# ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

**STUDY MATERIAL-11**

**SUBJECT – MATHEMATICS**

**Pre-test**

**Chapter: Continuity & Differentiability** **Class: XII**

**Topic: Continuity** **Date: 15.06.2020**

## -: Continuity :-

**Definition :-** **Let $f(x)$ be any single valued function of $x$ and $x = a$ be a point in the domain of definition of the function. $f(x)$ is said to be continuous at $x = a$, if**

$$\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x) = f(a)$$

$$\text{Or, } \lim_{h \to 0} f(a+h) = f(a).$$

- **The polynomial function is continuous for all real values.**
- **The functions $e^x$ , $\sin x, \cos x$ are continuous for all real x.**
- **The function $\log_e x$ is continuous for all positive values of x.**
- **The functions $\cot x, \operatorname{cosec} x$ are continuous for all real x other than $x = n\pi, where\ n\ is$ any integer.**
- **The functions $\tan x, \sec x$ are continuous for all real x other than $x = (2n+1)\frac{\pi}{2}, where\ n$ is any integer.**
- **If the functions $f(x)$ & $g(x)$ are continuous at $x = a$, then the functions $f(x) + g(x)$ ; $f(x) - g(x)$ ; $f(x) \cdot g(x)$ ; $\frac{f(x)}{g(x)}$ $\{When, g(x) \neq 0\}$ & $f(g(x))$ are also continuous at $x = a$.**

**1.** Prove that the function $f\ (x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$

Given $f(x) = 5x - 3$

**At $x = 0$**

*f(x)* is continuous at x = 0 if

$$\lim_{x\to 0} f(x) = f(0)$$

| **L.H.S** | **R.H.S** |
|---|---|
| $\lim_{x\to 0} f(x)$ | $f(0)$ |
| $= \lim_{x\to 0} (5x - 3)$ | $= 5(0) - 3$ |
| *Putting $x = 0$* | $= 0 - 3$ |
| $= 5(0) - 3$ | $= -3$ |
| $= -3$ | |

Since L.H.S = R.H.S

$$\therefore \lim_{x\to 0} f(x) = f(0)$$

Hence, $f$ **is continuous at $x = 0$**

**At x = −3**

*f(x)* is continuous at x = −3 if

$$\lim_{x\to -3} f(x) = f(-3)$$

| L.H.S | R.H.S |
|---|---|
| $\lim_{x \to 3} f(x)$ | $f(-3)$ |
| $= \lim_{x \to 3} (5x - 3)$ | $= 5(-3) - 3$ |
| *Putting* $x = -3$ | $= -15 - 3$ |
| $= 5(-3) - 3$ | $= -18$ |
| $= -18$ | |

Since, L.H.S = R.H.S

$$\therefore \lim_{x \to -3} f(x) = f(-3)$$

Hence, $f$ **is continuous at** $x = -3$

**At $x = 5$**

*f(x)* is continuous at x = 5 if

$$\lim_{x \to 5} f(x) = f(5)$$

| L.H.S | R.H.S |
|---|---|
| $\lim_{x \to 5} f(x)$ | $f(5)$ |
| $= \lim_{x \to 5} (5x - 3)$ | $= 5(5) - 3$ |
| *Putting* $x = 5$ | $= 25 - 3$ |
| $= 5(5) - 3$ | $= 22$ |
| $= 22$ | |

Since, L.H.S = R.H.S

$\therefore \lim_{x \to 5} f(x) = f(5)$

Hence, $\boldsymbol{f}$ **is continuous at** $\boldsymbol{x = 5}$

Thus the function is continuous at

$\boldsymbol{x = 0}$, **at** $\boldsymbol{x = -3}$ **& at** $\boldsymbol{x = 5}$

## 2. Examine the Following Function for continuity.

$f(x) = x - 5$

$f(x) = x - 5$

Since x – 5 is a polynomial.

$\therefore f(x)$ is defined for every real number c.

Let us check continuity at x = c

f(x) is is continuous at x = c if

$$\lim_{x \to c} f(x) = f(c)$$

| LHS | RHS |
|---|---|
| $\lim_{x \to c} f(x)$<br>$= \lim_{x \to c} x - 5$<br>$= c - 5$ | f (c) = c – 5 |

Since $\lim\limits_{x \to c} f(x) = f(c)$

So, $f$ is continuous for x = c, where c is a real number

$\therefore f$ is continuous for all real numbers

Hence, $f$ **is continuous for each x** $\in$ **R**

## 3. Examine the Following Function for continuity.

$f(x) = |x - 5|$

$f(x) = |x - 5|$

$$= \begin{cases} (x-5), & x \geq 5 \\ -(x-5), & x < 5 \end{cases}$$

**Since we need to find continuity at of the function**

We check continuity for different values of x

- When x = 5
- When x < 5
- When x > 5

**Case 1 : When x = 5**

*Since there are two different functions on the left & right of 5, we take LHL & RHL .*

$f$(x) is continuous at $x = 5$

if L.H.L = R.H.L = $f(5)$

if $\lim\limits_{x \to 5^-} f(x) = \lim\limits_{x \to 5^+} f(x) = f(5)$

**LHL at $x \to 5$**

$$\lim_{x \to 5^-} f(x) = \lim_{h \to 0} f(5-h)$$

$$= \lim_{h \to 0} -\left((5-h)-5\right)$$

$$= \lim_{h \to 0} -(-h)$$

$$= 0$$

**RHL at $x \to 5$**

$$\lim_{x \to 5^+} f(x) = \lim_{h \to 0} f(5+h)$$

$$= \lim_{h \to 0} \left((5+h)-5\right)$$

$$= \lim_{h \to 0} h$$

$$= 0$$

& $f(5) = (x-5)$

$= 5-5$

$= 0$

Hence, L.H.L = R.H.L = $f(5)$

$\therefore f$ is continuous at $x = 5$

**Case 2 : When x < 5**

For x < 5,

$f(x) = -(x-5)$

Since this a polynomial

It is continuous

$\therefore$ f(x) is continuous for x < 5

**Case 3 : When x > 5**

For x > 5,

$f(x) = (x - 5)$

Since this a polynomial

It is continuous

∴ f(x) is continuous for x > 5

Hence, $f(x) = |x - 5|$ is continuous at all points.

i.e. $\boldsymbol{f}$ **is continuous at** $\boldsymbol{x} \in$ **R.**

## 4. Examine the Following Function for continuity.

$f(x) = \frac{1}{x-5}$, $x \neq 5$

at x = 5, $f(x)$ is not defined.

Let us check continuity at x = c, where x ≠ 5

f(x) is continuous at x = c, x ≠ 5 if

$$\lim_{x \to c} f(x) = f(c)$$

| LHS | RHS |
|---|---|
| $\lim_{x \to c} f(x)$ <br> $= \lim_{x \to c} \frac{1}{x-5}$ <br> $= \frac{1}{c-5}$ | $f(c) = \frac{1}{C-5}$ |

Since $\lim\limits_{x \to c} f(x) = f(c)$

$f$ is continuous for all real numbers except 5

$\therefore$ **$f$ is continuous at each $x \in$ R – $\{5\}$**

**5.** Is the function $f$ defined by $f(x) = \begin{cases} x, & if\ x \leq 1 \\ 5, & if\ x > 1 \end{cases}$

continuous at $x = 0$ ? At $x = 1$ ? At $x = 2$ ?

Given $f(x) = \begin{cases} x, & if\ x \leq 1 \\ 5, & if\ x > 1 \end{cases}$

**At x = 0**

For x = 0,

f(x) = x

Since this a polynomial

It is continuous

$\therefore$ f(x) is continuous for x = 0

**At x = 1**

$f$(x) is continuous at $x = 1$

if L.H.L = R.H.L = $f(1)$

if $\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^+} f(x) = f(1)$

$$f(x) = \begin{cases} x, & if\ x \le 1 \\ 5, & if\ x > 1 \end{cases}$$

*Since there are two different functions on the left & right of 1, we take LHL & RHL .*

**LHL at x → 1**

$\lim\limits_{x\to1^-} f(x) = \lim\limits_{h\to0} f(1-h)$

$= \lim\limits_{h\to0} (1-h)$

$= 1 - 0$

$= 1$

**RHL at x → 1**

$\lim\limits_{x\to1^+} f(x) = \lim\limits_{h\to0} f(1+h)$

$= \lim\limits_{h\to0} 5$

$= 5$

Since L.H.L ≠ R.H.L

*f(x)* is not continuous at $x = 1$

**At x = 2**

For x = 2,

f(x) = 5

Since this a constant function

It is continuous

∴ f(x) is continuous for x = 2

**6.** Find all points of discontinuity of $f$, where $f$ is defined by

$$f(x) = \begin{cases} |x| + 3, & if\ x \leq -3 \\ -2x, & if\ -3 < x < 3 \\ 6x + 2, & if\ x \geq 3 \end{cases}$$

Since we need to find continuity at of the function

We check continuity for different values of x

- When x < −3
- When x = −3
- When −3 < x < 3
- When x = 3
- When x > 3

**Case 1 : When x < −3**

For x < −3,

$f(x) = |x| + 3$

$f(x) = -x + 3$ *(As x < −3, x is negative)*

Since this a polynomial

It is continuous

∴ f(x) is continuous for x < −3

**Case 2 : When x = −3**

$f(x)$ is continuous at $x = -3$

if L.H.L = R.H.L = $f(-3)$

if $\lim_{x\to-3^-} f(x) = \lim_{x\to-3^+} f(x) = f(-3)$

*Since there are two different functions on the left & right of −3, we take LHL & RHL .*

**LHL at x → −3**

$\lim_{x\to-3^-} f(x) = \lim_{h\to0} f(-3 - h)$

$= \lim_{h\to0} (|-3 - h| + 3)$

$= |-3 - 0| + 3$

$= |-3| + 3$

$= 3 + 3$

$= 6$

**RHL at x → −3**

$\lim_{x\to-3^+} f(x) = \lim_{h\to0} f(-3 + h)$

$= \lim_{h\to0} -2(-3 + h)$

$= \lim_{h\to0} 6 - 2h$

$= 6 - 0$

$= 6$

& $f(-3) = |-3| + 3$

$= 3 + 3$

$= 6$

Hence, L.H.L = R.H.L = $f(-3)$

$\therefore f$ is continuous at $x = -3$

**Case 3 : When −3 < x < 3**

For $-3 < x < 3$,

$f(x) = -2x$

Since this a polynomial

It is continuous

$\therefore$ f(x) is continuous for $-3 < x < 3$

**Case 4 : When x = 3**

*Since there are two different functions on the left & right of 3, we take LHL & RHL .*

$f(x)$ is continuous at $x = 3$

if L.H.L = R.H.L = $f(3)$

if $\lim\limits_{x\to3^-} f(x) = \lim\limits_{x\to3^+} f(x) = f(3)$

**LHL at x → 3**

$\lim\limits_{x\to3^-} f(x) = \lim\limits_{h\to0} f(3-h)$

$= \lim\limits_{h\to0} -2(3-h)$

$= \lim\limits_{h\to0} -6+2h$

$= -6 + 0$

$= -6$

**RHL at x → 3**

$\lim\limits_{x\to3^+} f(x) = \lim\limits_{h\to0} f(3+h)$

$= \lim\limits_{h\to0} 6(3+h)+2$

$= \lim\limits_{h\to0} 18+6h+2$

$= \lim\limits_{h\to0} 20+6h$

$= 20 + 0$

$= 20$

Since L.H.L ≠ R.H.L

*f(x)* is not continuous at $x = 3$

**Case 5: When x > 3**

For x > 3,

f(x) = 6x + 2

Since this a polynomial

It is continuous

∴ f(x) is continuous for x > 3

Hence, $f$ is discontinuous at only $x = 3$

Thus, $f$ is **continuous at all real numbers except 3.**

$f$ is continuous at $x \in \mathrm{R} - \{3\}$

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