ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-3

SUBJECT – MATHEMATICS

1st term

Chapter: Trigonometry **Class: XI**

Topic: Multiple angles **Date: 19.06.2020**

➢ Trigonometric ratios of multiple angles : -

Identities involving sin2A, cos2A, tan2A, sin3A etc., are called multiple angle identities.

(i) $\sin 2A = \sin(A+A) = \sin A\cos A + \cos A\sin A = 2\sin A\cos A.$

(ii) $\cos 2A = \cos(A+A) = \cos A\cos A - \sin A\sin A = \cos^2 A - \sin^2 A$

(iii) $\sin 3A = \sin(2A+A)$

$= \sin 2A\cos A + \cos 2A\sin A$

$= (2\sin A\cos A)\cos A + (1-2\sin^2 A)\sin A$

$= 2\sin A\cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A(1-\sin^2 A) + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

Thus we have the following multiple angles formulae

1. (i) $\sin 2A = 2\sin A\cos A$

   (ii) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

2. (i) $\cos 2A = \cos^2 A - \sin^2 A$

   (ii) $\cos 2A = 2\cos^2 A - 1$

   (iii) $\cos 2A = 1 - 2\sin^2 A$

   (iv) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

3. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

4. $\sin 3A = 3\sin A - 4\sin^3 A$

5. $\cos 3A = 4\cos^3 A - 3\cos A$

6. $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

➤ Solved examples :-

Example 1

Show that $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$

Solution

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Example 2

If $\tan A = \frac{1}{7}$ and $\tan B = \frac{1}{3}$, show that $\cos 2A = \sin 4B$

Solution

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{1}{49}}{1+\frac{1}{49}} = \frac{48}{49}\times\frac{49}{50} = \frac{24}{25} \quad \ldots (1)$$

Now $\sin 4B = 2\sin 2B \cos 2B$

$$= 2\frac{2\tan B}{1+\tan^2 B}\times\frac{1-\tan^2 B}{1+\tan^2 B} = \frac{24}{25} \quad \ldots (2)$$

From(1) and (2) we get,

$$\cos 2A = \sin 4B.$$

Example 3

If $\tan\alpha = \frac{1}{3}$ and $\tan\beta = \frac{1}{7}$ then prove that $(2\alpha+\beta) = \frac{\pi}{4}$.

Solution

$$\tan(2\alpha+\beta) = \frac{\tan 2\alpha + \tan\beta}{1-\tan 2\alpha\cdot\tan\beta}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\times\frac{1}{3}}{1-\frac{1}{9}} = \frac{3}{4}$$

$$\tan(2\alpha+\beta) = \frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}} = \frac{\frac{25}{28}}{\frac{25}{28}}$$

$$= 1 = \tan\frac{\pi}{4}$$

$$2\alpha+\beta = \frac{\pi}{4}$$

Example 4

If $\tan A = \frac{1-\cos B}{\sin B}$ then prove that $\tan 2A = \tan B$

Solution

Consider $\frac{1-\cos B}{\sin B} = \frac{2\sin^2\frac{B}{2}}{2\sin\frac{B}{2}\cos\frac{B}{2}} = \frac{\sin\frac{B}{2}}{\cos\frac{B}{2}} = \tan\frac{B}{2}$

$$\tan A = \frac{1-\cos B}{\sin B}$$

$$= \tan\frac{B}{2}$$

$$A = \frac{B}{2}$$

$$2A = B$$

$$\therefore \quad \tan 2A = \tan B$$

- **Homework** :-

1. Find the values of the following : (i) cosec15° (ii) $\sin(-105^\circ)$ (iii) $\cot 75^\circ$

2. Find the values of the following

   (i) $\sin 76^\circ \cos 16^\circ - \cos 76^\circ \sin 16^\circ$ (ii) $\sin\frac{\pi}{4}\cos\frac{\pi}{12} + \cos\frac{\pi}{4}\sin\frac{\pi}{12}$

   (iii) $\cos 70^\circ \cos 10^\circ - \sin 70^\circ \sin 10^\circ$ (iv) $\cos^2 15^\circ - \sin^2 15^\circ$

3. If $\sin A = \frac{3}{5}, 0 < A < \frac{\pi}{2}$ and $\cos B = \frac{-12}{13}, \pi < A < \frac{3\pi}{2}$ find the values of the following :

   (i) $\cos(A+B)$ (ii) $\sin(A-B)$ (iii) $\tan(A-B)$

4. If $\cos A = \frac{13}{4}$ and $\cos B = \frac{1}{7}$ where A, B are acute angles prove that $A - B = \frac{\pi}{3}$

5. Prove that $2\tan 80^\circ = \tan 85^\circ - \tan 5^\circ$

6. If $\cot\alpha = \frac{1}{2}$, $\sec\beta = \frac{-5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, find the value of $\tan(\alpha + \beta)$. State the quadrant in which $\alpha + \beta$ terminates.

7. If $A+B = 45^o$, prove that $(1+\tan A)(1+\tan B) = 2$ and hence deduce the value of $\tan 22\frac{1}{2}^\circ$

8. Prove that (i) $\sin(A+60^\circ) + \sin(A-60^\circ) = \sin A$

(ii) $\tan 4A \tan 3A \tan A + \tan 3A + \tan A - \tan 4A = 0$

9. (i) If $\tan\theta = 3$ find $\tan 3\theta$

(ii) If $\sin A = \frac{12}{13}$, find $\sin 3A$

10. If $\sin A = \frac{3}{5}$, find the values of $\cos 3A$ and $\tan 3A$

11. Prove that $\frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} + \frac{\sin(A-B)}{\cos A \cos B} = 0.$

12. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$ prove that $\cot(A-B) = \frac{1}{x} + \frac{1}{y}$.

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