



ST. LAWRENCE HIGH SCHOOL



A JESUIT CHRISTIAN MINORITY INSTITUTION

CLASS 8

SUBJECT :Algebra & Geometry

STUDY MATERIAL 5

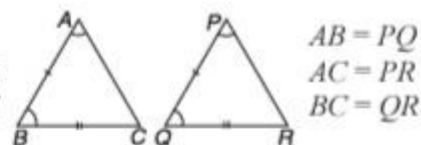
Congruency of Triangles

Date:8.5.2020

CONGRUENCY OF TRIANGLES

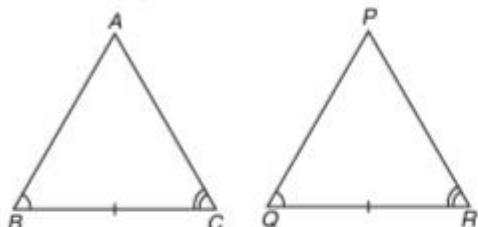
Tests of Congruency:

- (i) SAS Axiom (Side-angle-side): *If the two sides and the included angle of one triangle are respectively equal to the two sides and the included angle of the other, the triangles are congruent.*

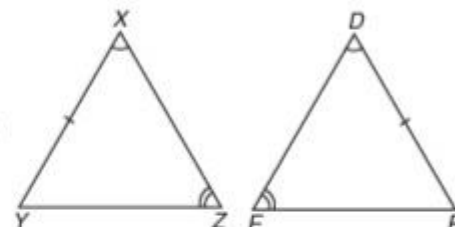


Note: The two equal sides must be opposite to angles which are known to be equal.

- (ii) ASA or AAS Axiom (Two angles, corresponding side): *If two angles and one side of a triangle are respectively equal to two angles and one side of the other triangles, the triangles are congruent. The side may be in included side.*

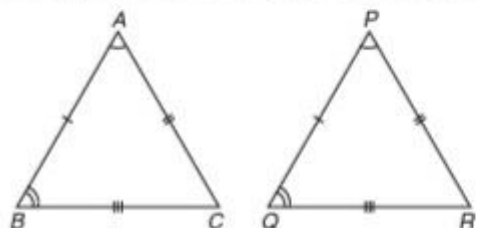


$\angle B = \angle Q$
 $\angle C = \angle R$
 $BC = QR$
 $\therefore \Delta ABC \cong \Delta PQR$ (ASA)



$\angle X = \angle D$
 $\angle Z = \angle E$
 $XY = DF$
 $\therefore \Delta XYZ \cong \Delta DEF$ (AAS)

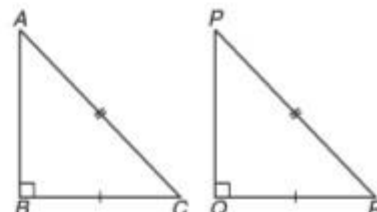
- (iii) SSS Axiom (Three sides): *If three sides of one triangle are respectively equal to the corresponding three sides of the other triangle, the triangles are congruent :*



$AB = PQ$
 $AC = PR$
 $BC = QR$
 $\Rightarrow \Delta ABC \cong \Delta PQR$ (SSS)

- (iv) RHS Axiom (Right angle-Hypotenuse-side): *If the hypotenuse and one side of a right angled triangle are respectively equal to the hypotenuse and corresponding side of the other right angled triangle, the two triangles are congruent.*

$\therefore AC = PR, BC = QR$ and $\angle B = \angle Q = 90^\circ \Rightarrow \Delta ABC \cong \Delta PQR$ (RHS)

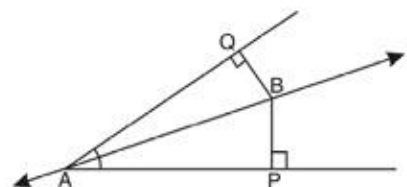


Solved Examples

Ex. 1. In the given figure, line l is the bisector of an angle A and B is any point on l . BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that B is equidistant from the arms of $\angle A$.

Sol. In ΔAPB and ABQ , we have

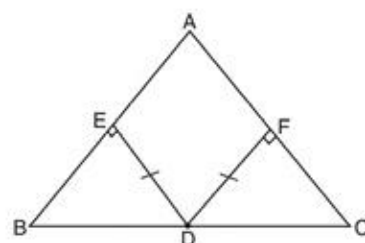
$$\begin{aligned} \angle APB &= \angle AQB && \text{(Each = } 90^\circ\text{)} \\ \angle PAB &= \angle QAB && \text{(AB bisects } \angle PAQ\text{)} \\ AB &= BA && \text{(common)} \\ \therefore \Delta APB &\cong \Delta ABQ && \text{(AAS)} \\ \Rightarrow BP &= BQ && \text{(cpct)} \\ \Rightarrow B &\text{ is equidistant from the arms of } \angle A. \end{aligned}$$



Ex. 2. ABC is a triangle and D is the mid-point of BC . The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Sol. Let DE and DF be the perpendiculars from D on AB and AC respectively.

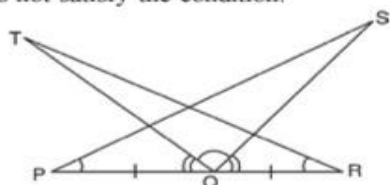
$$\begin{aligned} \text{In } \Delta BDE \text{ and } CDF, DE &= DF && \text{(Given)} \\ \angle BED &= \angle CFD = 90^\circ && \\ BD &= DC && (\because D \text{ is the mid-point of } BC) \\ \therefore \Delta BDE &\cong \Delta CDF && \text{(RHS)} \\ \Rightarrow \angle B &= \angle C && \text{(cpct)} \\ \Rightarrow AC &= AB && \text{(Sides opp. equal } \angle \text{ s are equal)} \\ \Rightarrow \Delta ABC &\text{ is isosceles.} \end{aligned}$$



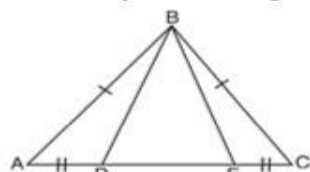
QUESTION BANK

Problems on Congruency

1. In the given figure, $\Delta RTQ \cong \Delta PSQ$ by ASA congruency condition. Which of the following pairs does not satisfy the condition.



- (a) $PQ = QR$ (b) $\angle P = \angle R$
 (c) $\angle TQP = \angle SQR$ (d) None of these
2. It is given that $AB = BC$ and $AD = EC$. The $\Delta ABE \cong \Delta CBD$ by _____ congruency.



- (a) SSS (b) ASA
 (c) SAS (d) AAS

3. $ABCD$ is a quadrilateral. AM and CN are perpendiculars to BD , $AM = CN$ and diagonals AC and BD intersect at O , then which one of the following is correct?

- (a) $AO = OC$ (b) $BO = OD$
 (c) $AO = BO$ (d) $CO = DO$

4. Squares $ABDE$ and $ACFH$ are drawn externally on the sides AB and AC respectively of a scalene ΔABC . Which one of the following is correct?

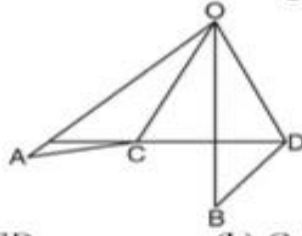
- (a) $BH = CE$ (b) $AD = AF$
 (c) $BF = CD$ (d) $DF = EH$

5. In the given figure, two sides AB and BC and the median AD drawn to side BC of ΔABC are equal to the two sides PQ and QR and the corresponding median PM of the other ΔPQR . Which of the following is not correct?

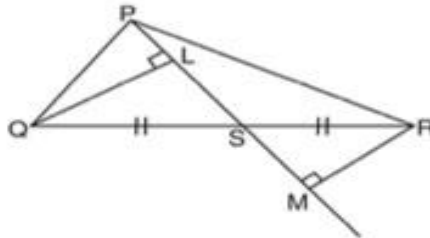


- (a) $\Delta ABD \cong \Delta PQM$ (b) $\Delta ABC \cong \Delta PQR$
 (c) $\Delta ABD \cong \Delta PMR$ (d) $\Delta ADC \cong \Delta PMR$

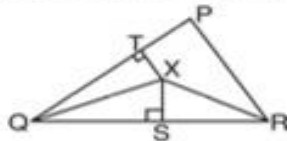
6. In the given figure, $OA = OB$, $OC = OD$, $\angle AOB = \angle COD$. Which of the following statements is true?



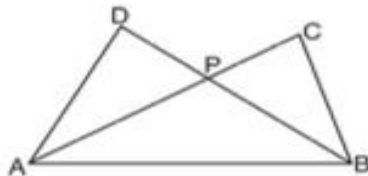
- (a) $AC = CD$ (b) $OA = OD$
 (c) $AC = BD$ (d) $\angle OCA = \angle ODC$
7. PS is a median and QL and RM are perpendiculars drawn from Q and R respectively on PS and PS produced. Then which of the following statements is correct?



- (a) $PQ = RM$ (b) $QL = RM$
 (c) $PL = SR$ (d) $PS = SM$
8. In the figure, QX and RX are the bisectors of angles Q and R respectively of $\triangle PQR$. If $XS \perp QR$ and $XT \perp PQ$, then $\triangle XTQ \cong \triangle XSQ$ by — congruency.



- (a) SAS (b) RHS
 (c) AAS (d) ASA
9. In the given figure, $AD = BC$, $AC = BD$. Then $\triangle PAB$ is



- (a) equilateral (b) right angled
 (c) scalene (d) isosceles
10. In a right angled triangle, one acute angle is double the other. The hypotenuse is — the smallest side.
- (a) $\sqrt{2}$ times (b) three times
 (c) double (d) 4 times

Hints and Solutions

1. (c) In $\Delta s RTQ$ and PSQ ,

$$\begin{aligned} QR &= PQ && \text{(Given)} \\ \angle P &= \angle R && \text{(Given)} \\ \angle TQR (\angle SQR + \angle SQT) & && \\ &= \angle PQS (\angle TQP + \angle SQT) && \end{aligned}$$

$$\therefore \Delta RTQ \cong \Delta PSQ \quad \text{(ASA)}$$

2. (c) Given, $AD = EC$

$$\Rightarrow AD + DE = DE + EC$$

$$\Rightarrow AE = DC$$

$$\text{Also, } AB = BC$$

$$\Rightarrow \angle BCA = \angle BAC \quad \text{(isos. } \Delta \text{ property)}$$

$$\Rightarrow \angle BCD = \angle BAE$$

- \therefore In $\Delta s ABE$ and CBD ,

$$AB = CB \quad \text{(Given)}$$

$$AE = DC \quad \text{(Proved above)}$$

$$\angle BAE = \angle BCD \quad \text{(Proved above)}$$

$$\therefore \Delta ABE \cong \Delta CBD \quad \text{(SAS)}$$

3. (a) In ΔAMO and ΔCNO



$$AM = CN \quad \text{(Given)}$$

$$\angle AMO = \angle CNO = 90^\circ$$

$$\angle AOM = \angle CON \quad \text{(vert. opp. } \angle s)$$

$$\therefore \Delta AMO \cong \Delta CNO \quad \text{(AAS)}$$

$$\Rightarrow AO = OC \quad \text{(cpct)}$$

4. (a) ΔABC is a scalene Δ .



$ACFH$ and $ABDE$ are squares drawn on sides AC and AB respectively.

$$\angle BAE = \angle CAH = 90^\circ \quad \text{(Angle of a square)}$$

$$\therefore \angle BAE + \angle BAC = \angle CAH + \angle BAC$$

$$\Rightarrow \angle CAE = \angle BAH$$

In $\Delta s EAC$ and HAB

$$EA = AB \quad \{ \text{sides of same square} \}$$

$$AC = AH$$

$$\angle CAE = \angle BAH \quad \text{(Proved)}$$

$$\therefore \Delta EAC \cong \Delta HAB \quad \text{(SAS)}$$

$$\Rightarrow EC = BH \quad \text{(cpct)}$$

5. (c) In $\Delta s ABD$ and PQM

$$AB = PQ$$

$$AD = PM$$

{ Given }

$$BD \left(\frac{1}{2} BC \right) = QM \left(\frac{1}{2} QR \right)$$

$$\therefore \Delta ABD \cong \Delta PQM \quad \text{(SSS)}$$

$$\angle B = \angle Q \quad \text{(cpct)}$$

In $\Delta s ABC$ and PQR

$$AB = PQ \quad \text{(Given)}$$

$$\angle B = \angle Q \quad \text{(Proved above)}$$

$$BC = QR \quad \text{(Given)}$$

$$\therefore \Delta ABC \cong \Delta PQR \quad \text{(SAS)}$$

$$AC = PR \quad \text{(cpct)}$$

In $\Delta s ADC$ and PMR

$$AD = PM \quad \text{(Given)}$$

$$DC \left(\frac{1}{2} BC \right) = MR \left(\frac{1}{2} QR \right) \quad \text{(Given)}$$

$$AC = PR \quad \text{(Proved above)}$$

$$\therefore \Delta ADC \cong \Delta PMR \quad \text{(SSS)}$$

$$\therefore \Delta ABD \cong \Delta PMR$$

6. (c) In $\Delta s AOC$ and BOD

$$OA = OB \quad \text{(Given)}$$

$$OC = OD \quad \text{(Given)}$$

$$\angle AOB - \angle COB = \angle COD - \angle COB,$$

$$\text{i.e., } \angle AOC = \angle BOD$$

$$\therefore \Delta AOC \cong \Delta BOD \quad \text{(SAS)}$$

$$\Rightarrow AC = BD \quad \text{(cpct)}$$

7. (b) In $\Delta s QLS$ and RMS ,

$$\angle QLS = \angle RMS = 90^\circ$$

$$\angle QSL = \angle RSM \quad \text{(Vert. opp. } \angle s)$$

$$QS = SR \quad \text{(PS is the median)}$$

$$\therefore \Delta QLS \cong \Delta RMS \quad \text{(AAS)}$$

$$\Rightarrow QL = RM \quad \text{(cpct)}$$

8. (c) In $\Delta s XTQ$ and XSQ .

$$XQ = XQ \quad \text{(Common)}$$

$$\angle XQT = \angle XQS \quad \text{(} QX \text{ bisects } \angle Q)$$

$$\angle XTQ = \angle XSQ = 90^\circ$$

$$\therefore \Delta XTQ \cong \Delta XSQ \quad \text{(AAS)}$$

9. (d) In Δs ADB and ACB

$$AD = BC \quad (\text{Given})$$

$$AC = BD$$

$$AB = BA \quad (\text{Common})$$

$$\therefore \Delta ADB \cong \Delta ACB \quad (\text{SSS})$$

$$\Rightarrow \angle ABD = \angle CAB \quad (\text{cpct})$$

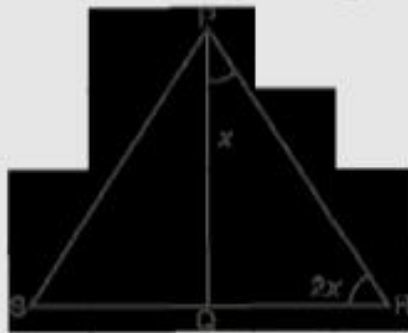
$$\Rightarrow \angle ABP = \angle PAB$$

$$\Rightarrow PA = PB \quad (\text{Sides opp. equal angles are equal})$$

$$\Rightarrow \Delta PAB \text{ is isosceles.}$$

10. (c) **Given :** A ΔPRQ in which

$$\angle Q = 90^\circ \text{ and } \angle PRQ = 2\angle QPR$$



Const. Produce RQ to S such that

$$RQ = QS. \text{ Join } PS.$$

In Δs PQS and PQR

$$QS = QR \quad (\text{By construction})$$

$$PQ = PQ \quad (\text{Common})$$

$$\angle PQS = \angle PQR \quad (\text{Each} = 90^\circ)$$

$$\therefore \Delta PQS \cong \Delta PQR \quad (\text{SAS})$$

$$\Rightarrow PS = PR \text{ and } \angle SPQ = \angle RPQ \text{ (cpct)}$$

$$\text{Let } \angle SPQ = x. \text{ Then } \angle PRQ = 2x \text{ (Given)}$$

$$\begin{aligned} \text{Then } \angle SPR &= \angle SPQ + \angle RPQ \\ &= x + x = 2x \end{aligned}$$

$$\Rightarrow \angle SPR = \angle PRQ \Rightarrow SR = PS$$

(Sides opp. equal $\angle s$ are equal)

$$\Rightarrow 2QR = PS \quad \{ \because SQ = QR \therefore SR = 2QR \}$$

$$\Rightarrow 2QR = PR \quad (\because PS = PR)$$

\Rightarrow The hypotenuse PR is **double** the smallest side QR .

SELF ASSESSMENT EXERCISE

MATHEMATICAL REASONING

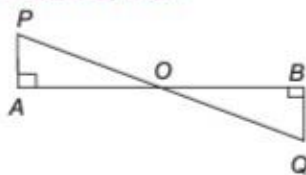
1. Which of the following statements is **INCORRECT** ?

- (A) Two triangles having same area are congruent.
- (B) If two sides and one angle of a triangle are equal to the corresponding two sides and the angle of another triangle, then the two triangles are congruent.
- (C) If the hypotenuse of one right angled triangle is equal to the hypotenuse of another right angled triangle, then the triangles are congruent.
- (D) All of these

2. In $\triangle ABC$, $AB = AC$ and AD is perpendicular bisector of BC . The property by which $\triangle ADB$ is not congruent to $\triangle ADC$ is _____.

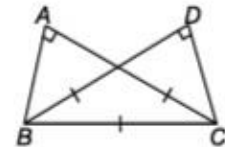
- (A) SAS property
- (B) SSS property
- (C) RHS property
- (D) AAA property

3. In the given figure, $PA \perp AB$, $QB \perp AB$ and $\triangle OAP \cong \triangle OBQ$, then



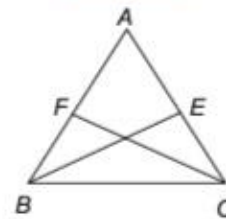
- (A) $PA = OB$
- (B) $AP = QB$
- (C) $OP = BQ$
- (D) $OA = OQ$

4. In the given figure, triangles ABC and DCB are right angled at A and D respectively and $AC = DB$, then $\triangle ABC \cong \triangle DCB$ by _____ criterion.



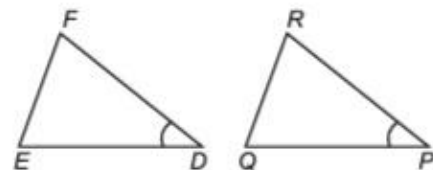
- (A) AAA
- (B) SAS
- (C) ASS
- (D) None of these

5. In the given figure, ABC is an isosceles triangle in which $AB = AC$. If E and F be the midpoints of AC and AB respectively, then BE is equal to _____.



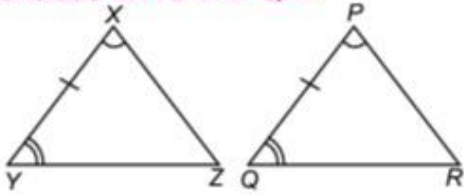
- (A) CF
- (B) AB
- (C) CE
- (D) BF

6. If you want to prove that $\triangle FED \cong \triangle RQP$ using the SAS criterion where $\angle D = \angle P$, then which of the following you need more?



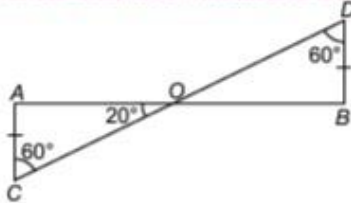
- (A) $FD = QP$
- (B) $QP = ED$
- (C) $FD = RP$
- (D) Both (B) and (C)

7. Which congruence criterion can be used to conclude $\triangle XYZ \cong \triangle PQR$?



- (A) SAS (B) SSS
(C) RHS (D) None of these

8. If $\triangle AOC \cong \triangle BOD$, then measure of $\angle OBD$ is



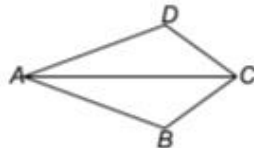
- (A) 100° (B) 50°
(C) 80° (D) None of these

9. If $\triangle ABC \cong \triangle PRQ$, then $\angle B$ and PQ are respectively equal to

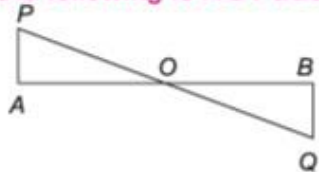
- (A) $\angle P$ and AC (B) $\angle R$ and BC
(C) $\angle R$ and AC (D) $\angle Q$ and AB

10. In the given figure, if $AB = AD$ and $CB = CD$, then which of the following is correct?

- (A) $\triangle ABC \cong \triangle ADC$
(B) $\angle BCA = \angle DCA$
(C) $\angle ADC = \angle ABC$
(D) All of these

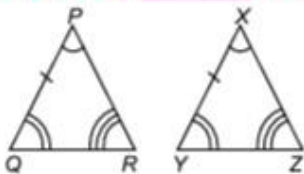


11. In the given figure, if $\triangle OAP \cong \triangle OBQ$, then which of the following is NOT true?



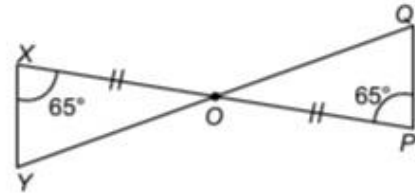
- (A) $AO = BO$ (B) $AP = BQ$
(C) $PO = BO$ (D) $\angle APO = \angle BQO$

12. $\triangle PQR \cong \triangle XYZ$ by _____ congruency rule.



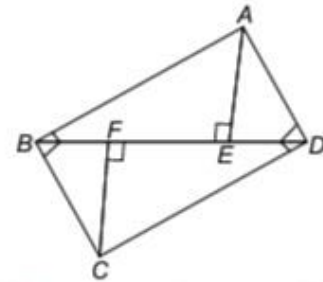
- (A) SSS (B) AAA
(C) SAS (D) ASA

13. Which congruence criterion can be used to state that $\triangle XOY \cong \triangle POQ$?



- (A) ASA (B) SAS
(C) SSS (D) RHS

DIRECTION (14 - 15) : Study the figure and information given below carefully and answer the following questions.



CF and AE are equal perpendiculars on BD and $BF = FE = ED$.

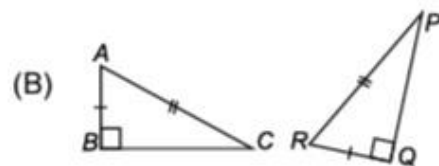
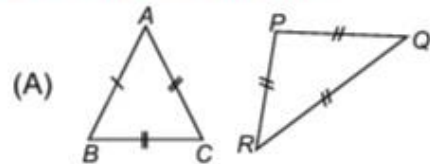
14. $\triangle ABE$ is congruent to

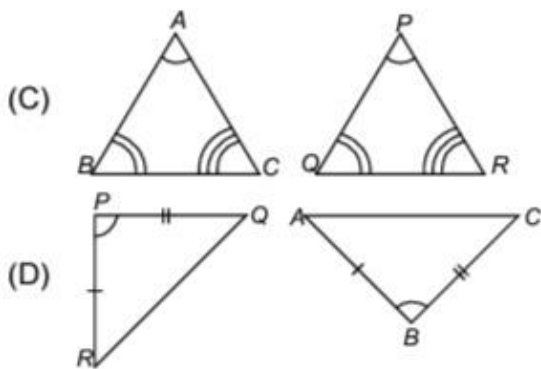
- (A) $\triangle AED$ (B) $\triangle BFC$
(C) $\triangle CDF$ (D) $\triangle BCD$

15. $\angle BAE = \dots\dots\dots$

- (A) $\angle BCD$ (B) $\angle CBA$
(C) $\angle ADC$ (D) $\angle DCF$

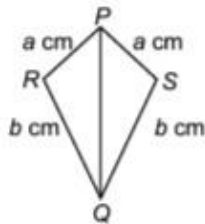
16. Select the odd one out.





17. If for $\triangle ABC$ and $\triangle DEF$, the correspondence $CAB \leftrightarrow EDF$ gives a congruence, then which of the following is NOT true?
- (A) $AC = DE$ (B) $AB = EF$
 (C) $\angle A = \angle D$ (D) $\angle C = \angle E$

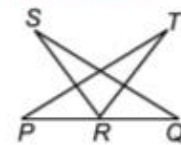
18. By which congruency criterion, $\triangle PQR \cong \triangle PQS$?



- (A) RHS (B) ASA
 (C) SSS (D) SAS

19. In two triangles PQR and LMN , $PQ = QR$, $\angle P = \angle M$ and $QR = LN$, then which of the following statements is CORRECT?
- (A) Triangles are congruent only.
 (B) Triangles are isosceles only.
 (C) Triangles are both congruent and isosceles.
 (D) None of these

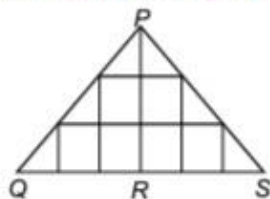
20. In the given figure, if $PR = QR$, $\angle SRP = \angle TRQ$ and $\angle SQP = \angle TPQ$, then



- (A) $\triangle SQR \cong \triangle PTR$ & $SR = TR$
 (B) $\triangle SQR \cong \triangle TPR$ & $SR = TR$
 (C) $\triangle RQS \cong \triangle TPR$ & $SR = TR$
 (D) $\triangle QRS \cong \triangle PRT$ & $SR = PT$

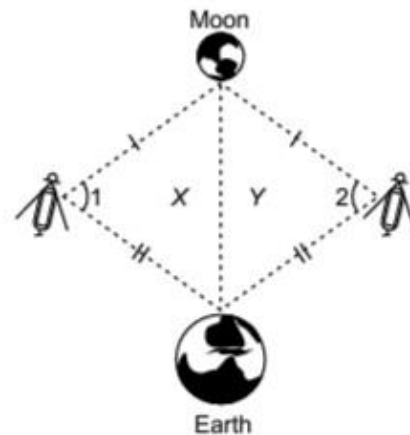
EVERYDAY MATHEMATICS

21. Ananya is designing the window shown in the figure. She wants to make $\triangle PRQ$ congruent to $\triangle PRS$. She designs the window so that $PR \perp QS$. Which of the following conditions will make the two triangles congruent?



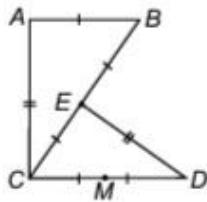
- (A) $RQ = RS$
 (B) $PQ = PS$
 (C) Both (A) and (B)
 (D) None of these

22. Two satellites are being launched such that their distance while moving in their respective orbits, are equal from Earth and Moon.



- Which of the following statements is/are true?
- (A) Figure X is not congruent to figure Y.
 (B) $\angle 1$ and $\angle 2$ are not equal.
 (C) Both (A) and (B)
 (D) None of these

23. Akira gave a problem to her sister Kiara. However, Kiara got stuck. Help Kiara identify whether the triangles are congruent and choose the correct option.

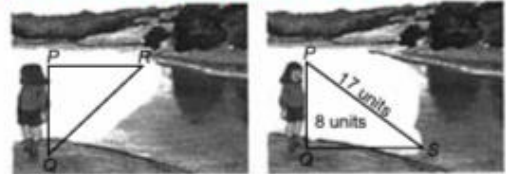


- (A) Yes, $\triangle ABC \cong \triangle DCE$
 (B) No, they are not congruent
 (C) Yes, $\triangle DCE \cong \triangle CAB$
 (D) Yes, $\triangle DEC \cong \triangle CAB$
24. Three students Pia, Sia and Tia wrote a statement on a blackboard.
 Pia wrote, "All rectangles are congruent".
 Sia wrote, "All equilateral triangles are congruent".
 Tia wrote, "All right angled triangles are congruent".

Who wrote the INCORRECT statement?

- (A) Pia (B) Sia
 (C) Tia (D) All of them

25. Tiara wants to know the width of the given river. While doing so, she stands on the edge of the river and look straight across to a point on the other edge without changing the inclination of the neck and head. She turns side ways until the vision is in line with a point on the side of the stream.



From the above description, find the value of QR.

- (A) 25 units
 (B) 12 units
 (C) 15 units
 (D) Can't be determined

ACHIEVERS SECTION (HOTS)

26. Match the figures in Column-I with their corresponding congruence criterion given in Column-II.

Column I	Column II
(i)	(a) ASA congruency
(ii)	(b) RHS congruency
(iii)	(c) SSS congruency
(iv)	(d) SAS congruency

- (A) (i) \rightarrow (b), (ii) \rightarrow (d), (iii) \rightarrow (a), (iv) \rightarrow (c)
 (B) (i) \rightarrow (c), (ii) \rightarrow (a), (iii) \rightarrow (b), (iv) \rightarrow (d)
 (C) (i) \rightarrow (b), (ii) \rightarrow (c), (iii) \rightarrow (a), (iv) \rightarrow (d)
 (D) (i) \rightarrow (a), (ii) \rightarrow (c), (iii) \rightarrow (b), (iv) \rightarrow (d)

27. Which of the following statements is CORRECT?

Statement-1 : Two triangles are said to be congruent if two sides and an angle of one triangle are respectively equal to the two sides and an angle of the other.

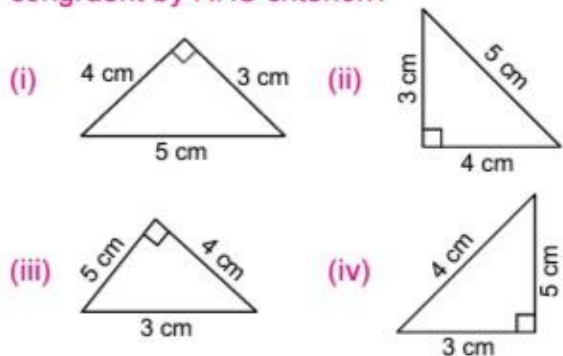
Statement-2 : Two triangles are congruent if two sides and the included angle of the one triangle equal to the corresponding two sides and included angle of the other.

- (A) Only Statement-1
 (B) Only Statement-2
 (C) Both Statement-1 and Statement-2
 (D) Neither Statement-1 nor Statement-2

28. Which of the following statements is CORRECT?

- (A) In an isosceles triangle, the angles opposite to equal sides are equal.
- (B) The bisector of the vertical angle of an isosceles triangle bisects the base at right angles.
- (C) If the hypotenuse and an acute angle of one right angled triangle is equal to the hypotenuse and the corresponding acute angle of another triangle, then the triangles are congruent.
- (D) All of these

29. Which of the following pair of triangles are congruent by RHS criterion?



- (A) (i) and (ii)
- (B) (iii) and (iv)
- (C) (i) and (iii)
- (D) (ii) and (iv)

30. State 'T' for true and 'F' for false.

1. To examine the congruency of plane figures, the superposition method is used.
2. If two line segments have different lengths, they are congruent.
3. The measure of two congruent angles is same.
4. Objects which are exact copies of one another are called plane objects.
5. If the corresponding angles of two triangles are equal, the triangles are said to be congruent.

	1	2	3	4	5
(A)	T	T	F	T	F
(B)	T	F	T	F	T
(C)	T	F	T	F	F
(D)	T	F	T	T	T



Darken your choice with HB Pencil

1.	(A) (B) (C) (D)	9.	(A) (B) (C) (D)	17.	(A) (B) (C) (D)	25.	(A) (B) (C) (D)
2.	(A) (B) (C) (D)	10.	(A) (B) (C) (D)	18.	(A) (B) (C) (D)	26.	(A) (B) (C) (D)
3.	(A) (B) (C) (D)	11.	(A) (B) (C) (D)	19.	(A) (B) (C) (D)	27.	(A) (B) (C) (D)
4.	(A) (B) (C) (D)	12.	(A) (B) (C) (D)	20.	(A) (B) (C) (D)	28.	(A) (B) (C) (D)
5.	(A) (B) (C) (D)	13.	(A) (B) (C) (D)	21.	(A) (B) (C) (D)	29.	(A) (B) (C) (D)
6.	(A) (B) (C) (D)	14.	(A) (B) (C) (D)	22.	(A) (B) (C) (D)	30.	(A) (B) (C) (D)
7.	(A) (B) (C) (D)	15.	(A) (B) (C) (D)	23.	(A) (B) (C) (D)		
8.	(A) (B) (C) (D)	16.	(A) (B) (C) (D)	24.	(A) (B) (C) (D)		

ANSWERS : 1-D, 2-D, 3-B, 4-D, 5-A, 6-D, 7-D, 8-A, 9-C, 10-D, 11-C, 12-D, 13-A, 14-C, 15 -D, 16-A, 17-B, 18-C, 19-D, 20-B, 21-C, 22-D, 23-D, 24-D, 25-C, 26-C, 27-B, 28-D, 29-A, 30-C

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