ST. LAWRENCE HIGH SCHOOL

TOPIC- FACTORISATION

Sub: Mathematics **CLASS-9**

Study Material-5 **Date: 8.5.2020**

USEFUL FORMULAE :

1. a) $a^2 - b^2 = (a+b)(a-b)$

   b) $a^2 + b^2 = (a+b)^2 - 2ab = (a-b)^2 + 2ab$

   c) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

   d) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

   e) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

   f) If $a+b+c=0$ then $a^3 + b^3 + c^3 = 3abc$

   g) $x^2 + (p+q)x + pq = (x+p)(x+q)$

   h) $x^2 - (p+q)x + pq = (x-p)(x-q)$

   i) $x^2 + (p-q)x - pq = (x+p)(x-q)$

   j) $x^2 - (p-q)x - pq = (x-p)(x-q)$

2. If in an expression putting x=a, its value becomes zero then (x-a) is a factor of the expression. Its converse is also true.

3. If in an expression putting x=-a, its value becomes zero then (x+a) is a factor of the expression. Its converse is also true.

SOLVED SUMS :

1. Factorise : $128x^4 + 162$

   Ans: $= 2(64x^4 + 81)$

   $= 2\{(8x^2)^2 + (9)^2\}$

$= 2\{(8x^2+9)^2 - 2.8x^2.9\}$

$=2\{(8x^2+9)^2 - (12x)^2\}$

$=2(8x^2+9+12x)(8x^2+9-12x).$

2. Factorise : $ax^2 - (a^2-1)x - a$

   Ans : $=ax^2 - a^2x + x - a$

   $= ax(x-a) + 1(x-a)$

   $=(ax+1)(x-a)$

3. What is the sum of the factors of $a^2 - 5a - 150$?

   Ans: $a^2 - 5a - 150$

   $= a^2 - 15a + 10a - 150$

   $=a(a-15)+10(a-15)$

   $= (a-15)(a+10)$

Therefore the sum of the factors is $(a-15)+(a+10) = 2a - 5$.

4. If the two factors of a quadratic expression are (2x-3) and (3x+2) then find the expression ?

   Ans : $(2x-3)(3x+2)$

   $= 6x^2 - 9x + 4x - 6$

   $= 6x^2 - 5x - 6.$

   Hence the required expression is $6x^2 - 5x - 6$.

5. Factorise : $x^3 - 6x + 4$

   Ans : If in the above expression we put x=2 the value of the expression becomes 0.
   Therefore, (x-2) is a factor of the given expression.

   Hence the given expression

   $=x^3 - 2x^2 + 2x^2 - 4x - 2x + 4$

   $=x^2(x-2) + 2x(x-2) - 2(x-2)$

   $=(x-2)(x^2+2x-2)$

6. Factorise : $x^3 - 12x - 16$

   Ans : if in the above expression we put x=-2, the value of the expression becomes 0.
   Therefore ,(x+2) is a factor of the given expression.

Hence the given expression

$= x^3+2x^2 -2x^2 -4x -8x -16$

$= x^2(x+2) -2x(x+2) -8(x+2)$

$= (x+2)(x^2 -2x -8)$

$= (x+2)(x -4)(x+2).$

7. Factorise : $(x-1)(x-2)(x+3)(x+4) -36$

Ans : Grouping the above expression we get ,

$=\{(x-1)(x+3)\}\{(x-2)(x+4)\} -36$

$= (x^2 +2x -3)(x^2 +2x -8) -36$

$= (y -3)(y -8) -36$ [putting $x^2 +2x=y$]

$=y^2 -11y +24 -36$

$=y^2 -11y -12$

$=y^2 -(12-1)y -12$

$=y^2 -12y +y -12$

$=y(y -12) +1(y -12)$

$= (y -12)(y+1)$

$=(x^2 +2x -12)(x^2+2x+1)$ [putting $y=x^2+2x$]

$=(x^2+2x -12)(x+1)^2$

8. If $a^3 +b^3 +c^3 -3abc =0$ and $a+b+c$ not equal to 0 then show that, $a=b=c$.

Ans : $a^3 +b^3 +c^3 -3abc =0$

Or ½ $(a+b+c)\{(a-b)^2 +(b-c)^2+(c-a)^2\} =0$

Or $(a-b)^2 +(b-c)^2 +(c-a)^2 =0$ [since $a+b+c$ is not equal to 0]

Since the sum of three squares is zero,
Each of them is equal to zero.

Therefore $(a-b)^2 =0$ or $a-b=0$ 0r $a=b$.

Again $(b-c)^2 =0$ or $b-c =0$ or $b=c$.

Again $(c-a)^2 =0$ or $c-a=0$ or $c=a$.

Hence $a=b=c$ [proved].

9. If $a^2 -b^2 =11 \times 7$ where a and b are positive, find the value of a and b ?

Ans : $a^2 -b^2 =11 \times 7$

Or $(a+b)(a-b) = 11 \times 7$

Since $a>0$, $b>0$, and $a>b$,

Therefore $(a+b) = 11$ and $(a-b)=7$

Solving we get, $a=9$ and $b=2$.

10. If the area of a rectangle is $(x^2 - x - 20)$ sqcm and its breadth is $(x-5)$cm then what is its length ?

Ans : $x^2 - x - 20$

$= x^2 - 5x + 4x - 20$

$= x(x-5) + 4(x-5)$

$= (x-5)(x+4)$

We know area =length x breadth.

If breadth is $(x-5)$, then the length is $(x+4)$.

DEBJANI DAS.