ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-28

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Integration **Class: XII**

Topic: Indefinite integrals **Date: 18.07.2020**

Solved Examples (Part 2)

18. $\int e^{2x^2+\ln x}\,dx$ is equal to

(A) $\frac{e^{2x^2}}{4}+c$ **(B)** $\frac{e^{2x^2}}{2}+c$

(C) $\frac{e^{2x^2}}{4}+\frac{x^2}{2}+c$ **(D)** $\frac{xe^{2x^2}}{4}+c$

Solution:

$$I=\int e^{2x^2+\ln x}\,dx \Rightarrow xe^{2x^2}\,dx$$

Let $x^2=t$. Then

$$2x\,dx=dt$$

$$\Rightarrow I=\frac{1}{2}\int e^{2t}\,dt=\frac{e^{2t}}{4}+c=\frac{e^{2x^2}}{4}+c$$

Hence, the correct answer is option (A).

19. $\int \frac{2^{x+1}-5^{x-1}}{10^x}dx$ is equal to

(A) $\frac{2}{\ln 5}5^x+\frac{1}{5\ln 2}2^x+c$ **(B)** $\frac{-2}{\ln 5}5^{-x}+\frac{1}{5\ln 2}2^{-x}+c$

(C) $\frac{1}{2\ln 5}5^{-x}\frac{1}{5\ln 2}2^{-x}+c$ **(D)** None of these

Solution:

$$I=\int \frac{2^{x+1}-5^{x-1}}{10^x}dx=\int\left[2\left(\frac{1}{5}\right)^x-\frac{1}{5}\left(\frac{1}{2}\right)^x\right]dx$$

$$=\frac{2\left(\frac{1}{5}\right)^x}{\ln\frac{1}{5}}-\frac{\frac{1}{5}\left(\frac{1}{2}\right)^x}{\ln\frac{1}{2}}+c$$

$$=\frac{-2}{\ln 5}5^{-x}+\frac{1}{5\ln 2}2^{-x}+c$$

Hence, the correct answer is option (B).

20. If $I_n=\int(\ln x)^n dx$, then I_n+nI_{n-1} is equal to

(A) $x(\ln x)^{n-1}$ **(B)** $x(\ln x)^n$

(C) $nx(\ln x)^n$ **(D)** None of these

Solution:

$$I_n=\int(\ln x)^n dx$$

$$I_n=x(\ln x)^n-n\int(\ln x)^{n-1}dx$$

$$I_n=x(\ln x)^n-n\,I_{n-1}$$

$$I_n+nI_{n-1}=x(\ln x)^n$$

Hence, the correct answer is option (B).

21. If $\int\frac{dx}{x-x^3}=A\ln\left|\frac{x^2}{1-x^2}\right|+c$, then A is equal to

(A) 2 **(B)** 1/2

(C) 2/3 **(D)** 1/4

Solution:

$$I=\int\frac{dx}{x-x^3}=\int\frac{dx}{x^3\left(\frac{1}{x^2}-1\right)}$$

Let $\frac{1}{x^2}=t$. Then $-\frac{2}{x^3}\,dx=dt$. Therefore,

$$I=-\frac{1}{2}\int\frac{dt}{(t-1)}=\frac{1}{2}\ln|t-1|+c=-\frac{1}{2}\ln\left|\frac{1-x^2}{x^2}\right|+c$$

$$=\frac{1}{2}\ln\left|\frac{x^2}{1-x^2}\right|+c\Rightarrow A=\frac{1}{2}$$

Hence, the correct answer is option (B).

22. $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ is equal to

(A) $xe^{\tan^{-1}x}+c$ **(B)** $x^2e^{\tan^{-1}x}+c$

(C) $\frac{1}{xe^{\tan^{-1}x}}+c$ **(D)** $\frac{1}{x^2}e^{\tan^{-1}x}+c$

Solution:

$$I=\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$$

Let $p=\tan^{-1}x$. Then

$$x=\tan p\Rightarrow dx=\sec^2 p\,dp$$

$$\Rightarrow I=\int e^p(\sec^2 p+\tan p)\,dp$$

$$=e^p\tan p=x\,e^{\tan^{-1}x}+c$$

Hence, the correct answer is option (A).

23. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$ is equal to

(A) $\frac{x^2}{2}+\log|x|+c$ **(B)** $\frac{x^2}{2}+\log|x|+2x+c$

(C) $\frac{1}{3}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^3+c$ **(D)** $\frac{x^2}{2}+\log x-2x+c$

Solution:

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx=\int\left(x+\frac{1}{x}-2\right)dx=\frac{x^2}{2}+\log x-2x+c$$

Hence, the correct answer is option (D).

24. $\int\frac{\cos 2x}{\cos x+\sin x}dx$ is equal to

(A) $\sin x-\cos x+c$ **(B)** $-\sin x+\cos x+c$

(C) $\sin x+\cos x+c$ **(D)** None of these

Solution:

$$\int\frac{\cos 2x}{\cos x+\sin x}dx=\int\frac{(\cos x+\sin x)(\cos x-\sin x)}{\cos x+\sin x}dx$$

$$=\sin x+\cos x+c$$

Hence, the correct answer is option (C).

25. If $\int\frac{\sqrt{\cot x}}{\sin x\cos x}dx=A\sqrt{\cot x}+B$, then A is equal to

(A) 1 **(B)** 2

(C) −1 **(D)** −2

Solution:

$$\int \frac{\sqrt{\cot x}}{\sin x \cos x} dx = \int \frac{\sqrt{\cot x}}{\cot x} \cdot \operatorname{cosec}^2 x \, dx$$

$$= \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cot x}} dx = -2\sqrt{\cot x} + B = A\sqrt{\cot x} + B$$

$$\Rightarrow A = -2$$

Hence, the correct answer is option (D).

26. The value of $\int \frac{\ln(x/e)}{(\ln x)^2} dx$ is

(A) $\frac{x+1}{(\ln x)^2} + c$ **(B)** $\frac{x-1}{(\ln x)^2} + c$

(C) $\frac{x}{\ln x} + c$ **(D)** $\frac{\ln x}{x} + c$

Solution:

$$I = \int \frac{\ln(x/e)}{(\ln x)^2} dx = \int \frac{\ln(x)-1}{(\ln x)^2} dx$$

Put $\ln x = t$. Then

$$x = e^t \Rightarrow dx = e^t dt$$

$$I = \int e^t \left(\frac{t-1}{t^2}\right) dt = \int e^t \left(\frac{1}{t} - \frac{1}{t^2}\right) dt = \frac{e^t}{t} + c = \frac{x}{\ln x} + c$$

Hence, the correct answer is option (C).

27. $I = \int \frac{(10x^9 + 10^x \log_e 10)}{(x^{10} + 10^x)} dx$ is equal to

(A) $10^x + x^{10} + c$ **(B)** $10^x - x^{10} + c$

(C) $10^x + x^{10} + c$ **(D)** $\ln(10^x + x^{10}) + c$

Solution: If $p = x^{10} + 10^x$, then

$$(10x^9 + 10^x \log_e 10) dx = dp$$

$$\Rightarrow I = \int dp = p + c$$

$$\Rightarrow I = \ln(x^{10} + 10^x) + c$$

Hence, the correct answer is option (D).

28. The value of the integral $\int e^{\sin^2 x}(\cos x + \cos^3 x) \sin x \, dx$ is

(A) $\frac{1}{2} e^{\sin^2 x}(3 - \sin^2 x) + c$

(B) $e^{\sin^2 x}\left(1 + \frac{1}{2}\cos^2 x\right) + c$

(C) $e^{\sin^2 x}(3\cos^2 x + 2\sin^2 x) + c$

(D) $e^{\sin^2 x}(2\cos^2 x + 3\sin^2 x) + c$

Solution:

$$\mathrm{I} = \int e^{\sin^2 x}(\cos x + \cos^3 x) \sin x dx = \int e^{\sin^2 x}(2 - \sin^2 x) \cos x \sin x dx$$

Put $t = \sin^2 x$. Then

$$dt = 2\sin x \cos x \, dx$$

The integral reduces to

$$\Rightarrow I = \frac{1}{2}\int e^t (2-t) dt = \frac{3}{2}e^t - \frac{te^t}{2} + c$$

$$= \frac{1}{2} e^{\sin^2 x}(3 - \sin^2 x) + c \qquad \text{(Option A)}$$

$$= e^{\sin^2 x}\left(1 + \frac{1}{2}\cos^2 x\right) + c \qquad \text{(Option B)}$$

Hence, the correct answers are options (A) and (B).

29. $\int \frac{x^3 - 3x + 7}{x^2 + 4} dx$ is equal to

(A) $\frac{x^2}{2} + \frac{7}{2}\ln(x^2+4) + c$

(B) $\frac{x^2}{2} + \frac{7}{2}\tan^{-1}\frac{x}{2} - \frac{7}{2}\ln(x^2+4) + c$

(C) $-\frac{x^2}{2} + \frac{7}{2}\tan^{-1}\frac{x^2}{2} + \frac{7}{2}\ln\left(x^2+4\right) + c$

(D) $\frac{x}{2} + \frac{7}{2}\tan^{-1}\frac{x}{2} + c$

Solution:

$$\frac{x^3 - 3x + 7}{x^2 + 4} = x - \frac{7(x-1)}{x^2+4}$$

$$\int \frac{x^3 - 3x + 7}{x^2 + 4} dx = \int x dx - 7\int \frac{(x-1)}{x^2+4} dx$$

$$\int \frac{x^3 - 3x + 7}{x^2 + 4} dx = \frac{x^2}{2} - 7\int \frac{(x-1)}{x^2+4} dx$$

$$= \frac{x^2}{2} + \frac{7}{2}\tan^{-1}\frac{x}{2} - \frac{7}{2}\ln(x^2+4) + c$$

Hence, the correct answer is option (B).

30. $\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$ is equal to

(A) $\frac{1}{e}\ln(x^e + e^x) + c$ **(B)** $\frac{1}{e}\ln(x + e) + c$

(C) $\frac{1}{e}\ln(x^{-e} + e^{-x}) + c$ **(D)** None of these

Solution:

$$I = \int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$$

Let $e^x + x^e = t$. Then

$$(e^x + e \cdot x^{e-1}) dx = dt \Rightarrow e(e^{x-1} + x^{e-1}) dx = dt$$

Therefore,

$$I = \frac{1}{e}\int \frac{dt}{t} = \frac{1}{e}\ln(e^x + x^e) + c$$

Hence, the correct answer is option (A).

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