ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-10

SUBJECT – MATHEMATICS

1st term

Chapter : Sequence & Series **Class : XI**

Topic: Arithmetic Progression **Date: 02.07.2020**

Progression :-

When terms of a sequence are written under specific conditions, then the sequence is called a **progression**.

A progression is represented as $t_1, t_2, \ldots, t_n$ or $a_1, a_2, \ldots, a_n$ where t_1 or a_1 means the first term, and t_n or a_n means the n^{th} term. t_k or a_k is called the general term of the progression and its n^{th} term is always expressible in terms of n. The number of terms of progression can be finite or infinite.

Arithmetic Progression (AP) :-

A sequence is called an arithmetic progression (AP) if its terms continually increase or decrease by the same number. The fixed number by which terms increase or decrease is called the common difference.

OR

A sequence of numbers $\{a_n\}$ is called an AP if there is a number d, such that $d = a_n - a_{n-1}$ for all n. d is called the common difference (CD) of the AP.

Example: Common difference of

(a) 2, 6, 10, 14 is 4
(b) 10, 5, 0, –5, –10 is –5
(c) $a, a+d, a+2d, a+3d$ is d.

General term of an AP :-

Let a be the first term and d the difference on an AP. Let T_1, T_2, T_3, …., T_n denote 1st, 2nd, 3rd, …., nth terms, respectively. Then we have

$$T_2 - T_1 = d$$

$$T_3 - T_2 = d$$

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$$T_n - T_{n-1} = d$$

Upon adding these, we get

$$T_n - T_1 = (n-1)d \Rightarrow T_n = T_1 + (n-1)d$$

But $T_1 = a$. Therefore, general term $= T_n = a + (n-1)d$.

Thus, if a is the first term and d is the common difference of an AP, then the AP is $a, a+d, a+2d, \ldots, a+(n-1)d$ or $a, a+d, a+2d, \ldots,$ accordingly as it is finite or infinite.

If the number of terms of an AP is n and the value of the last term is l, then

$$l = T_n = a + (n-1)d$$

If a is the first term and d is the common difference, then AP can be written as

$$a, a+d, a+2d, \ldots, a+(n-1)d$$

The n^{th} term of AP is

$$T_n = a + (n-1)\, d$$

where $d = T_n - T_{n-1}$.

The n^{th} term of this AP from the last, if last term l is given is

$$T'_n = l - (n-1)d$$

If the n^{th} term of AP from stating is T_n and from last is T'_n, then

$$T_n + T'_n = a + l$$

Examples :-

Example 1. The n^{th} term of an AP is $4n - 1$. Write down the first 4 terms and the 18^{th} term of the AP.

Solution: Given $T_n = 4n - 1$. Putting $n = 1, 2, 3, 4, \ldots, 18$, we get

$$T_1 = 3, T_2 = 7, T_3 = 11, T_4 = 15 \text{ and } T_{18} = 71$$

Example 2. The 8^{th} term of a series in the AP is 23 and the 102^{th} term is 305 in the series. Find the series.

Solution: Given

$$T_8 = a + 7d = 23$$

$$T_{102} = a + 101d = 305$$

Solving the two equations, we get

$$a = 2, d = 3$$

Now the series is $2, 5, 8, 11, \ldots$

Example 3. If p times the p^{th} term of an AP is equal to q times the q^{th} term, show that the $(p+q)^{th}$ term is zero.

Solution: Given that $p \cdot t_p = q \cdot t_q$.

If a is the first term and d is the common difference then

$$p[a+(p-1)\,d] = q[a+(q-1)d]$$
$$\Rightarrow pa + p(p-q)d = qa + q(q-1)d$$
$$\Rightarrow (p-q)a = q^2d - qd - p^2d + pd$$
$$\Rightarrow (p-q)a = d(q^2-p^2) - d\,(q-p)$$
$$\Rightarrow (p-q)a = d(q+p)\,(q-p) - d\,(q-p)$$
$$\Rightarrow -a = d(q+p-1)$$
$$\Rightarrow a + [(q+p)-1]d = 0$$
$$\Rightarrow t_{p+q} = 0$$

Example 4. If a sequence of numbers $a_1, a_2, \ldots, a_n$ satisfies the relation $a_{n+1}^2 = a_n \cdot a_{n+2} + (-1)^n$ then find a_3, if $a_1 = 2$ and $a_2 = 5$.

Solution: Put $n = 1$ in the given relation. We get

$$a_2^2 = a_1a_3 + (-1)^1 \Rightarrow 5^2 = 2a_3 - 1 \Rightarrow 2a_3 = 26 \Rightarrow a_3 = 13$$

Example 5. If a, b and c are the x^{th}, y^{th} and z^{th} terms of an AP, show that

(A) $a(y-z)+b(z-x)+c(x-y)=0$

(B) $x(b-c)+y(c-a)+z(a-b)=0$

Solution: Let A be the first term and D be the common difference. The x^{th}, y^{th}, z^{th} terms are given by

$$T_x = A+(x-1)D = a \qquad (1)$$

$$T_y = A+(y-1)D = b \qquad (2)$$

$$T_z = A+(z-1)D = c \qquad (3)$$

Equation (2) – Eq. (3), Eq. (3) – Eq. (1) and Eq. (1) – Eq. (2), respectively, give

$$(b-c)=(y-z)D \Rightarrow (y-z)=\frac{b-c}{D},$$

$$(c-a)=(z-x)D \Rightarrow (z-x)=\frac{c-a}{D},$$

$$(a-b)=(x-y)D \Rightarrow (x-y)=\frac{a-b}{D}$$

(A) Now substituting the values of $(y-z)$, $(z-x)$ and $(x-y)$ in LHS of the expression (A), we get

$$\text{LHS} = \frac{a(b-c)}{D}+\frac{b(c-a)}{D}+\frac{c(a-b)}{D}$$

$$= \frac{ab-ac+bc-ab+ca-cb}{D} = 0 = \text{RHS}$$

(B) Now substituting the values of $(b-c)$, $(c-a)$ and $(a-b)$ in LHS of the expression (B), we get

$$\text{LHS} = x(y-z)D+y(z-x)D+z(x-y)D$$

$$= \{xy-xz+yz-xz+zx-zy)\}D = 0 = \text{RHS}$$

-Prepared by

Mr. Sukumar Mandal