ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-9

SUBJECT – MATHEMATICS

Pre-test

Chapter: Limit **Class: XII**

Topic: Limit **Date: 11.06.2020**

-: LIMIT :-

➢ Some important limits:-

1. If n is a rational number, then $\lim_{x\to 0}\frac{(1+x)^n-1}{x} = n$.
2. $\lim_{x\to 0}\frac{e^x-1}{x} = 1$
3. $\lim_{x\to 0}\frac{1}{x}\log_e(1+x) = 1$
4. $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$
5. $\lim_{x\to \infty}(1+\frac{1}{x})^x = e$
6. $\lim_{x\to 0}\frac{a^x-1}{x} = \log_e a \ [a > 0]$
7. $\lim_{x\to 0}\frac{\sin x}{x} = 1$
8. $\lim_{x\to 0}\frac{x^n-a^n}{x-a} = na^{n-1}$; where n is a rational number.

➢ Some Important Solved Problems :-

Example 1. $\lim_{h\to 0}\frac{e^{\tan h}-1}{h}=?$

Soln :- $\lim_{h\to 0}\frac{e^{\tan h}-1}{\tan h}\cdot\frac{\tan h}{h}=\lim_{h\to 0}\frac{e^{\tan h}-1}{\tan h}\cdot\lim_{h\to 0}\frac{\tan h}{h}=1\cdot\lim_{h\to 0}\frac{\tan h}{h}=\lim_{h\to 0}\frac{\frac{\sin h}{\cos h}}{h}$

$=\lim_{h\to 0}\frac{\sin h}{h}\cdot\lim_{h\to 0}\frac{1}{\cos h}=1\cdot 1=1$ (Ans.)

Example 2. $\lim_{x\to 0}\frac{(e^x-1)\log_e(1+x)}{\sin x}=?$

Soln :- $\lim_{x\to 0}\left(\frac{\frac{(e^x-1)}{x}\cdot\frac{\log_e(1+x)}{x}\cdot x}{\frac{\sin x}{x}}\right)=\frac{\lim_{x\to 0}\frac{(e^x-1)}{x}\cdot\lim_{x\to 0}\frac{\log_e(1+x)}{x}\cdot\lim_{x\to 0}x}{\lim_{x\to 0}\frac{\sin x}{x}}=\frac{1\times 1\times 0}{1}=0$ (Ans.)

Example 3. $\lim_{x\to 0}\frac{\sqrt{(1+x)}-1}{\log_e(1+x)}=?$

Soln :- $\lim_{x\to 0}\frac{\sqrt{(1+x)}-1}{\log_e(1+x)}=\lim_{x\to 0}\left(\frac{\frac{\sqrt{(1+x)}-1}{x}}{\frac{\log_e(1+x)}{x}}\right)=\left(\frac{\lim_{x\to 0}\frac{\sqrt{(1+x)}-1}{x}}{\lim_{x\to 0}\frac{\log_e(1+x)}{x}}\right)=\lim_{x\to 0}\frac{\sqrt{(1+x)}-1}{x}$

$=\lim_{x\to 0}\frac{(\sqrt{(1+x)}-1)(\sqrt{(1+x)}+1)}{x(\sqrt{(1+x)}+1)}=\lim_{x\to 0}\frac{1}{(\sqrt{(1+x)}+1)}=\frac{1}{2}$ (Ans.)

Example 4. $\lim_{x\to 0}\frac{\sin\log(1+x)}{x}=?$

Soln :- $\lim_{x\to 0}\frac{\sin\log(1+x)}{x}=\lim_{x\to 0}\frac{\sin\log(1+x)}{\log(1+x)}\cdot\lim_{x\to 0}\frac{\log(1+x)}{x}$

$[\text{Now, let } \log(1+x)=y.\ \therefore \textit{when } x\to o,\ y\to 0]$

$=\lim_{y\to 0}\frac{\sin y}{y}\cdot\lim_{x\to 0}\frac{\log(1+x)}{x}=1\times 1=1$ (Ans).

Example 5. $\lim_{x\to 1}\frac{4^x-1}{x-1}=?$

Soln :- $\lim_{x\to 1}\frac{4^x-1}{x-1}=\lim_{x\to 1}\frac{4(4^{x-1}-1)}{x-1}=4\cdot\lim_{(x-1)\to 0}\frac{(4^{x-1}-1)}{x-1}=4\cdot\log_e 4=8\log_e 2$ (Ans.)

Example 6. $\lim\limits_{x\to 2}\frac{\log(2x-3)}{2(x-2)} = ?$

Soln :- $\lim\limits_{x\to 2}\frac{\log(2x-3)}{2(x-2)} = \lim\limits_{2x\to 4}\frac{\log\{(2x-4)+1\}}{(2x-4)} = \lim\limits_{2x-4\to 0}\frac{\log\{(2x-4)+1\}}{(2x-4)} = 1$ **(Ans.)**

Example 7. $\lim\limits_{x\to 1} x^{\frac{1}{1-x}} = ?$

Soln :- $\lim\limits_{x\to 1} x^{\frac{1}{1-x}} = \lim\limits_{x\to 1}(1+x-1)^{\frac{1}{x-1}\cdot(-1)} = \left[\lim\limits_{x-1\to 0}\{1+(x-1)\}^{\frac{1}{x-1}}\right]^{-1} = e^{-1}$ **(Ans.)**

Example 8. $\lim\limits_{x\to 0}(1+4x)^{\frac{x+2}{x}} = ?$

Soln :- $\lim\limits_{x\to 0}(1+4x)^{\frac{x+2}{x}} = \lim\limits_{x\to 0}(1+4x)^{\frac{1}{4x}\cdot(4x+8)} = \left[\lim\limits_{4x\to 0}(1+4x)^{\frac{1}{4x}}\right]^{\lim\limits_{x\to 0}(4x+8)} = e^{8}$ **(Ans.)**

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