St. Lawrence High School
A Jesuit Christian Minority Institution
Study Material – 3
Term : 1st

Class – XI | Subject – Physics
Chapter – Motion in plane and Vectors | Date – 17.08.20

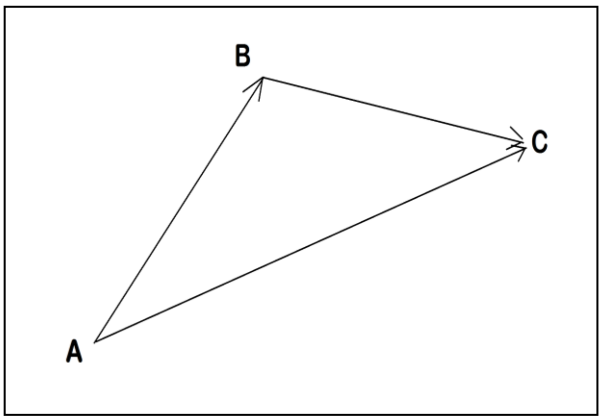
❖ Different types of vector

- **Position vector** – the vector that defines the position of a point w.r.t the origin of a coordinate system.
- **Polar vector** – these are the vectors which have a starting point or a point of application. E.g – Displacement, force etc
- **Axial vector -** The vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule are called axial vectors *e.g.*, torque, angular momentum., etc.
- **Equal vectors -** Two vectors are said to be equal if they have the same magnitude and direction.
- **Negative vector -** The negative of a vector is defined as another vector having the same magnitude but having an opposite direction.
- **Zero vector or null vector -** A vector having zero magnitude and an arbitrary direction is called a zero or null Vector.
- **Collinear vectors -** The vectors which either act along the same line or along parallel lines are called collinear vectors.
- **Coplanar vectors -** The vectors which act in the same plane are called coplanar vectors.
- **Unit vector -** A unit vector is a vector of unit magnitude drawn in the direction of a given vector. A unit vector in the direction of $\vec{A}$ is given by $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

❖ Vector addition –

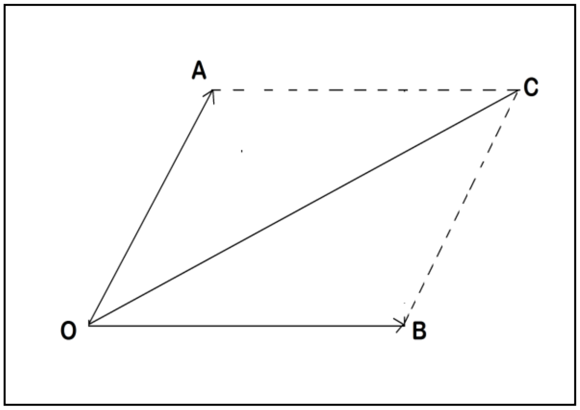
- **Triangular law –**
  If two vectors can be re presented both in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant is represented completely both in magnitude and direction by the third side of the triangle taken in the opposite order.

  So, as shown in the figure –
  $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$
  If $|\overrightarrow{AB}| = a$, $|\overrightarrow{BC}| = b$ and $|\overrightarrow{AC}| = $ c
  And the angle between two vectors $\overrightarrow{AB}$ *and* $\overrightarrow{BC}$ be θ
  Then, $c^2 = a^2 + b^2 + 2ab\cos\theta$



- **Parallelogram law –** If two vectors acting simultaneously at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented completely both in magnitude and direction by the diagonal of the parallelogram drawn from that point.

$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$
If $|\overrightarrow{OA}| = a$, $|\overrightarrow{OB}| = b$ and $|\overrightarrow{OC}| = c$
And the angle between two vectors $\overrightarrow{OB}$ *and* $\overrightarrow{OB}$ be θ
Then, $c^2 = a^2 + b^2 + 2ab\cos\theta$
And also, if the angle between two vectors $\overrightarrow{OC}$ and $\overrightarrow{OB}$ is φ
Then – $\tan\varphi = \frac{a\sin\theta}{b+a\cos\theta}$



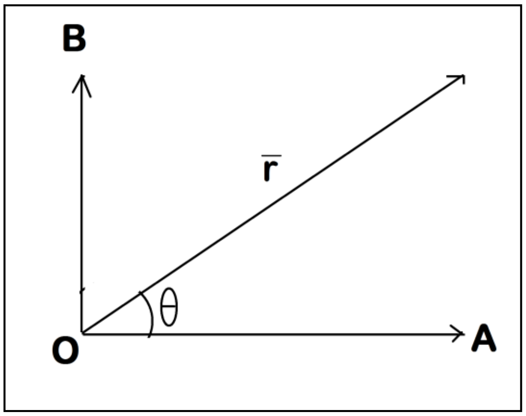
- **Polygon law of vector addition –**
If a number of vectors arc represented both in magnitude and direction by the sides of an open polygon taken in the same order, then their resultant represented both in magnitude and direction by the closing side of the polygon taken in opposite order.

❖ Representation of vectors in Cartesian coordinate system

$\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ are the unit vectors considered respectively along X-axis, Y – axis and Z – axis.
Hence, $|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1$
And the position vector of the point (x, y, z) can be taken as $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
$x\hat{\imath}$ is the x-component of the vector $\vec{r}$
$y\hat{\jmath}$ is the y-component of the vector $\vec{r}$
$z\hat{k}$ is the z-component of the vector $\vec{r}$

❖ Resolution of a vector in two mutually perpendicular components

A vector can be resolved to have its components along any two mutually perpendicular directions. As shown in the figure the components of the vector $\vec{r}$ along OA and OB will be of magnitudes $r\cos\theta$ and $r\sin\theta$ respectively.



❖ Product of vectors

- **Dot product or scalar product** – The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and cosine of the angle θ between them. Thus –

$\vec{A}\bullet\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$
So, $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$ and $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$
Again, for two vector $\vec{A} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{B} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$,

$\vec{A}\bullet\vec{B} = x_1x_2 + y_1y_2 + z_1z_2$

- **Cross product or vector product** – the cross product of two vectors is defined as
  $\vec{A} \times \vec{B} = |\vec{A}|.|\vec{B}| \sin\theta\, \hat{n}$, where $\hat{n}$ is the unit vector perpendicular to both the vectors $\vec{A}$ and $\vec{B}$.
  So, $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$ and
  $\hat{\imath}.\hat{\jmath} = \hat{k}$
  $\hat{\jmath} \times \hat{k} = \hat{\imath}$
  $\hat{k} \times \hat{\imath} = \hat{\jmath}$
  Again, for $\vec{A} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{B} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$
  $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

❖ <u>Projectile motion</u>

if a point mass is projected with initial velocity u, making θ angle with the horizontal, then –

i) The time of flight is $T = \frac{2u \sin\theta}{g}$

ii) Maximum height achieved $H = \frac{u^2 \sin^2\theta}{2g}$

iii) Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

iv) Equation of trajectory is $y = x\tan\theta - \frac{g}{2\,u^2 \cos^2\theta}\, x^2$

v) The horizontal range will be maximum for angle of projection $\theta = 45^0$

vi) For a fixed initial speed of projection, the horizontal range is equal for two complementary angles of projection.

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