ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL - 2 (PART - II)

Subject : PHYSICS

Date : 13.5.20

CLASS : XII

Chapter : Electrostatics

Topic : Torque on dipole, potential, potential energy, equipotential surface, potential of a charged spherical conductor

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D : Short Notes

1. **Electric Potential :** The amount of work done to bring unit positive charge from infinity to that point against the electric field of a given charge without changing its kinetic energy or velocity.

   $$V = \int_{\infty}^{r} -E.dx = \frac{Q}{4\pi\varepsilon_0 r}$$

   It is a scalar quantity and its unit is volt, 1 volt = $\frac{1J}{1C}$. Its dimensional formula is $[ML^3T^{-3}A^{-1}]$.

2. **Potential Difference :** $\Delta V = V_2 - V_1 \quad = \int_{r_1}^{r_2} -E.dr \quad = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

3. **Potential Energy :** $U = qV = \frac{Qq}{4\pi\varepsilon_0 r}$

4. **Equipotential Surface** is the surface, where potential is equal at every point. For a point charge, a sphere will be equipotential surface with point charge at the centre at the centre of the sphere. Equipotential surface for a long line charge is a cylinder with line charge along its axis.

   The work done in carrying a charge from one point to another along an equipotential surface is zero.

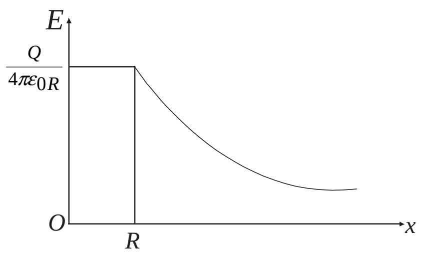
   The electric field lines are always perpendicular to the equipotential surface.

   Every conductor (metal) is an equipotential surface and hence electric field lines will emerge perpendicular to it.

   Electric field and surface charge density are maximum at pointed ends of a conductor.

   Note $\oint \vec{E}.\vec{dl} = 0$ and $\int_a^b \vec{E}.\vec{dl} = V_a - V_b$

5. **Electric potential due to a spherical shell (radius R, charge Q)**



   $V_{inside} = \frac{Q}{4\pi\varepsilon_0 R} = V_{surface} \qquad x \le R$

   $V_{outside} = \frac{Q}{4\pi\varepsilon_0 x} \qquad x > R$

6. **Electric potential along axial line**

   $V_{axial} = \frac{P}{4\pi\varepsilon_0 (x^2 - l^2)}$ $\qquad V_{axial} = \frac{P}{4\pi\varepsilon_0 x^2}$ due to a short dipole.

7. **Electric potential at any point along equatorial line :** $V_{equatorial} = 0$

8. **Electric potential due to a dipole at any point**

$$V_{any\ point} = \frac{p\cos\theta}{4\pi\varepsilon_0(x^2 - l^2\cos^2\theta)} \qquad V_{any\ point} = \frac{p\cos\theta}{4\pi\varepsilon_0 x^2}\text{, due to a short dipole.}$$

x = distance between mid point of the dipole and the point of observation.

l = Half length of the diapole.

9. **Torque experienced by a dipole when placed in a uniform electric field E**

$\Sigma F = 0$, i.e., no linear motion is possible

$\vec{\tau} = \vec{P} \times \vec{E} = pE \sin\theta$ is illustrated in the following figure.

If $\theta = 0,\ \tau = 0$, equilibrium is stable.

if $\theta = 90^0,\ \tau = pE$ and is maximum.

If $\theta = 180^0,\ \tau = 0$ equilibrium is unstable.

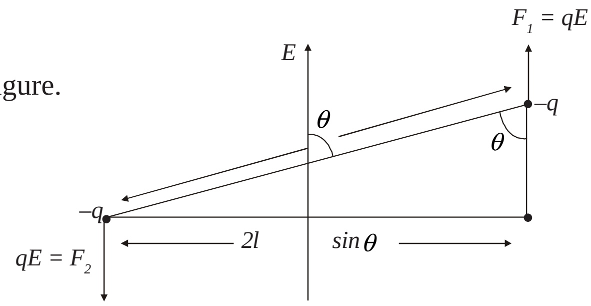


Fig. Torque due to a dipole

Work done W = $\int_{\theta_1}^{\theta_2} \tau . d\theta = pE(\cos\theta_1 - \cos\theta_2)$

If $\theta_1 = 0,\ \theta_2 = 180^0$ (i.e., dipole is reversed) when $W = 2pE$

If $\theta_1 = 0,\ \theta_2 = 90^0$

$W = pE$

Potential energy due to a dipole $U = -pE \cos\theta$

10. **Potential Energy (PE) _** It is the amount of work done to bring a charge q from infinity to that point against the electric field of a given charge Q without changing its KE.

$PE\ U = \frac{qQ}{4\pi\varepsilon_0 r} = qV$ Since the electrostatic force is conservative, therefore work done $W = \Delta PE$

$$W = U_f - U_i = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right] = q\left[V_{final} - V_{initial}\right]$$

11. **Charged particle in motion :** Force $F = qE$ $\quad ma = qE \quad$ *or,* $a = \frac{qE}{m}$

Velocity v after travelling a distance of using $v^2 = 2ad$ is $v = \sqrt{\frac{2qEd}{m}}$

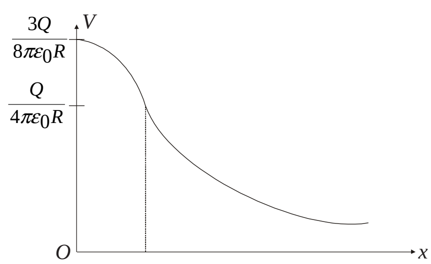
Velocity after time t if it starts from rest $v = at = \frac{qEt}{m}$

12. **Potential due to a uniformly charged sphere**

$V_{out} = \frac{Q}{4\pi\varepsilon_0 x}$ for $x > R$

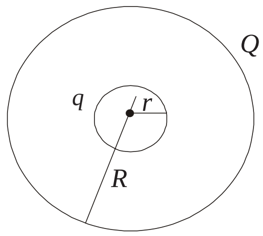
$V_{inside} = \int_R^x \frac{-Qx^2 dx}{4\pi\varepsilon_0 R^3} + \frac{Q}{4\pi\varepsilon_0 R}$ for $x < R$

$V_{surface} = \int_R^x \frac{Q}{4\pi\varepsilon_0 R}$ for $x = R$



E. Exercise Problems :

1. Two concentric shells carry charges q and Q. Their radius are and R. The potential difference between the two is —

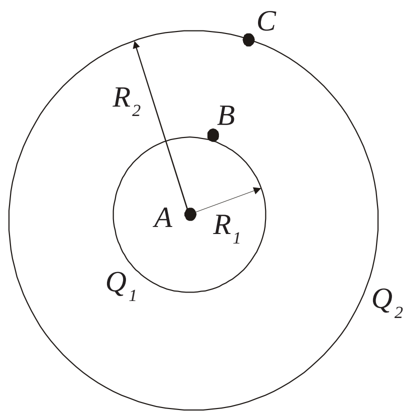


Ans. : $V_R = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R}+\frac{q}{R}\right]$ and $V_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}+\frac{Q}{R}\right]$

$$\Delta V = V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R}\right]$$

2. Two concentric spherical shells of radius R_1 and $R_2(R_2>R_1)$ are having uniformly distributed charges Q_1 and Q_2 respectively. Find out potentialC

(A) at point A
(B) at surface of smaller shell (i.e. at point B)
(C) at surface of larger shell (i.e. at point C)
(D) at $r \le R_1$
(E) at $R_1 \le r \le R_2$
(F) at at $r \ge R_2$



Solution : Using the results of hollow sphere as

(A) $V_A = \frac{KQ_1}{R_1}+\frac{KQ_2}{R_2}$

(B) $V_B = \frac{KQ_1}{R_1}+\frac{KQ_2}{R_2}$

(C) $V_C = \frac{KQ_1}{R_2}+\frac{KQ_2}{R_2}$

(D) For $r \le R_1$

$$V = \frac{KQ_1}{R_1}+\frac{KQ_2}{R_2}$$

(E) For $R_1 \le r \le R_2$

$$V = \frac{KQ_1}{r}+\frac{KQ_2}{R_2}$$

(F) For $r \ge R_2$

$$V = \frac{KQ_1}{r}+\frac{KQ_2}{r}$$

3. Two hollow concentric non-conducting spheres of radius a and b $(a > b)$ contains charges Q_a and Q_b respectively. Prove that potential difference between two spheres is independent of charge on outer sphere. If outer sphere is given an extra charge, is there any change in potential difference?

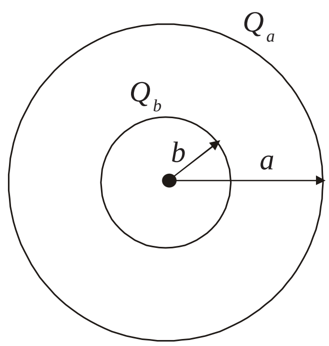
Solution :

$$V_{inner\ sphere} = \frac{KQ_b}{b}+\frac{KQ_a}{a}$$

$$V_{outer\ sphere} = \frac{KQ_b}{a}+\frac{KQ_a}{a}$$

$$V_{inner\ sphere} - V_{outer\ sphere} = \frac{KQ_b}{b} - \frac{KQ_b}{a}$$

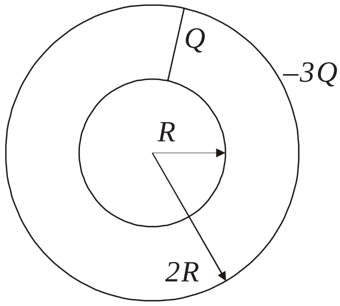
$$\Delta V = KQ_b\left[\frac{1}{b}-\frac{1}{a}\right]$$



which is independent of charge on outer sphere.

If outer sphere in given any extra charge, then there will be no change in potential difference.

4. The two conducting spherical shells are joined by a conducting wire and cut after some time when charge stops flowing. Find out the charge on each sphere after that.

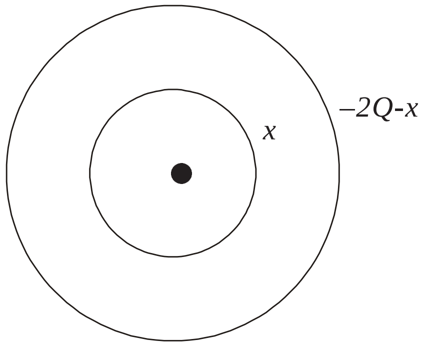


Solution : After cutting the wire, the potential of both the shells is equal.

Thus, potential of inner shell

$$V_{in} = \frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$$

and potential of outer shell $V_{out} = \frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$



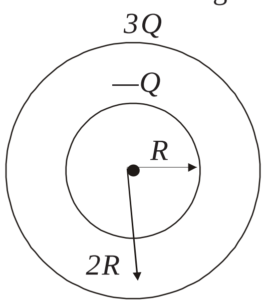
As $\qquad V_{out} = V_{in}$

$\Rightarrow \qquad \frac{-KQ}{R} = \frac{K(x-2Q)}{2R}$

$\Rightarrow \qquad -2Q = x - 2Q$

$\Rightarrow \qquad x = 0.$ So charge on inner spherical shell = 0 and outer spherical shell = $-2Q$.

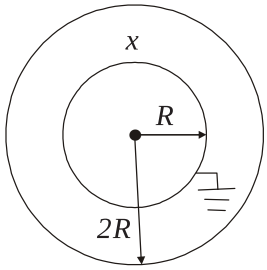
5. Two conducting hollow spherical shells of radii R and $2R$ carry charges $-Q$ and $3Q$ respectively. How much charge will flow into the earth if inner shell is grounded?



Solution : When inner shell is grounded to the earth then the potential of inner shell will become zero, because potential of the earth is taken to be zero

$$\frac{Kx}{R} + \frac{K3Q}{2R} = 0$$

$$x = \frac{-3Q}{2}$$



the charge that has increased

$$= \frac{-3Q}{2} - (-Q)\frac{2p}{r^3} = \frac{-Q}{2}$$

Hence charge flows into the earth = $\frac{Q}{2}$