ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-9

SUBJECT – STATISTICS

Pre-test

Chapter: THEORITICAL PROBABILITY DISTRIBUTION **Class: XII**

Topic: BINOMIAL PROBABILITY DISTRIBUTION **Date: 20.06.20**

PROBABILITY DISTRIBUTION

PART 3

1. Mean and standard deviation of a binomial distribution are respectively, 4 and $\sqrt{\frac{8}{3}}$. Find the values of n and p.

Solution: $np = 4$ and $np(1 - p) = \frac{8}{3}$

$$1 - p = \frac{2}{3} \Rightarrow p = \frac{1}{3}$$

So, $n = 12$

2. If a random variable x follows a binomial distribution with mean $\frac{5}{3}$ and $P(X = 1) = P(X = 2)$, find P(X is at most 1) and P (X is at least 1).

Solution: $np = \frac{5}{3}$

$$n_{C_1}\, p^1(1-p)^{n-1} = n_{C_2}\, p^2(1-p)^{n-2}$$

$$\Rightarrow n\, p\, (1-p)^{n-1} = \frac{n(n-1)}{2}\, p^2(1-p)^{n-2}$$

$$\Rightarrow 1 - p = \frac{n-1}{2}\, p$$

$$\Rightarrow 2 - 2p = np - p$$

$$\Rightarrow p = 2 - \frac{5}{3} = \frac{1}{3}$$

$$\Rightarrow n = 5$$

P(X is at most 1) = $f(0) + f(1) = (\frac{1}{3})^5 + 5.\frac{1}{3}\left(\frac{2}{3}\right)^4 = \frac{81}{243}$

P (X is at least 1) = $1 - f(0) = 1 - (\frac{2}{3})^5 = \frac{211}{243}$

3. If X is a symmetric binomial variable with n=36, calculate E(X(X–1)).

Solution: For a a symmetric binomial $p = \frac{1}{2}$

From the expression of factorial moments E(X(X–1)) = n(n-1)p^2

$$= 36 * 35 * (\frac{1}{2})^2$$

$$= 315.$$

If X follows binomial distribution with parameters n and p, then prove that P(X is even) = $\frac{1}{2}\{1+(q-p)^n\}$, where p+q=1

Solution: $(p+q)^n = n_{C_0}\, p^0 q^{n-0} + n_{C_1}\, p^1 q^{n-1} + n_{C_2}\, p^2 q^{n-2} + \ldots\ldots\ldots\ldots + n_n\, p^n q^0$

$$(q-p)^n = n_{C_0}\, p^0 q^{n-0} - n_{C_1}\, p^1 q^{n-1} + n_{C_2}\, p^2 q^{n-2} - \ldots\ldots\ldots\ldots + (-1)^n n_n\, p^n q^0$$

$$(p+q)^n + (q-p)^n = 2\{ n_{C_0}\, p^0 q^{n-0} + n_{C_2}\, p^2 q^{n-2} + n_{C_4}\, p^4 q^{n-4} + \ldots\ldots\ldots\ldots$$

$$\Rightarrow \frac{1}{2}\{ 1 + (q-p)^n \} = f(0) + f(2) + f(4) + \ldots$$

$$= P(X = \text{even})$$

4. If a random variable X follows a binomial distribution with parameters n and p, calculate the value of $Cov\left(\frac{X}{n}, \frac{n-X}{n}\right)$.

Solution: $cov\left(\frac{X}{n}, \frac{n-x}{n}\right) = -v(\frac{x}{n})$

$$= \frac{n\,p(1-p)}{n^2} = \frac{p(1-p)}{n}.$$

5. If X denotes the number of heads in a single toss of 4 unbiased coins, then calculate the value of $P(1 \le X \le 3)$.

Solution: $P(1 \le X \le 3) = 1 - \{ f(0) + f(4)\}$

$$= 1 - \frac{2}{2^4} = \frac{7}{8}.$$

6. Suppose that in a large metropolitan area 40% of all households have a microwave oven. Consider selecting a group of six households. What is the probability that there are at most two of the six households have a microwave oven?

Solution: Define X : Number of households have a microwave oven

So $X \sim Bin\ (6, 0.4)$

$P(\, x = atmost\ 2)$

$= f(0) + f(1) + f(2)$

$= (0.6)^6 + 6 * (0.4)(0.6)^5 + 15 * (0.4)^2(0.6)^4$

= 0.046656 +0.186624 + 0.31104 = 0.54432

7. In a certain population, 85% of the people have covid19 positive blood. Suppose that the two people from this population get married. What is the probability that they are both covid19 negative, thus making it inevitable that their children will be covid19-negative?

Solution: Define X: Number of persons have covid19 negative

$X \sim Bin\ (2, 0.15)$

Required probability = f(2)

$= (0.85)^2 = 0.7225$

8. The number, X of switches that fail some inspection has binomial distribution with parameters n = 10, p=0.01. What is the probability that no more than 1 switch will fail?

Solution: X : Number of switches that fail some inspection

$X \sim Bin\ (10, 0.01)$

Required probability = f(0) + f(1)

$$= 10_{C_0} 0.01^0 0.09^{10} + 10_{C_1} 0.01^1 0.09^9$$

$$= \frac{9^9}{10^{19}}$$

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