ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-15

SUBJECT – MATHEMATICS

1st - Term

Chapter: Sequence & Series | **Class: XI**

Topic: Complex numbers (Basics) | **Date: 10.07.2020**

Part 1

A complex number is a number that can be expressed in the form p + iq, where p and q are real numbers, and i is a solution of the equation $x^2 = -1$. $\sqrt{-1} = i$ or $i^2 = -1$. Examples of complex numbers: 8 − 2i, 2 +31i, $2 + \frac{4}{5}i$, etc. Complex numbers are denoted by 'z'.

General form of Complex Number: z = p + iq

Where,

- p is known as the real part, denoted by Re z
- q is known as the imaginary part, denoted by Im z

If z = 12 + 35i, then Re z = 12 and Im z = 35. If z1 and z2 are two **complex numbers** such that z1 = p + iq and z2 = r + is. z1 and z2 are equal if p = r and q = s.

Algebra of Complex Numbers

- **Addition of complex numbers**

Let z1 = m + ni and z2 = o + ip be two complex numbers. Then, z1 + z2 = z = (m + o) + (n + p)i, where z = resultant complex number. For example, (12 + 13i) + (-16 +15i) = (12 − 16) + (13 + 15)i = -4 + 28i.

1. The sum of complex numbers is always a complex number (closure law)
2. For complex numbers z1 and z2: z2 + z1= z1 + z2 (commutative law) For complex numbers z1, z2, z3: (z1 + z2) + z3 = z1 + (z2 + z3) [associative law].
3. For every complex number z, z + 0 = z [additive identity]
4. To every complex number z = p + qi, we have the complex number -z = -p + i(-q), called the negative or additive inverse of z. [z + (−z) = 0]

- **Difference of complex numbers**

Let z1 = m + ni and z2 = o + ip be two complex numbers, then z1 − z2 = z1 + (-z2). For example, (16 + 13i) − (12 − 1i) = (16 + 13i) + (-12 + 1i) = 4 + 14i and (12 − 1i) − (16 + 13i) = (12 − 1i) + (-16 − 13i) = -4 − 14i

- **Multiplication of complex numbers**

Let z1 = m + ni and z2 = o + ip be two complex numbers then, z1 × z2 = (mo − np) + i(no + pm). For example, (2 + 4i) (1 + 5i) = (2 × 1 − 4 × 5) + i(2 × 5 + 4 × 1) = -22 + 14i The product of two complex numbers is a complex number (closure law)

- For complex numbers z1 and z2, z1 × z2 = z2 × z1 (commutative law).
- For complex numbers z1, z2, z3, (z1 × z2) × z3 = z1 × (z2 × z3) [associative law].

Let z1 = m + in and z2 = o + ip. Then,

- z1 + z2 = (m + o) + i (n + p)
- z1 z2 = (mo − np) + i(mp + on)
- The conjugate of the complex number z = m + in, denoted by $\overline{z}$, is given by z = m − in.

- Division of complex numbers

We multiplied both sides by the **conjugate** of the denominator, which is a number with the same real part and the opposite imaginary part.

example:

$$\frac{20-4i}{3+2i}$$

$$=\frac{20-4i}{3+2i}\cdot\frac{3-2i}{3-2i}$$

$$=\frac{(20-4i)(3-2i)}{(3+2i)(3-2i)}$$

$$=\frac{52-52i}{13}$$

$$=\frac{52}{13}-\frac{52}{13}i$$

$$=4-4i$$

The Modulus and Conjugate of Complex Numbers

Let z = m + in be a complex number. Then, the modulus of z, denoted by |z| = $\sqrt{m^2-n^2}$ and the conjugate of z, denoted by $\overline{z}$ is the complex number m − ni.In the Argand plane, the modulus of the complex number m + in = $\sqrt{m^2-n^2}$ is the distance between the point (m, n) and the origin (0, 0). The x-axis is termed as the real axis and the y-axis is termed as the imaginary axis.

Prepared By –

Mr. Sukumar Mandal (SkM)