ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-11

SUBJECT – STATISTICS

Pre-test

Chapter: THEORITICAL PROBABILITY DISTRIBUTION **Class: XII**

Topic: BINOMIAL PROBABILITY DISTRIBUTION **Date: 23.06.20**

PROBABILITY DISTRIBUTION

PART 5

1. A fair die is to be tossed 600 times. Let W denote the number of times a ‘2’ occurs. Find the expected value of W.

Solution:

X : Number of times 2 appears as the face value

$X \sim Bin\ (\ 600, \frac{1}{6}\ )$

$So,\ E(\ X) = 600 * \frac{1}{6} = 100.$

2. A hat contains ten marbles, three of which are red. If you pick five marbles from the hat with replacement, what is the probability distribution of the number of red marbles? Find the mean and variance of this probability distribution.

Solution:

X: Number of red marbles picked

$X \sim Bin\ (\ 5, \frac{3}{10}\ )$

Mean $= 5 * \frac{3}{10} = \frac{3}{2} = 1.5$

Variance $= 5 * \frac{3}{10} * \frac{7}{10} = 1.05$

3.. For a binomial distribution mean and variance are 3 and 2, respectively. Calculate the probabilities of getting (i) a non-zero value, (ii) values at most 1

Solution:

Let $X \sim Bin\ (\ n, p\ )$

According to the problem $np = 3\ and\ np(1-p) =\ 2$

$p = \frac{1}{3}\ and\ n = 9$

So $X \sim Bin\ (\ 9, \frac{1}{3}\ )$

(i) P(non-zero value) = 1 – f(0) = 1 – $(\frac{2}{3})^9 = 0.974$

(ii) P (at most 1) = P (X = 0) + P (X= 1)

$= \ (\left(\frac{2}{3}\right)^9 + 9 * (\frac{1}{3})(\frac{2}{3})^{\ 8}\ = 0.143$

4. Children inherit their blood type from their parents, with probability that reflects the parents' genetic makeup. Each of 4 children of Mr. and Mrs. Kulkarni has probability $\frac{1}{4}$ of having blood type O and inherit independently of each other.

(i) What is the distribution of the children having blood type O?

(ii) What is the probability that none of them has blood type O?

(iii) What is the mean number of children with type O blood? What is its s.d.?

Solution:

X : Number of children having blood type O

(i) Therefore $X \sim Bin\ (\ 4, \frac{1}{4}\ )$

(ii) P(X= 0) = f(0) = $\left(\frac{3}{4}\right)^4 = 0.3$

(iii) Mean = $4 * \frac{1}{4} = 1$

Variance = $4 * \frac{1}{4} * \frac{3}{4} = \frac{3}{4}$

5. Seventy percent of all trucks undergoing a brake inspection at a certain inspection facility pass the inspection. What is the probability that out of five randomly selected trucks exactly three trucks fail the inspection?

Solution:

Define X: Number of trucks fail the inspection

So $X \sim Bin\ (\ 5, \frac{3}{10}\ )$

Required probability = f(3)

$$= 5_{c_3}(\frac{3}{10})^3(\frac{7}{10})^2$$

= 0.1323

6. A new surgical procedure is said to be successful 80% of the time. Suppose the operation is performed five times and the results are assumed to be independent of one another. What are the probabilities of these event :

(i) All five operations are successful?

(ii) Exactly four are successful?

(iii) Less than two are successful?

Solution:

Define X: Number of operations are successful

So $X \sim Bin\ (5, \frac{8}{10}\ )$

(i) P (X = 5) $= (\frac{8}{10})^5 = 0.32$

(ii) P (x = 4) $= 5 * \left(\frac{8}{10}\right)^4 * \frac{2}{10}$

$= 0.4096$

(iii) $P\ (X < 2) = f(0) + f(1)$

$= \left(\frac{2}{10}\right)^5 + 5 * (\frac{2}{10})^4 * \frac{8}{10} = 0.0067$

7. An experiment consists of throwing 25 darts at a dart board. The probability of hitting the bull's eye (the target) is 0.4. Assuming that the throws are independent, find the probability of hitting the bull's eye exactly five times.

Solution:

Define X: Number of hits at the bull's eye

So $X \sim Bin\ (\ 25, \frac{4}{10}\ )$

Required probability = f(5)

$$= \ 25_{C_5}(0.4)^5(0.6)^{20}$$

$$= 0.01989$$

8. Assume that the probability that a bomb dropped from an airplane will strike a certain target is $\frac{1}{5}$. If six bombs are dropped, find the probability that (i) exactly two will strike the target

(ii) at least two will strike the target.

Solution:

Define X: Number of bombs strike the target

So, $X \sim Bin\ (\ 6, \frac{1}{5}\ )$

(i) P(exactly two will strike the target)

= f(2)

$= 6_{C_2}(0.2)^2(0.8)^4$

$= 0.245$

(ii) P(atleast two will strike the target)

$= 1 - (f(0) + f(1))$

$= 1 - ((0.8)^6 + 6 * (0.2) * (0.8)^5)$

$= 0.344$

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