ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-9

SUBJECT – MATHEMATICS

1st term

Chapter & Topic : Sequence & Series

Class: XI **Date: 01.07.2020**

Sequence :-

A sequence is a function of natural numbers with co-domain that is the set of real numbers and its terms are in a definite order.

$$f: N \to R \quad \text{defined as } t_n = f(n), \quad n \in N$$

is called a sequence and denoted by

$$\{t_1, t_2, t_3, \ldots\} = \{f(1), f(2), f(3), \ldots\}$$

Some more examples of sequences:

(a) $2, 4, 6, 8, \ldots$

(b) $5, 3, 1, -1, \ldots$

(c) $1, 3, 9, 27, \ldots$

(d) $32, 16, 8, 4, \ldots$

A sequence is said to be finite or infinite accordingly as it has the finite or infinite number of terms.

Examples :-

Example 1. If $f: N \to R$ where $f(n) = \frac{n}{(2n+1)^2}$, find the sequence in an ordered pair form.

Solution:

$$t_n = \frac{n}{(2n+1)^2}$$

Putting $n = 1, 2, 3, 4, \ldots$ successively, we get

$$t_1 = \frac{1}{(2.1+1)^2} = \frac{1}{3^2} = \frac{1}{9}$$

$$t_2 = \frac{2}{(2.2+1)^2} = \frac{2}{5^2} = \frac{2}{25}$$

$$t_3 = \frac{3}{(2.3+1)^2} = \frac{3}{7^2} = \frac{3}{49}$$

$$t_4 = \frac{4}{(2.4+1)^2} = \frac{4}{9^2} = \frac{4}{81}$$

Hence, it sequences in an ordered pair form

$$\left\{\left(1, \frac{1}{9}\right), \left(2, \frac{2}{25}\right), \left(3, \frac{3}{49}\right), \left(4, \frac{4}{81}\right), \ldots\right\}$$

Example 2. Write down the sequence whose n^{th} terms are

(A) $\left(\frac{2n+2}{4}\right)$ **(B)** $(-1)^n\left(\frac{3n+2}{5}\right)$

(C) $\frac{1}{n^2}\sin\left(\frac{n\pi}{3}\right)$

Solution:

(A) $t_n = \left(\frac{2n+2}{4}\right)$

Putting $n = 1, 2, 3, 4, \ldots$ successively, we get

$$1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$$

which is the required sequence.

(B) $t_n = (-1)^n\left(\frac{3n+2}{5}\right)$

Putting $n = 1, 2, 3, 4, \ldots$ successively, we get

$$t_1 = -1, \frac{8}{5}, -\frac{11}{5}, \frac{14}{5} \ldots$$

which is the required sequence.

(C) $t_n = \frac{1}{n^2}\sin\left(\frac{n\pi}{3}\right)$

Putting $n = 1, 2, 3, 4, \ldots$ successively, we get

$$t_1 = \frac{1}{1^2}\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$t_2 = \frac{1}{2^2}\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{8}$$

$$t_3 = \frac{1}{3^2}\sin\left(\frac{3\pi}{3}\right) = 0$$

$$t_4 = \frac{1}{4^2}\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{32}$$

Hence, the required sequence is

$$\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{8}, 0, -\frac{\sqrt{3}}{32}$$

Example 3. A sequence of numbers a_1, a_2, a_3 satisfies the relation $a_{n+1} = a_n + a_{n-1}$ for $n \geq 2$. Find a_4 if $a_1 = a_2 = 1$.

Solution: Put $n = 2$. Then

$$a_3 = a_2 + a_1 = 1 + 1 = 2$$

Again using $n = 3$, we get

$$a_4 = a_3 + a_2 = 2 + 1 = 3$$

Example 4. If a sequence of numbers $a_1, a_2, \ldots, a_n$ satisfies the relation $a_{n+1}^2 = a_n \cdot a_{n+2} + (-1)^n$ then find a_3, if $a_1 = 2$ and $a_2 = 5$.

Solution: Put $n = 1$ in the given relation. We get

$$a_2^2 = a_1 a_3 + (-1)^1 \Rightarrow 5^2 = 2a_3 - 1 \Rightarrow 2a_3 = 26 \Rightarrow a_3 = 13$$

Example 5. A sequence of numbers u_0, u_1, u_2, u_3 satisfies the relation $u_{n+1} = 3u_n - 2u_{n-1}$. Find u_2 if $u_0 = 2$ and $u_1 = 3$.

Solution: Put $n = 1$ in the given relation. We get

$$u_2 = 3u_1 - 2u_0 = 3.3 - 2.2 = 9 - 4 = 5$$

Series :-

If $a_1, a_2, a_3, a_4,$ .... is a sequence, then the corresponding series is given by

$S_N = a_1+a_2+a_3 + .. + a_N$

Note: The series is finite or infinite depending if the sequence is finite or infinite.

We denote $S_n = \sum_{r=1}^{n} a_r$ as finite series.

We denote $S_n = \sum_{r=1}^{\infty} a_r$ as infinite series.

Examples :-

Example 6. From the Sequence {5, 7, 9, 11, . . . }, prepare the series of 1st n terms.

Solution: $u_1 = 5 = 2 \times 1 + 3$

$u_2 = 7 = 2 \times 2 + 3$

Hence, nth term = $u_n = 2 \times n + 3$

Series of 1st n terms is $5 + 7 + 9 + 11 + . . . + (2n+3)$.

- **Prepared by**

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