ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-8

SUBJECT – STATISTICS

Pre-test

Chapter: THEORITICAL PROBABILITY DISTRIBUTION **Class: XII**

Topic: BINOMIAL PROBABILITY DISTRIBUTION **Date: 19.06.20**

PROBABILITY DISTRIBUTION

PART 2

A discrete random variable X $\sim Bin\ (n, p\ )$

PROPERTIES:

5. Mean deviation about the mean

$MD_{np}\,(X) = E\ |X - np|$

$= 2\ \sum_{x=k+1}^{n}(x - np)f(x)$, where k=[np], ie, highest integer

contained in the value of np.

$= 2\ \sum_{x=k+1}^{n}(x(p + 1 - p) - np)f(x)$

$= 2\ \sum_{x=k+1}^{n}(x(1 - p) - (n - x)p)f(x)$

$= 2\ \sum_{x=k+1}^{n}\{\ x(1 - p)\frac{n!\,p^{x}(1-p)^{n-x}}{(n-x)!x!} - (n - x)p\ \frac{n!\ p^{x}(1-p)^{n-x}}{(n-x)!x!}\}$

$= 2\sum_{x=k+1}^{n}\{\ \frac{n!\,p^{x}(1-p)^{n-x+1}}{(n-x)!(x-1)!} - \frac{n!\,p^{x+1}(1-p)^{n-x}}{(n-x-1)!x!}\}$ ……………(*)

$Let\ \gamma_x = \frac{n!\,p^{x}(1-p)^{n-x+1}}{(n-x)!(x-1)!} \Rightarrow \gamma_{x+1} = \frac{n!\,p^{x+1}(1-p)^{n-x}}{(n-x-1)!x!}$

So from (*) $MD_{np}\,(X) = 2\sum_{x=k+1}^{n}\{\ \gamma_x - \gamma_{x+1}\}$

$= 2\gamma_{k+1}$

$= \frac{n!\,p^{k+1}(1-p)^{n-k}}{(n-k-1)!k!}$

5. MODE OF THE RANDOM VARIABLE X.

Mode is defined as observation which contains maximum probability.

So at mode $\mathrm{f(x)} \geq \mathrm{f(x-1)}$ and $\mathrm{f(x)} \geq \mathrm{f(x+1)}$

Simplifying $\mathrm{f(x)} \geq \mathrm{f(x-1)}$

$$\Rightarrow n_{C_x} p^x (1-p)^{n-x} \geq n_{C_{x-1}} p^{x-1} (1-p)^{n-x+1}$$

$$\Rightarrow \frac{n!\,p^x (1-p)^{n-x}}{(n-x)!x!} \geq \frac{n!\,p^{x-1} (1-p)^{n-x+1}}{(n-x+1)!(x-1)!}$$

$$\Rightarrow \frac{p}{x} \geq \frac{1-p}{n-x+1}$$

$\Rightarrow (n+1)p \geq x$ ....(*)

Simplifying $\mathrm{f(x)} \geq \mathrm{f(x+1)}$

$$\Rightarrow n_{C_x} p^x (1-p)^{n-x} \geq n_{C_{x+1}} p^{x+1} (1-p)^{n-x-1}$$

$$\Rightarrow \frac{n!\,p^x (1-p)^{n-x}}{(n-x)!x!} \geq \frac{n!\,p^{x+1} (1-p)^{n-x-1}}{(n-x-1)!(x+1)!}$$

$$\Rightarrow \frac{1-p}{n-x} \geq \frac{p}{x+1}$$

$\Rightarrow (n+1)p - 1 \leq x$ ....(**)

Combining (*) and (**) , $(n+1)p - 1 \leq x \leq (n+1)p$

Case1: When the distribution is unimodal and mode lies atX =k , then

$$f(1) < f(2) < \cdots < f(k-1) < f(k) > f(k+1) > \cdots > f(n-1) > f(n)$$

Then mode is at X=K where, k= [(n+1)p]

Case2: When the distribution is bimodal and mode lies atX =k , then

$$f(1) < f(2) < \cdots < f(k-1) = f(k) > f(k+1) > \cdots > f(n-1) > f(n)$$

Then mode is at X=K-1 and X= k where, k= (n+1)p which is an integer.

6. RECCURSSION RELATION OF CENTRAL MOMENTS

f(x) $= n_{C_x} p^x (1-p)^{n-x}$

$$\Rightarrow \frac{d}{dp} f(x) = n_{C_x} (xp^{x-1}(1-p)^{n-x} - (n-x)p^x(1-p)^{n-x-1})$$

$$= n_{C_x} p^x (1-p)^{n-x} \left(\frac{x}{p} - \frac{n-x}{1-p} \right)$$

$$= \frac{(x-np)}{p(1-p)} f(x) \ldots\ldots\ldots\ldots\ldots \text{(*)}$$

Now $\mu_r = E(X-np)^r$

$$= \sum_{x=0}^{n} (x-np)^r \,.\, f(x)$$

$$\frac{d}{dp} \mu_r = \sum_{x=0}^{n} \{ -nr\,(x-np)^{r-1} \,.\, f(x) + (x-np)^{r-1} \frac{(x-np)}{p(1-p)} f(x) \} \text{ (from (*))}$$

$$= -nr\, \mu_{r-1} + \frac{1}{p(1-p)} \mu_{r+1} \ldots\ldots\ldots\ldots\ldots \text{(**)}$$

Putting r = 2 in (**), we know that $\mu_2 = np(1-p) and\ \mu_1 = 0$

So $\mu_3 = np(1-p)(1-2p)$

Hence the measure of skewness $\gamma_1 = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{(1-2p)}{\sqrt{np\,(1-p)}}$

$\gamma_1 < 0 \Rightarrow p > \frac{1}{2} \Rightarrow$ X is negatively skewed.

$\gamma_1 > 0 \Rightarrow p < \frac{1}{2} \Rightarrow$ X is positively skewed

$\gamma_1 = 0 \Rightarrow p = \frac{1}{2} \Rightarrow$ X is symmetric.

Putting x=3 in (**), we get $\mu_4 = 3n^2p^2(1-p)^2 + np(1-p)(1-6np(1-p))$

So, kurtosis $\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3 = \frac{(1-6np\,(1-p))}{np\,(1-p)}$

So depending upon the values of n and p we can determine the kurtosis of the distribution.

7. MAXIMUM VARIANCE OF PROPORTION OF SUCCESS

In case of Bin(n , p), the proportion of success is $\frac{x}{n}$.

$$V\left(\frac{x}{n}\right) = \frac{np(1-p)}{n^2} = v\ (say).$$

Now $\frac{d}{dp}v = \frac{1-2p}{n}$

$$\frac{d^2v}{dp^2} = -\frac{2}{n} < 0$$

$\frac{d}{dp}v = 0 \Rightarrow \frac{1-2p}{n} \Rightarrow 0 \Rightarrow p = \frac{1}{2}$.

So at $p = \frac{1}{2}$. V is maximum.

So maximum v = $\frac{1}{4\mathrm{n}}$.

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