $\int\dfrac{\log(x/e)}{(\log x)^2}dx$ is equal to

(A) $\dfrac{x+1}{(\log x)^2}+c$ **(B)** $\dfrac{x-1}{(\log x)^2}+c$

(C) $\dfrac{x}{\log x}+c$ **(D)** $\dfrac{\log x}{x}+c$

44. $\int\dfrac{dx}{\cos(x-a)\cos(x-b)}=$

(A) $\operatorname{cosec}(a-b)\log\dfrac{\sin(x-a)}{\sin(x-b)}+c$

(B) $\operatorname{cosec}(a-b)\log\dfrac{\cos(x-a)}{\cos(x-b)}+c$

(C) $\operatorname{cosec}(a-b)\log\dfrac{\sin(x-b)}{\sin(x-a)}+c$

(D) $\operatorname{cosec}(a-b)\log\dfrac{\cos(x-b)}{\cos(x-a)}+c$

45. $\int\dfrac{dx}{\sqrt{x+a}+\sqrt{x+b}}=$

(A) $\dfrac{2}{3(b-a)}\left[(x+a)^{3/2}-(x+b)^{3/2}\right]+c$

(B) $\dfrac{2}{3(a-b)}\left[(x+a)^{3/2}-(x+b)^{3/2}\right]+c$

(C) $\dfrac{2}{3(a-b)}\left[(x+a)^{3/2}+(x+b)^{3/2}\right]+c$

(D) None of these

46. $\int\dfrac{3\cos x+3\sin x}{4\sin x+5\cos x}dx=$

(A) $\dfrac{27}{41}x-\dfrac{3}{41}\log(4\sin x+5\cos x)+c$

(B) $\dfrac{27}{41}x+\dfrac{3}{41}\log(4\sin x+5\cos x)+c$

(C) $\dfrac{27}{41}x-\dfrac{3}{41}\log(4\sin x-5\cos x)+c$

(D) None of these

47. If $\int(\sin 2x+\cos 2x)dx=\dfrac{1}{\sqrt{2}}\sin(2x-c)+a$, then the value of a and c is

(A) $c=\pi/4$ and $a=k$ (an arbitrary constant)

(B) $c=-\pi/4$ and $c=\pi/2$ $a=\pi/2$

(C) $c=\pi/2$ and a is an arbitrary constant

(D) None of these

48. $\int\dfrac{x^3-x-2}{(1-x^2)}dx=$

(A) $\log\left(\dfrac{x+1}{x-1}\right)-\dfrac{x^2}{2}+c$ **(B)** $\log\left(\dfrac{x-1}{x+1}\right)+\dfrac{x^2}{2}+c$

(C) $\log\left(\dfrac{x+1}{x-1}\right)+\dfrac{x^2}{2}+c$ **(D)** $\log\left(\dfrac{x-1}{x+1}\right)-\dfrac{x^2}{2}+c$

Solutions

31. $\int \frac{e^x\left[x(x^2+1)+1\right]}{(x^2+1)^{3/2}}dx$

$$= \int e^x\left[\frac{x}{(x^2+1)^{1/2}} + \frac{1}{(x^2+1)^{3/2}}\right]dx$$

(Since, $e^x[f'(x)+f(x)] = e^x f(x) + c$)

$$= \frac{xe^x}{(x^2+1)^{1/2}} + c$$

32. $\int \frac{2\cos^2 x - 1}{\cos x}dx$

$$= 2\int \cos x - \int \sec x \, dx$$

$$= 2\sin x - \log|(\sec x + \tan x)| + c$$

33. $\cos x = t \Rightarrow -\sin x \, dx = dt$

$$I = -\int t^{-3/2}dt = -\frac{t^{-3/2+1}}{-\frac{3}{2}+1} + c = \frac{2}{\sqrt{t}} + c = 2\sqrt{\sec x} + c$$

34. $\int e^x\left(\frac{x+2}{x+4}\right)^2 dx=\int e^x\left(1-\frac{2}{x+4}\right)^2 dx$

$$=\int e^x\left[1-\frac{4}{x+4}+\frac{4}{(x+4)^2}\right]dx$$

$$=e^x+\int e^x\left[-\frac{4}{x+4}+\frac{4}{(x+4)^2}\right]dx$$

$$=e^x-\frac{4e^x}{x+4}=\frac{xe^x}{x+4}+c$$

35. $\int\sqrt{\frac{x-a}{b-x}}dx$

Let $x=a\sin^2\theta+b\cos^2\theta$. Then

$dx=(2a\sin\theta\cos\theta-2b\cos\theta\sin\theta)d\theta=(a-b)\sin2\theta\cdot d\theta$

$$=\int\sqrt{\frac{\cos^2\theta(b-a)}{(b-a)\sin^2\theta}}\times(a-b)\sin2\theta\,d\theta=2\int(a-b)(\cos^2\theta)d\theta$$

$$=(a-b)\left(\theta+\frac{1}{2}\sin2\theta\right)+c$$

36. $\left[\frac{\sin4x}{4x}-\int\frac{-\sin4x}{4x^2}dx\right]-\int\frac{a}{x^2}\sin4x\,dx$

$$=\frac{b}{4}\frac{\sin4x}{x}+\left(\frac{b-a}{4}\right)\frac{\sin4x}{x^2}dx$$

Hence, $a=1, b=4$.

37. $\int\frac{dx}{x\sqrt{1-x^3}}=\int\frac{x^2dx}{x^3\sqrt{1-x^3}}$

Let $1-x^3=t^2$. Then $-3x^2dx=2tdt$.

$$I=-\frac{2}{3}\int\frac{tdt}{(1-t^2)t}=-\frac{2}{3}\int\frac{dt}{1-t^2}=\frac{2}{3}\int\frac{dt}{t^2-1}$$

$$=\frac{1}{3}\log\frac{t-1}{t+1}=\frac{1}{3}\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1+x^3}+1}\right|$$

$$\Rightarrow a=\frac{1}{3}$$

38. $\int x\log\left(1+\frac{1}{x}\right)dx=P(x)\ln\left(1+\frac{1}{x}\right)+\frac{x}{2}-\frac{1}{2}\ln(1+x)+c$

$$\text{LHS}=\log\left(1+\frac{1}{x}\right)\cdot\frac{x^2}{2}-\int\frac{-\frac{1}{x^2}}{\left(1+\frac{1}{x}\right)}\cdot\frac{x^2}{2}dx$$

$$=\log\left(1+\frac{1}{x}\right)\cdot\frac{x^2}{2}+\frac{1}{2}\int\frac{x}{x+1}dx=\frac{x^2}{2}\ln\left(1+\frac{1}{x}\right)+\frac{x}{2}-\frac{1}{2}\ln(1+x)+c$$

Hence, $p(x)=\frac{x^2}{2}$.

39. Let $\sqrt{x}=t$. Then $\frac{1}{2\sqrt{x}}dx=dt$.

$$I=2\int\cos dt=2\sin t+c$$

$$=2\sin\sqrt{x}+c$$

40. $x+\log\sec x=t\Rightarrow\left(1+\frac{\sec x\tan x}{\sec x}\right)dx=dt$

Hence, $I=\log(x+\log\sec x)+c$.

41. Let $\tan^{-1}\left(\frac{x^2+1}{x}\right)=\theta$. Then

$$\frac{1}{1+\left(\frac{x^2+1}{x}\right)^2}\times\frac{x(2x)-(x^2+1)}{x^2}dx=d\theta$$

$$\Rightarrow\frac{(x^2-1)}{(x^4+3x^2+1)}dx=d\theta$$

$$\Rightarrow I=\log\left|\tan^{-1}\left(\frac{x^2+1}{x}\right)\right|+c$$

Hence, the correct answer is (1).

42. $\int\frac{\ln x}{(1+\ln x)^2}dx=\int\frac{1}{1+\ln x}dx-\int\frac{1}{(1+\ln x)^2}dx$

$$=\frac{x}{1+\ln x}+\int\frac{x\cdot\frac{1}{x}dx}{(1+\ln x)^2}-\int\frac{1}{(1+\ln x)^2}dx$$

$$=\frac{x}{1+\ln x}+c$$

43. $\int\frac{\log(x/e)}{(\log x)^2}dx=\int\left[\frac{1}{\log x}-\frac{1}{(\log x)^2}\right]dx$

$$=\frac{x}{\log x}+\int\frac{x}{x(\log x)^2}dx-\int\frac{1}{(\log x)^2}dx$$

$$=\frac{x}{\log x}+c$$

44. $\int\frac{dx}{\cos(x-a)\cos(x-b)}$

$$=\frac{1}{\sin(a-b)}\int\frac{\sin((x-b)-(x-a))}{\cos(x-a)\cos(x-b)}dx$$

$$=\frac{1}{\sin(a-b)}\int\left(\frac{\sin(x-b)}{\cos(x-b)}-\frac{\sin(x-a)}{\cos(x-a)}\right)dx$$

$$=\frac{1}{\sin(a-b)}\ln\left|\frac{\cos(x-a)}{\cos(x-b)}\right|+c$$

45. $\int\frac{dx}{\sqrt{x+a}+\sqrt{x+b}}=\int\frac{(\sqrt{x+a}-\sqrt{x+b})dx}{(x+a-x-b)}$

$$=\frac{1}{(a-b)}\int(\sqrt{x+a}-\sqrt{x+b})dx=\frac{2}{3(a-b)}((x+a)^{3/2}-(x+b)^{3/2})+c$$

46. $3\cos x+3\sin x=a(4\sin x+5\cos x)+b\frac{d}{dx}(4\sin x+5\cos x)$

$$3\cos x+3\sin x=\cos x(5a+4b)+\sin x(4a-5b)$$

Compare the coefficients of $\sin x$ and $\cos x$ on the both sides

$$(5a+4b)=3,\ (4a-5b)=3$$

$$a=\frac{27}{41},\ b=-\frac{3}{41}$$

$$\int \frac{3\cos x + 3\sin x}{4\sin x + 5\cos x} dx = \int \frac{27}{41} dx - \frac{3}{41} \int \frac{4\cos x - 5\sin x}{4\sin x + 5\cos x} dx$$

$$\int \frac{3\cos x + 3\sin x}{4\sin x + 5\cos x} dx = \frac{27}{41} x - \frac{3}{41} \ln|4\sin x + 5\cos x| + c$$

47. $\int (\sin 2x + \cos 2x) dx = -\frac{\cos 2x}{2} + \frac{\sin 2x}{2} + k$

$$= \frac{1}{\sqrt{2}}\left(-\frac{\cos 2x}{\sqrt{2}} + \frac{\sin 2x}{\sqrt{2}}\right) + k = \frac{1}{\sqrt{2}} \sin\left(2x - \frac{\pi}{4}\right) + k$$

So, $c = \frac{\pi}{4}$ and $a = k$, an arbitrary constant.

48. $\int \frac{x^3 - x - 2}{1 - x^2} dx = \int \frac{x(x^2 - 1)}{1 - x^2} dx - \int \frac{2}{1 - x^2} dx$

$$= -\int x\, dx + \int \frac{2}{x^2 - 1} dx = -\frac{x^2}{2} + \ln\left|\frac{x-1}{x+1}\right| + c$$

COMPILED BY –

MR. SUKUMAR MANDAL (SKM)