ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-27

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Integration **Class: XII**

Topic: Indefinite integrals **Date: 13.07.2020**

Solved Examples (Part 1)

1. $\int \frac{dx}{x(\ln^2 x + 4\ln x - 1)}$ is equal to

(A) $\frac{1}{\sqrt{5}}\ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x + 2 + \sqrt{5}}\right| + c$

(B) $\frac{1}{2}\ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x + 2 + \sqrt{5}}\right| + c$

(C) $\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x - 2 + \sqrt{5}}\right| + c$

(D) $\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x + 2 + \sqrt{5}}\right| + c$

Solution:

$$I = \int \frac{dx}{x(\ln^2 x + 4\ln x - 1)}$$

Let $\ln x = t$. Then $\frac{1}{x}dx = dt$. Therefore,

$$I = \int \frac{dt}{(t^2 + 4t - 1)} = \int \frac{dt}{((t+2)^2 - 5)} = \frac{1}{2\sqrt{5}}\ln\left|\frac{t + 2 - \sqrt{5}}{t + 2 + \sqrt{5}}\right| + c$$

$$\Rightarrow I = \frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x + 2 - \sqrt{5}}{\ln x + 2 + \sqrt{5}}\right| + c$$

Hence, the correct answer is option (D).

2. $\int x^x(1+\ln x)dx$ is equal to

(A) $x^x \ln x + c$

(B) $x^x + c$

(C) $x \ln x + c$

(D) None of these

Solution:

$$I = \int x^x(1+\ln x)dx$$

Let $x^x = t$. Then

$$x \ln x = \ln t$$

$$\Rightarrow \left(1\cdot \ln x + x\cdot\frac{1}{x}\right)dx = \frac{dt}{t} \Rightarrow dx\,(1+\ln x)x^x = dt$$

Therefore,

$$I = \int dt \Rightarrow I = t + c = x^x + c$$

Hence, the correct answer is option (B).

3. $\int \frac{\sin x + \cos x}{9 + 16\sin 2x}dx$ is equal to

(A) $\frac{1}{40}\ln\left|\frac{5+4(\sin x - \cos x)}{5-4(\sin x - \cos x)}\right| + c$

(B) $\frac{1}{40}\ln\left|\frac{5-4(\sin x - \cos x)}{5+4(\sin x - \cos x)}\right| + c$

(C) $\frac{1}{40}\ln\left|\frac{5+4(\sin x + \cos x)}{5-4(\sin x + \cos x)}\right| + c$

(D) $\frac{1}{40}\ln\left|\frac{5-4(\sin x + \cos x)}{5+4(\sin x + \cos x)}\right| + c$

Solution:

$$I = \int \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$$

Let $t = \sin x - \cos x$. Then $t^2 = 1 - \sin 2x \Rightarrow \sin 2x = (1 - t^2)$

Also,

$$dt = \cos x + \sin x$$

Therefore,

$$= \int \frac{dt}{9 + 16 - 16t^2} = \int \frac{dt}{25 - 16t^2}$$

$$= \frac{1}{16}\int \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2} = \frac{1}{16} \cdot \frac{1}{2 \cdot \frac{5}{4}} \ln\left|\frac{\left(\frac{5}{4} + t\right)}{\left(\frac{5}{4} - t\right)}\right| + c$$

$$= \frac{1}{40}\ln\left|\frac{5 + 4t}{5 - 4t}\right| + c = \frac{1}{40}\ln\left|\frac{5 + 4(\sin x - \cos x)}{5 - 4(\sin x - \cos x)}\right| + c$$

Hence, the correct answer is option (A).

4. For what value of a and b, the equation

$\int(\sin 2x - \cos 2x)\, dx = \frac{1}{\sqrt{2}}\sin(2x - a) + b$ holds good.

(A) $a=-\frac{5\pi}{4}$, b is any arbitrary constant

(B) $a=\frac{5\pi}{4}$, b is any arbitrary constant

(C) $a=-\frac{\pi}{4}$, b is any arbitrary constant

(D) $a=\frac{\pi}{4}$, b is any arbitrary constant

Solution:

$$\int(\sin 2x-\cos 2x)\,dx=\int\sqrt{2}\left(\frac{\sin 2x}{\sqrt{2}}-\frac{\cos 2x}{\sqrt{2}}\right)dx=$$

$$-\int\sqrt{2}\cos\left(2x+\frac{\pi}{4}\right)dx$$

$$=-\frac{\sqrt{2}}{2}\sin\left(2x+\frac{\pi}{4}\right)+c=\frac{1}{\sqrt{2}}\sin\left(2x+\frac{5\pi}{4}\right)+c$$

$a=-\frac{5\pi}{4}$, b is any arbitrary constant.

Hence, the correct answer is option (A).

5. For what value of a and b, the equation $\int\frac{dx}{1+\sin x}=\tan\left(\frac{x}{2}+a\right)+b$ holds good.

(A) $a=-\frac{5\pi}{4}$, b is any arbitrary constant

(B) $a=\frac{5\pi}{4}$, b is any arbitrary constant

(C) $a=-\frac{\pi}{4}$, b is any arbitrary constant

(D) $a=\frac{\pi}{4}$, b is any arbitrary constant

Solution:

$$\int\frac{dx}{1+\sin x}=\int\frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)}=\frac{1}{2}\int\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-\frac{1}{2}}+c=-\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)+c=\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)+c$$

$a=-\frac{\pi}{4}$, b any arbitrary constant.

Hence, the correct answer is option (C).

6. $\int\cos\left(\ln\frac{x}{a}\right)dx$ is equal to

(A) $x\left(\cos\left(\ln\frac{x}{a}\right)-\sin\left(\ln\frac{x}{a}\right)\right)+c$

(B) $\frac{x}{2}\left(\cos\left(\ln\frac{x}{a}\right)+\sin\left(\ln\frac{x}{a}\right)\right)+c$

(C) $\frac{x}{2}\left(\cos\left(\ln\frac{x}{a}\right)-\sin\left(\ln\frac{x}{a}\right)\right)+c$

(D) $x\left(\cos\left(\ln\frac{x}{a}\right)+\sin\left(\ln\frac{x}{a}\right)\right)+c$

Solution:

$$I=\int\cos\left(\ln\frac{x}{a}\right)dx$$

Let $\ln\frac{x}{a}=t$. Then

$$x=a\cdot e^t\Rightarrow dx=ae^t\,dt$$

$$I=a\int e^t\cos t\,dt=\frac{ae^t}{2}(\cos t+\sin t)+c$$

$$=\frac{x}{2}\left(\cos\left(\ln\frac{x}{a}\right)+\sin\left(\ln\frac{x}{a}\right)\right)+c$$

Hence, the correct answer is option (B).

7. $\int\frac{\ln(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}dx$ is equal to

(A) $\frac{1}{2}\ln^2(x-\sqrt{x^2+1})+c$ **(B)** $\ln^2(x-\sqrt{x^2+1})+c$

(C) $\frac{1}{2}\ln^2(x+\sqrt{x^2+1})+c$ **(D)** $\ln^2(x+\sqrt{x^2+1})+c$

Solution:

$$I=\int\frac{\ln(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}dx$$

Let $\ln(x+\sqrt{x^2+1})=t$. Then

$$\frac{1}{(x+\sqrt{x^2+1})}\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)dx=dt\Rightarrow\frac{dx}{\sqrt{x^2+1}}=dt$$

$$\Rightarrow I=\int t\,dt=\frac{t^2}{2}+c$$

$$I=\frac{1}{2}\left[\ln\ (x+\sqrt{x^2+1})\right]^2+c$$

Hence, the correct answer is option (C).

8. $\int\frac{x\cos x}{(x\sin x+\cos x)^2}dx$ is equal to

(A) $\frac{1}{(x\sin x+\cos x)^2}+k$ **(B)** $-\frac{1}{(x\sin x+\cos x)}+k$

(C) $\frac{1}{(x\sin x+\cos x)^3}+k$ **(D)** $\frac{1}{(x\sin x+\cos x)^4}+k$

Solution:

$$I=\int\frac{x\cos x}{(x\sin x+\cos x)^2}dx$$

Let $\frac{1}{x\sin x+\cos x}=t$. Then

$$\frac{(x\sin x+\cos x)\cdot 0-1(x\cos x+\sin x-\sin x)}{(x\sin x+\cos x)^2}=\frac{dt}{dx}$$

$$\Rightarrow\frac{-x\cos x}{(x\sin x+\cos x)^2}=\frac{dt}{dx}$$

Therefore,

$$I=-\int dt=-\frac{1}{x\sin x+\cos x}+c$$

Hence, the correct answer is option (B).

9. $\int(\sqrt{\tan x}+\sqrt{\cot x})\,dx$ is equal to

(A) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+c$ **(B)** $\frac{1}{\sqrt{2}}\sin^{-1}(\sin x-\cos x)+c$

(C) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$ **(D)** $\frac{1}{\sqrt{2}}\sin^{-1}(\sin x+\cos x)+c$

Solution:

$$I=\int(\sqrt{\tan x}+\sqrt{\cot x})\,dx$$

$$=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{2\sin x\cdot\cos x}}\,dx$$

$$=\sqrt{2}\int\frac{(\sin x+\cos x)}{\sqrt{\sin 2x}}\,dx$$

Let $t=\sin x-\cos x$. Then

$$t^2=1-\sin 2x\Rightarrow dt=(\cos x+\sin x)\,dx$$

Therefore,

$$I=\sqrt{2}\int\frac{dt}{\sqrt{1-t^2}}=\sqrt{2}\sin^{-1}(t)+c$$

$$I=\sqrt{2}\sin^{-1}(\sin x-\cos x)+c.$$

Hence, the correct answer is option (C).

10. $\int\frac{1}{x^2(x^4+1)^{3/4}}dx$ is equal to

(A) $\left(1+\frac{1}{x^4}\right)^{1/4}+c$ **(B)** $-\left(1+\frac{1}{x^4}\right)^{1/4}+c$

(C) $-\left(1-\frac{1}{x^4}\right)^{1/4}+c$ **(D)** $\left(1-\frac{1}{x^4}\right)^{1/4}+c$

Solution:

$$I=\int\frac{1}{x^2(x^4+1)^{3/4}}dx=\int\frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$$

Let $1+x^{-4}=t$. Then

$$\frac{-4}{x^5}\,dx=dt\Rightarrow\frac{1}{x^5}\,dx=-\frac{1}{4}\,dt$$

$$\Rightarrow I=-\frac{1}{4}\int\frac{dt}{t^{3/4}}=-\frac{1}{4}\int t^{-3/4}dt=-\frac{1}{4}\times 4t^{1/4}+c$$

$$\Rightarrow \text{I}=-\left(1+\frac{1}{x^4}\right)^{1/4}+c$$

Hence, the correct answer is option (B).

11. $\int\frac{\sqrt{5+x^{10}}}{x^{16}}\,dx$ is equal to

(A) $-\frac{1}{75}\left(1+\frac{5}{x^{10}}\right)^{3/2}+c$ **(B)** $-\frac{1}{75}\left(1-\frac{5}{x^{10}}\right)^{3/2}+c$

(C) $\frac{1}{75}\left(1-\frac{5}{x^{10}}\right)^{3/2}+c$ **(D)** $\frac{1}{75}\left(1+\frac{5}{x^{10}}\right)^{3/2}+c$

Solution:

$$I=\int\frac{\sqrt{5+x^{10}}}{x^{16}}\,dx=\int\frac{x^5\sqrt{\frac{5}{x^{10}}+1}}{x^{16}}\,dx=\int\frac{\sqrt{\frac{5}{x^{10}}+1}}{x^{11}}\,dx$$

Let $1+\frac{5}{x^{10}}=t$. Then

$$5\left(\frac{-10}{x^{11}}\right)dx=dt\Rightarrow\frac{1}{x^{11}}dx=-\frac{1}{50}\,dt$$

$$\Rightarrow I=-\frac{1}{50}\int\sqrt{t}dt=-\frac{1}{50}\times\frac{2}{3}t^{3/2}=-\frac{1}{75}\left(1+\frac{5}{x^{10}}\right)^{3/2}+c$$

Hence, the correct answer is option (A).

12. $\int\text{cosec}^6 x\,dx$ is equal to

(A) $-\cot x-\frac{\cot^5 x}{5}-\frac{2\cot^5 x}{3}+k$

(B) $-\frac{\cot x}{3}+\frac{2\cot^5 x}{5}+2\cot^{-3}x+k$

(C) $\frac{\tan^3 x}{3}-\frac{\tan x}{5}+2\tan^3 x+k$

(D) None of these

Solution:

$$I=\int\text{cosec}^6 x\,dx=\int\text{cosec}^4 x\cdot\text{cosec}^2 x\,dx$$

$$=\int(1+\cot^2 x)^2\cdot\text{cosec}^2 x\,dx=\int(1+\cot^4 x+2\cot^2 x)\cdot\text{cosec}^2 x\,dx$$

Let $\cot x=t$. Then

$$-\text{cosec}^2 x\,dx=dt$$

$$\Rightarrow I=-\int(1+t^4+2t^2)\,dt=-t-\frac{t^5}{s}-\frac{2}{3}t^3+c$$

$$\Rightarrow I=-\cot x-\frac{\cot^5 x}{5}-\frac{2}{3}\cot^3 x+c$$

Hence, the correct answer is option (A).

13. $\int\frac{(\sqrt{x^2+1})\{\ln(x^2+1)-2\ln x\}}{x^4}dx$ is equal to

(A) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(B) $\frac{(x^2+1)\sqrt{x^2+1}}{9x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(C) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(D) $\frac{(x^2+1)\sqrt{x^2+1}}{9x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

Solution:

$$I=\int\frac{(\sqrt{x^2+1})\{\ln(x^2+1)-2\ln x\}}{x^4}\,dx=\int\frac{\left(\frac{\sqrt{x^2}+1}{x^2}\right)\left\{\ln\left(\frac{x^2+1}{x^2}\right)\right\}}{x^3}\,dx$$

Let $\dfrac{x^2+1}{x^2}=t$. Then

$$-\frac{2}{x^3}dx=dt$$

$$\Rightarrow I=-\frac{1}{2}\int\sqrt{t}\ln t\,dt$$

$$=-\frac{1}{2}\left[(\ln t)\cdot\frac{2t^{3/2}}{3}-\frac{2}{3}\int\frac{1}{t}\cdot t^{3/2}dt\right]$$

$$=\frac{1}{3}\left[\int t^{1/2}dt-t^{3/2}(\ln t)\right]$$

$$=\frac{1}{9}t^{3/2}[2-3\ln t]+c$$

$$=\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$$

Hence, the correct answer is option (D).

14. $\displaystyle\int\frac{\sqrt{1+x^2}}{x^4}dx$ is equal to

(A) $\dfrac{1}{3}\left(1+\dfrac{1}{x^2}\right)^{3/2}+c$ **(B)** $\dfrac{1}{3}\left(1-\dfrac{1}{x^2}\right)^{3/2}+c$

(C) $-\dfrac{1}{3}\left(1+\dfrac{1}{x^2}\right)^{3/2}+c$ **(D)** $-\dfrac{1}{3}\left(1-\dfrac{1}{x^2}\right)^{3/2}+c$

Solution:

$$I=\int\frac{\sqrt{1+x^2}}{x^4}dx=\int\frac{\sqrt{1+\frac{1}{x^2}}}{x^3}dx$$

Let $1+\dfrac{1}{x^2}=t$. Then

$$-\frac{2}{x^3}dx=dt$$

$$\Rightarrow I=-\frac{1}{2}\int\sqrt{t}\,dt$$

$$=-\frac{1}{2}.\frac{2}{3}t^{3/2}+c$$

$$=-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}+c$$

Hence, the correct answer is option (C).

15. $\displaystyle\int\frac{dx}{5+4\cos x}$ is equal to

(A) $\dfrac{2}{3}\tan^{-1}\left(\dfrac{1}{3}\tan\dfrac{x}{2}\right)+c$ **(B)** $-\dfrac{2}{3}\tan^{-1}\left(\dfrac{1}{3}\tan\dfrac{x}{2}\right)+c$

(C) $\dfrac{1}{3}\tan^{-1}\left(\dfrac{1}{3}\tan\dfrac{x}{2}\right)+c$ **(D)** None of these

Solution:

$$I=\int\frac{dx}{5+4\cos x}=\int\frac{dx}{5+4\left[\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]}$$

Let $t=\tan\dfrac{x}{2}$. Then

$$dt=\frac{1}{2}\sec^2\frac{x}{2}dx$$

$$\Rightarrow dx=\frac{2dt}{1+\tan^2\frac{x}{2}}$$

$$dx=\frac{2dt}{1+t^2}$$

$$\Rightarrow I=\int\frac{\frac{2dt}{1+t^2}}{5+4\left(\frac{1-t^2}{1+t^2}\right)}=2\int\frac{dt}{t^2+9}$$

$$=\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$$

Hence, the correct answer is option (A).

16. $\displaystyle\int\frac{dx}{(2x+3)\sqrt{4x+5}}$ is equal to

(A) $\tan^{-1}\sqrt{4x-5}+c$ **(B)** $\tan^{-1}\sqrt{4x+5}+c$

(C) $\tan^{-1}\sqrt{5x+4}+c$ **(D)** $\tan^{-1}\sqrt{5x-4}+c$

Solution:

$$I=\int\frac{dx}{(2x+3)\sqrt{4x+5}}$$

Put $4x+5=t$. Then

$$x=\frac{t-5}{4}\Rightarrow dx=\frac{dt}{4}$$

$$\Rightarrow I=\frac{1}{4}\int\frac{dt}{\left(\frac{2t-10}{4}+3\right)\sqrt{t}}=\frac{1}{2}\int\frac{dt}{(t+1)\sqrt{t}}$$

Let $\sqrt{t}=u$. Then $\dfrac{1}{2\sqrt{t}}dt=du$.

Therefore,

$$I=\int\frac{du}{u^2+1}=\tan^{-1}\sqrt{t}+c$$

$$\Rightarrow I=\tan^{-1}\sqrt{4x+5}+c.$$

Hence, the correct answer is option (B).

17. $\displaystyle\int e^{ax}\cos bx\,dx$ is equal to

(A) Real part of $\int e^{(a+bi)x}dx$

(B) Imaginary part of $\int e^{(a+bi)x}dx$

(C) Neither real nor imaginary part of $\int e^{(a+bi)x}dx$

(D) None of these

Solution:

$$I=\int e^{ax}\cos bx\,dx$$

$$=\text{real part of}\int e^{ax}\,e^{ibx}dx$$

$$=\text{real part of}\int e^{ax+ibx}dx$$

Hence, the correct answer is option (A).

COMPILED BY –

MR. SUKUMAR MANDAL

(SKM)