



ST. LAWRENCE HIGH SCHOOL



A JESUIT CHRISTIAN MINORITY INSTITUTION

Sub: Algebra and Geometry

Class: 7

Date: 05.05.20

STUDY MATERIAL: LINES AND ANGLES

Concepts

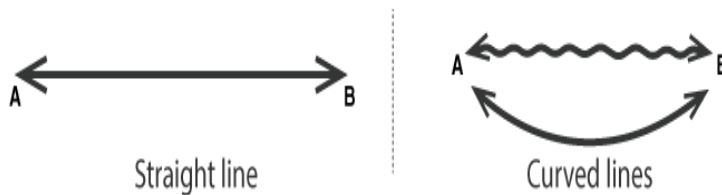
Lines and angles introduction starts with introducing Geometry first. Geometry is derived from two Greek words, Geo, which means Earth and Metry, which mean measurement. So basically, Geometry deals with the measurement of different figures like Point, Line, Line Segment, Angles, Circles, Squares etc.

Geometry begins with a point which denotes a location and is represented by a dot(.). A point has no dimensions i.e. no length, no width and no height, but with the help of this point, we can draw lines and angles. Here, we are going to discuss the various types of angle, its measure, parallel lines angle formed using lines, etc.

Lines and Angles

Line:

A line is a straight figure which doesn't have an endpoint and extends infinitely in opposite directions.



Ray:

A ray is a straight line, which starts from a fixed point and moves in one direction.



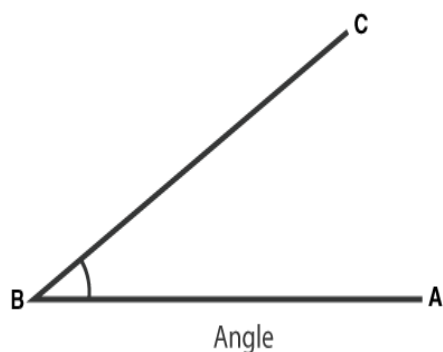
Line Segment

A portion of the line formed with two definite points is called a Line Segment. A line is a one-dimensional figure and has no thickness.



Angle

When we join two line segments at a single point, an angle is formed or we can say, an Angle is a combination of two line segments at a common endpoint. This common point is called Vertex of the angle and the two line segments are sides or arms of the angle formed.



Types of Angles

There are basically 6 types of angles which are:

1. **Acute Angle:** If an angle is less than 90 degrees, then it is called an Acute angle
2. **Obtuse Angle:** If an angle is more than 90 degrees, then it is called Obtuse Angle
3. **Right Angle:** If an angle is exactly at 90 degrees, then it is called Right Angle.
4. **Straight Angle:** If an angle is exactly 180 degrees, then it is called Straight Angle.
5. **Reflex Angle:** If the angle is more than 180 degree but less than 270 degrees, it is denoted as a Reflex angle.
6. **Full Angle:** A 360-degree angle is called a Full angle.

Related Angles

Apart from this, the different angles are:

Complementary Angle: The sum of the measures of two angles is 90°

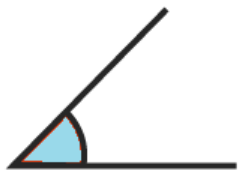
Supplementary Angle: The sum of the measures of two angles is 180°

Adjacent Angle: Adjacent angles have a common vertex and a common arm but no common interior points

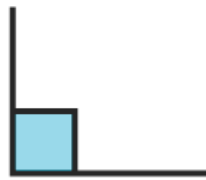
Linear pair: A linear pair is a pair of adjacent angles whose non-common sides are opposite rays

Vertically Opposite Angles: when two lines intersect, the vertically opposite angles so formed are equal.

TYPES OF ANGLES



ACUTE ANGLE
Less than 90°



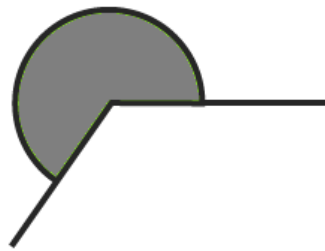
RIGHT ANGLE
Exact 90°



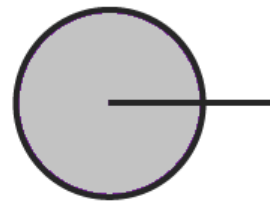
OBTUSE ANGLE
Greater than 90°
and less than 180°



STRAIGHT ANGLE
Exact 180°



REFLEX ANGLE
Greater than 180°



COMPLETE ANGLE
Exact 360°

Pairs of Lines

Intersecting lines: Two lines intersect if they have a point in common. This common point O is their point of intersection.

Transversal: A line that intersects two or more lines at distinct points is called a transversal.

Angles made by the transversal: There are different angles formed when the transversal cuts the lines. They are:

- Interior angles
- Exterior angles
- Pairs of Alternate interior angles
- Pairs of Alternate exterior angles
- Pairs of Corresponding angles
- Pairs of interior angles on the same side of the transversal

When a transversal cuts two lines, such that pairs of corresponding angles are equal, then the lines have to be parallel.

When a transversal cuts two lines, such that pairs of alternate interior angles are equal, the lines have to be parallel.

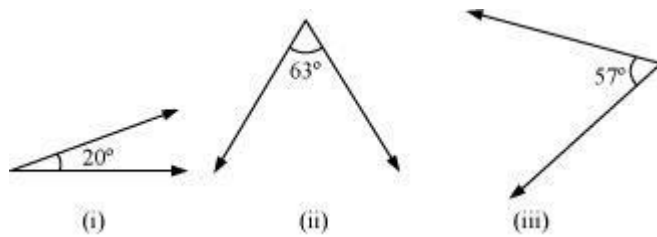
When a transversal cuts two lines, such that pairs of interior angles on the same side of the transversal are supplementary, the lines have to be parallel.

Pairs of Angles	Condition
Two complementary angles	Measures add up to 90°
Two supplementary angles	Measures add up to 180°
Two adjacent angles	Have a common vertex and a common arm but no common interior
Linear pair	Adjacent and supplementary

Solved Numericals

Question 1:

Find the complement of each of the following angles:



ANSWER:

The sum of the measures of complementary angles is 90° .

(i) 20°

$$\text{Complement} = 90^\circ - 20^\circ$$

$$= 70^\circ$$

(ii) 63°

$$\text{Complement} = 90^\circ - 63^\circ$$

$$= 27^\circ$$

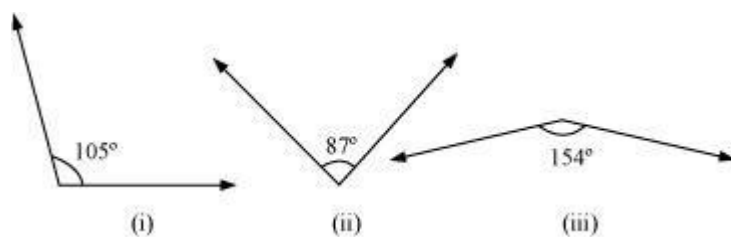
(iii) 57°

$$\text{Complement} = 90^\circ - 57^\circ$$

$$= 33^\circ$$

Question 2:

Find the supplement of each of the following angles:



ANSWER:

The sum of the measures of supplementary angles is 180° .

(i) 105°

$$\text{Supplement} = 180^\circ - 105^\circ$$

$$= 75^\circ$$

(ii) 87°

$$\text{Supplement} = 180^\circ - 87^\circ$$

$$= 93^\circ$$

(iii) 154°

$$\text{Supplement} = 180^\circ - 154^\circ$$

$$= 26^\circ$$

Question 3:

Identify which of the following pairs of angles are complementary and which are supplementary.

(i) 65° , 115° (ii) 63° , 27°

(iii) 112° , 68° (iv) 130° , 50°

(v) 45° , 45° (vi) 80° , 10°

ANSWER:

The sum of the measures of complementary angles is 90° and that of supplementary angles is 180° .

(i) 65° , 115°

$$\text{Sum of the measures of these angles} = 65^\circ + 115^\circ = 180^\circ$$

∴ These angles are supplementary angles.

(ii) $63^\circ, 27^\circ$

Sum of the measures of these angles = $63^\circ + 27^\circ = 90^\circ$

∴ These angles are complementary angles.

(iii) $112^\circ, 68^\circ$

Sum of the measures of these angles = $112^\circ + 68^\circ = 180^\circ$

∴ These angles are supplementary angles.

(iv) $130^\circ, 50^\circ$

Sum of the measures of these angles = $130^\circ + 50^\circ = 180^\circ$

∴ These angles are supplementary angles.

(v) $45^\circ, 45^\circ$

Sum of the measures of these angles = $45^\circ + 45^\circ = 90^\circ$

∴ These angles are complementary angles.

(vi) $80^\circ, 10^\circ$

Sum of the measures of these angles = $80^\circ + 10^\circ = 90^\circ$

∴ These angles are complementary angles.

Question 4:

Find the angle which is equal to its complement.

ANSWER:

Let the angle be x .

Complement of this angle is also x .

The sum of the measures of a complementary angle pair is 90° .

$$\therefore x + x = 90^\circ$$

$$2x = 90^\circ$$

$$x = \frac{90^\circ}{2} = 45^\circ$$

Question 5:

Find the angle which is equal to its supplement.

ANSWER:

Let the angle be x .

Supplement of this angle is also x .

The sum of the measures of a supplementary angle pair is 180° .

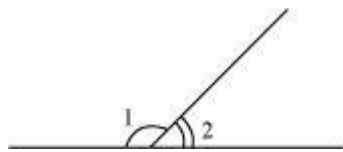
$$\therefore x + x = 180^\circ$$

$$2x = 180^\circ$$

$$x = 90^\circ$$

Question 6:

In the given figure, $\angle 1$ and $\angle 2$ are supplementary angles. If $\angle 1$ is decreased, what changes should take place in $\angle 2$ so that both the angles still remain supplementary.

**ANSWER:**

$\angle 1$ and $\angle 2$ are supplementary angles.

If $\angle 1$ is reduced, then $\angle 2$ should be increased by the same measure so that this angle pair remains supplementary.

Question 7:

Can two angles be supplementary if both of them are:

(i) Acute? (ii) Obtuse? (iii) Right?

ANSWER:

(i) No. Acute angle is always lesser than 90° . It can be observed that two

angles, even of 89° , cannot add up to 180° . Therefore, two acute angles cannot be in a supplementary angle pair.

(ii) No. Obtuse angle is always greater than 90° . It can be observed that two angles, even of 91° , will always add up to more than 180° . Therefore, two obtuse angles cannot be in a supplementary angle pair.

(iii) Yes. Right angles are of 90° and $90^\circ + 90^\circ = 180^\circ$

Therefore, two right angles form a supplementary angle pair together.

Question 8:

An angle is greater than 45° . Is its complementary angle greater than 45° or equal to 45° or less than 45° ?

ANSWER:

Let A and B are two angles making a complementary angle pair and A is greater than 45° .

$$A + B = 90^\circ$$

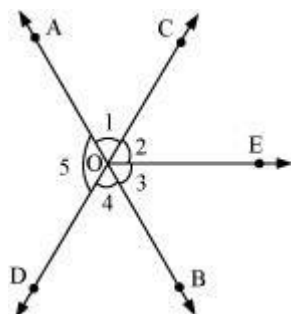
$$B = 90^\circ - A$$

Therefore, B will be lesser than 45° .

Question 9:

In the adjoining figure:

- (i) Is $\angle 1$ adjacent to $\angle 2$?
- (ii) Is $\angle AOC$ adjacent to $\angle AOE$?
- (iii) Do $\angle COE$ and $\angle EOD$ form a linear pair?
- (iv) Are $\angle BOD$ and $\angle DOA$ supplementary?
- (v) Is $\angle 1$ vertically opposite to $\angle 4$?
- (vi) What is the vertically opposite angle of $\angle 5$?



ANSWER:

(i) Yes. Since they have a common vertex O and also a common arm OC. Also, their non-common arms, OA and OE, are on either side of the common arm.

(ii) No. They have a common vertex O and also a common arm OA. However, their non-common arms, OC and OE, are on the same side of the common arm. Therefore, these are not adjacent to each other.

(iii) Yes. Since they have a common vertex O and a common arm OE. Also, their non-common arms, OC and OD, are opposite rays.

(iv) Yes. Since $\angle BOD$ and $\angle DOA$ have a common vertex O and their non-common arms are opposite to each other.

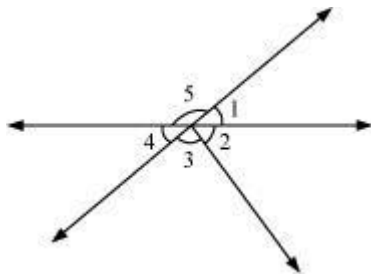
(v) Yes. Since these are formed due to the intersection of two straight lines (AB and CD).

(vi) $\angle COB$ is the vertically opposite angle of $\angle 5$ as these are formed due to the intersection of two straight lines, AB and CD.

Question 10:

Indicate which pairs of angles are:

(i) Vertically opposite angles. (ii) Linear pairs.



ANSWER:

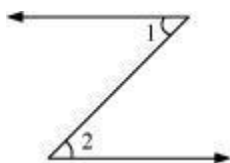
(i) $\angle 1$ and $\angle 4$, $\angle 5$ and $\angle 2 + \angle 3$ are vertically opposite angles as these are formed due to the intersection of two straight lines.

(ii) $\angle 1$ and $\angle 5$, $\angle 5$ and $\angle 4$ as these have a common vertex and also

have non-common arms opposite to each other.

Question 11:

In the following figure, is $\angle 1$ adjacent to $\angle 2$? Give reasons.

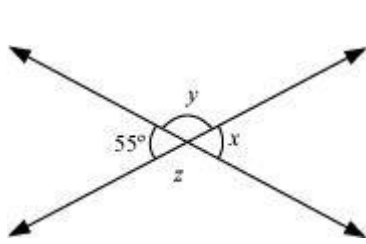


ANSWER:

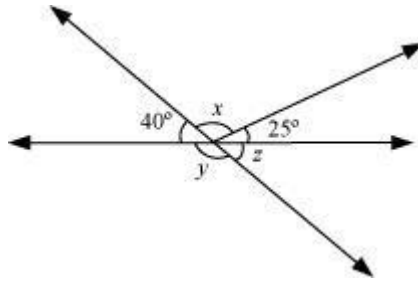
$\angle 1$ and $\angle 2$ are not adjacent angles because their vertex is not common.

Question 12:

Find the value of the angles x , y , and z in each of the following:



(i)



(ii)

ANSWER:

(i) Since $\angle x$ and $\angle 55^\circ$ are vertically opposite angles,

$$\angle x = 55^\circ$$

$$\angle x + \angle y = 180^\circ \text{ (Linear pair)}$$

$$55^\circ + \angle y = 180^\circ$$

$$\angle y = 180^\circ - 55^\circ = 125^\circ$$

$$\angle y = \angle z \text{ (Vertically opposite angles)}$$

$$\angle z = 125^\circ$$

(ii) $\angle z = 40^\circ$ (Vertically opposite angles)

$$\angle y + \angle z = 180^\circ \text{ (Linear pair)}$$

$$\angle y = 180^\circ - 40^\circ = 140^\circ$$

$$40^\circ + \angle x + 25^\circ = 180^\circ \text{ (Angles on a straight line)}$$

$$65^\circ + \angle x = 180^\circ$$

$$\angle x = 180^\circ - 65^\circ = 115^\circ$$

Question 13:

Fill in the blanks:

(i) If two angles are complementary, then the sum of their measures is _____.

(ii) If two angles are supplementary, then the sum of their measures is _____.

(iii) Two angles forming a linear pair are _____.

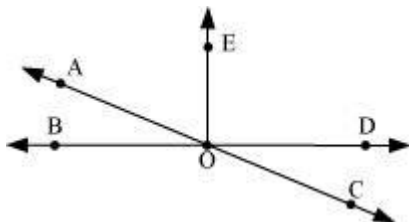
- (iv) If two adjacent angles are supplementary, they form a _____.
- (v) If two lines intersect at a point, then the vertically opposite angles are always _____.
- (vi) If two lines intersect at a point, and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are _____.

ANSWER:

- (i) 90°
- (ii) 180°
- (iii) Supplementary
- (iv) Linear pair
- (v) Equal
- (vi) Obtuse angles

Question 14:

In the adjoining figure, name the following pairs of angles.



- (i) Obtuse vertically opposite angles
- (ii) Adjacent complementary angles
- (iii) Equal supplementary angles
- (iv) Unequal supplementary angles
- (v) Adjacent angles that do not form a linear pair

ANSWER:

- (i) $\angle AOD$, $\angle BOC$
- (ii) $\angle EOA$, $\angle AOB$
- (iii) $\angle EOB$, $\angle EOD$

(iv) $\angle EOA$, $\angle EOC$

(v) $\angle AOB$ and $\angle AOE$, $\angle AOE$ and $\angle EOD$, $\angle EOD$ and $\angle COD$

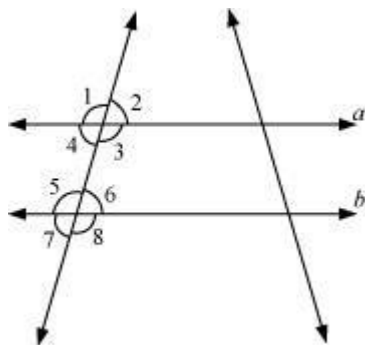
Question 15:

State the property that is used in each of the following statements?

(i) If $a \parallel b$, then $\angle 1 = \angle 5$

(ii) If $\angle 4 = \angle 6$, then $a \parallel b$

(iii) If $\angle 4 + \angle 5 = 180^\circ$, then $a \parallel b$



ANSWER:

(i) Corresponding angles property

(ii) Alternate interior angles property

(iii) Interior angles on the same side of transversal are supplementary.

Question 16:

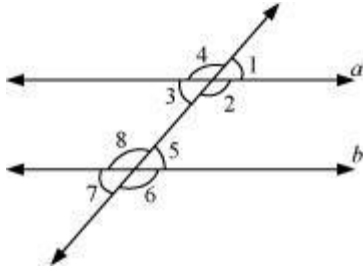
In the adjoining figure, identify

(i) The pairs of corresponding angles

(ii) The pairs of alternate interior angles

(iii) The pairs of interior angles on the same side of the transversal

(iv) The vertically opposite angles

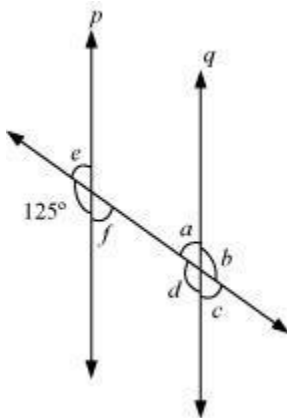


ANSWER:

- (i) $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
- (ii) $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 5$
- (iii) $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$
- (iv) $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$, $\angle 5$ and $\angle 7$, $\angle 6$ and $\angle 8$

Question 17:

In the adjoining figure, $p \parallel q$. Find the unknown angles.

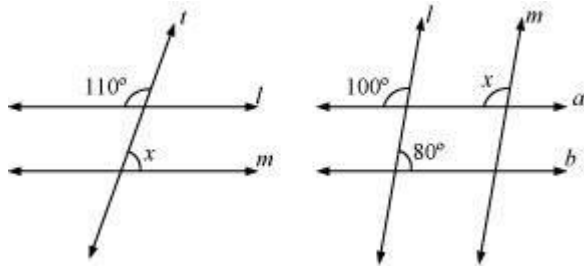


ANSWER:

- $\angle d = 125^\circ$ (Corresponding angles)
- $\angle e = 180^\circ - 125^\circ = 55^\circ$ (Linear pair)
- $\angle f = \angle e = 55^\circ$ (Vertically opposite angles)
- $\angle c = \angle f = 55^\circ$ (Corresponding angles)
- $\angle a = \angle e = 55^\circ$ (Corresponding angles)
- $\angle b = \angle d = 125^\circ$ (Vertically opposite angles)

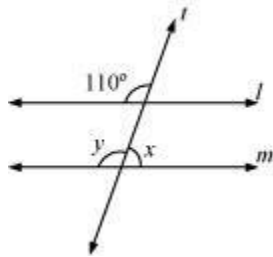
Question 18:

Find the value of x in each of the following figures if $l \parallel m$.



ANSWER:

(i)



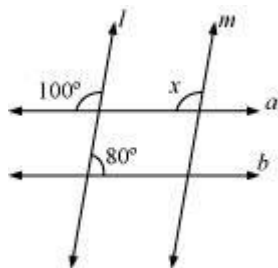
$\angle y = 110^\circ$ (Corresponding angles)

$\angle x + \angle y = 180^\circ$ (Linear pair)

$\angle y = 180^\circ - 110^\circ$

$= 70^\circ$

(ii)



$\angle x = 100^\circ$ (Corresponding angles)

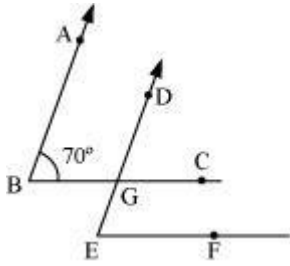
Question19:

In the given figure, the arms of two angles are parallel.

If $\angle ABC = 70^\circ$, then find

(i) $\angle DGC$

(ii) $\angle DEF$



ANSWER:

(i) Consider that $AB \parallel DG$ and a transversal line BC is intersecting them.

$\angle DGC = \angle ABC$ (Corresponding angles)

$\angle DGC = 70^\circ$

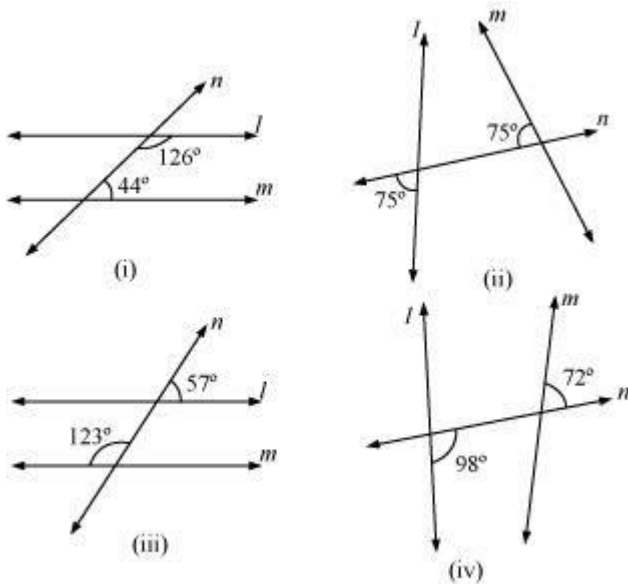
(ii) Consider that $BC \parallel EF$ and a transversal line DE is intersecting them.

$\angle DEF = \angle DGC$ (Corresponding angles)

$\angle DEF = 70^\circ$

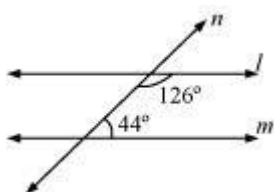
Question 20:

In the given figures below, decide whether l is parallel to m .



ANSWER:

(i)

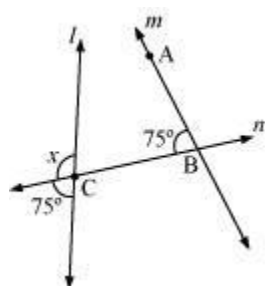


Consider two lines, l and m , and a transversal line n which is intersecting them.

Sum of the interior angles on the same side of transversal = $126^\circ + 44^\circ = 170^\circ$

As the sum of interior angles on the same side of transversal is not 180° , therefore, l is not parallel to m .

(ii)

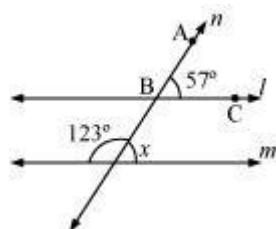


$x + 75^\circ = 180^\circ$ (Linear pair on line l)

$$x = 180^\circ - 75^\circ = 105^\circ$$

For l and m to be parallel to each other, corresponding angles ($\angle ABC$ and $\angle x$) should be equal. However, here their measures are 75° and 105° respectively. Hence, these lines are not parallel to each other.

(iii)

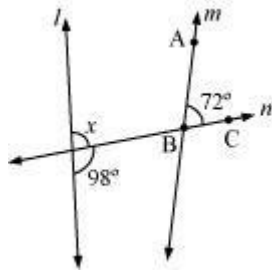


$\angle x + 123^\circ = 180^\circ$ (Linear pair)

$$\angle x = 180^\circ - 123^\circ = 57^\circ$$

For l and m to be parallel to each other, corresponding angles ($\angle ABC$ and $\angle x$) should be equal. Here, their measures are 57° and 57° respectively. Hence, these lines are parallel to each other.

iv.



$$98 + \angle x = 180^\circ \text{ (Linear pair)}$$

$$\angle x = 82^\circ$$

For l and m to be parallel to each other, corresponding angles ($\angle ABC$ and $\angle x$) should be equal. However, here their measures are 72° and 82° respectively. Hence, these lines are not parallel to each other.

Previous Years Solution

2019

1st Term

(iv) If two lines are parallel then the alternate interior angles are:
(a) equal; (b) not equal; (c) complementary; (d) supplementary.

Ans: (a) equal

(v) The measures of two complementary angles are in the ratio of 1:2, what is the measure of the larger angle?

(a) 30° ; (b) 45° ; (c) 60° ; (d) 120°

Ans: (c) 60°

3. State True or False:

(i) If two angles are equal and supplementary, then each angle is 90° .

Ans: True

(ii) The sum of two acute angles can be 180° .

Ans: False

3. State True or False:

(i) If two angles are equal and supplementary, then each angle is 90° .

Ans: True

(ii) The sum of two acute angles can be 180° .

Ans: False

5. Write Yes or No:

1x5=5

(i) A transversal is a line that intersects two coplanar lines at two different points.

Ans: Yes

(ii) Supplementary angles whose sum is 108° .

Ans: No

(iii) Adjacent angles means two angles with no common arm.

Ans: No

(iv) An unlimited number of lines can be drawn through a given point.

Ans: Yes

(v) Three or more points which lie on the same line are called collinear points.

Ans: Yes.

(iv) Find the complement of 54° .

Ans: The sum of two complementary angles is 90°

So the complement of 54° is $90^\circ - 54^\circ = 36^\circ$

(v) Define complete angle.

Ans: An angle whose measure is 360° is called complete angle.

(iv) What do you mean by adjacent angles? Draw the diagram.

Ans: Two angles with a common vertex, a common arm and the other arms lying on the opposite sides of the common arm form a pair of adjacent angles.

2nd Term

iii) Define obtuse triangle. Ans- Any one angle is obtuse (>90 but < 180)

iv) Find the complementary angle of 64° . Ans- $(90-64) = 26^\circ$.

3rd Term

3) Write true or false : ($1 \times 5 = 5$)

i) The sum of two acute angles can be 180°

Ans: false

ii) The two angles are equal and complementary, then each angle is 45°

Ans: true

ii) Three or more points which lie on the same line.

Ans: collinear points

2018

1st Term

iii) Complementary angle of 77° is

b) 13°

iv) A straight line cuts two other straight lines and the following are the pairs of alternate angles in different cases. Show in which of these cases the two straight lines are parallel.

b) $80^\circ, 80^\circ$ d) $119^\circ, 119^\circ$

iii) Three or more points which lie on the same line are called collinear points

iv) A line that divides an angle into two equal angles is called Bisector.

v) If two coplanar lines are cut by a transversal and they are parallel, then alternate interior angles are Equal

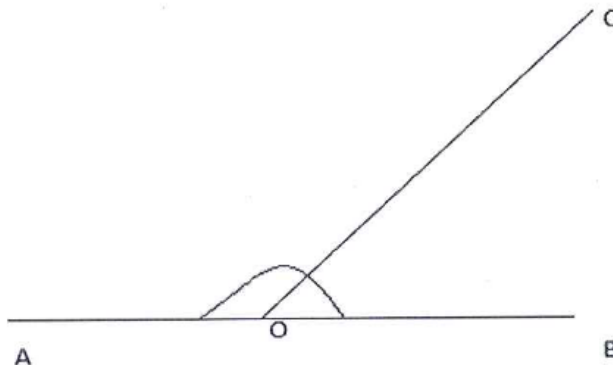
iv) Corresponding angles lie on the opposite sides of the transversal. False

(iv) Supplement of $84^\circ = 180^\circ - 84^\circ = 96^\circ$

(iv) Adjacent angles whose two non-common arms are opposite rays, form a linear

(v) pair.

refl



(vi) Let two complementary angles be $4x$ and $5x$.

BTP, $4x + 5x = 90^\circ$

$\Rightarrow 9x = 90^\circ$

$\Rightarrow x = \frac{90^\circ}{9} = 10^\circ$

\therefore the angles are: $4 \times 10^\circ = 40^\circ$ and $5 \times 10^\circ = 50^\circ$.

2nd
Term

i) The angles formed by two intersecting lines having no common arm are called vertically opposite angles. true

v) If two angles are equal and supplementary, then each angle is 90°

i) What is bisector of an angle? Ans: the line segment that divides an angle into two equal halves

ii) What is Exterior angle? Ans: when any side of a triangle is extended beyond the vertex it forms an exterior angle with the other side at the same vertex.

iv) Find the complement of 58° .

Ans 32°

iv) An angle is 40° less than three times its supplement. Find the angles.

Or

Two complementary angles are in the ratio 4:5. Find the angles.

iv) Let one angle be x . supplement of this angle is $(180-x)$ according to the question

$x = 3(180-x) - 40$

or, $x = 540 - 3x - 40$

or, $x + 3x = 500$

or, $4x = 500$

or, $x = 125^\circ$

supplement = $180^\circ - 125^\circ = 55^\circ$

Required angles are 125° and 55°

Or

Let the angles be $4x$ and $5x$

The given angles are complementary $4x + 5x = 90^\circ$

Or, $9x = 90$

Or, $x = 10$

Therefore $4x = 40^\circ$ and $5x = 50^\circ$

3rd Term

iii) Supplement angle of 84° is

b) 96°

vi) Find out supplement of $129^\circ 43' 24''$.

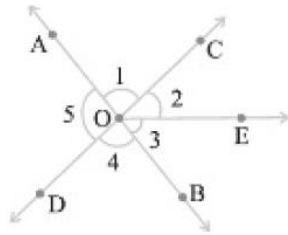
Ans. $179^\circ 59' 60'' - 129^\circ 43' 24'' = 50^\circ 16' 36''$

Exercise Problems

How many points a line segment have?

- a. 2
- b. 1
- c. 3
- d. 0

2. In the following figure which angle is adjacent to $\angle 1$?

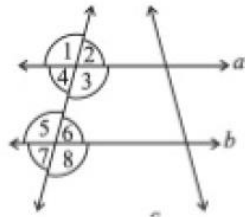


- a. $\angle 3$
- b. $\angle 2$
- c. $\angle 5$
- d. both b and c

3. If a line is a transversal to three lines, how many points of intersections are there?

- a. 1
- b. 2
- c. 3
- d. 4

4. State the property that is used below: If $a \parallel b$, then $\angle 1 = \angle 5$.



- a. alternate interior angles
- b. pair of interior angle
- c. vertically opposite angles
- d. corresponding angles

Fill in the blanks:

5. When the sum of the measures of two angles is 90° , the angles are called _____.

6. _____ angles have a common vertex and a common arm but no common interior points.

7. When two lines intersect, the vertically opposite angles so formed are _____.

8. Two lines l and m intersect if they have a point in _____.

9. When a transversal cuts two lines, such that pairs of corresponding angles are equal, then the lines have to be _____.

10. In the following figure, identify the pairs of corresponding angles

