ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

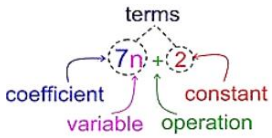
Sub: Algebra and Geometry **Class: 7** **Date: 07.05.20**

STUDY MATERIAL: LINEAR EQUATIONS IN ONE VARIABLE

Concepts and Explanations

Algebraic Expressions

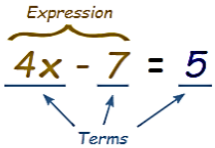
Any expression involving constant, variable and some operations like addition, multiplication etc is called **Algebraic Expression**.



- A **variable** is an unknown number and generally, it is represented by a letter like x, y, n etc.
- Any number without any variable is called **Constant**.
- A number followed by a variable is called **Coefficient** of that variable.
- A **term** is any number or variable separated by operators.

Equation

A statement which says that the two expressions are equal is called **Equation**.



Linear Expression

A linear expression is an expression whose highest power of the variable is one only.

Example

2x + 5, 3y etc.

The expressions like $x^2 + 1$, $z^2 + 2z + 3$ are not the linear expressions as their highest power of the variable is greater than 1.

Linear Equations

The equation of a straight line is the linear equation. It could be in one variable or two variables.

Linear Equation in One Variable

If there is only one variable in the equation then it is called a linear equation in one variable.

The general form is

$ax + b = c$, where a, b and c are real numbers and $a \neq 0$.

Example

$x + 5 = 10$

$y – 3 = 19$

These are called linear equations in one variable because the highest degree of the variable is one and there is only one variable.

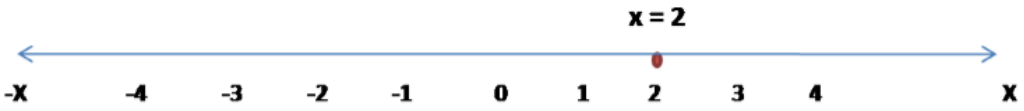
Some Important points related to Linear Equations

- There is an equality sign in the linear equation. The expression on the left of the equal sign is called the LHS (left-hand side) and the expression on the right of the equal sign is called the RHS (right-hand side).
- In the linear equation, the LHS is equal to RHS but this happens for some values only and these values are the solution of these linear equations.

Graph of the Linear Equation in One Variable

We can mark the point of the linear equation in one variable on the number line.

x = 2 can be marked on the number line as follows-



Solving Equations which have Linear Expressions on one Side and Numbers on the other Side

There are two methods to solve such type of problems-

1. Balancing Method

In this method, we have to add or subtract with the same number on both the sides without disturbing the balance to find the solution.

Example

Find the solution for $3x – 10 = 14$

Solution

Step 1: We need to add 10 to both the sides so that the numbers and variables come on the different sides without disturbing the balance.

$3x – 10 +10 =10+14$

$3x = 24$

Step 2: Now to balance the equation, we need to divide by 3 into both the sides.

$3x/3 = 24/3$

$x = 8$

Hence x = 8 is the solution of the equation.

We can **recheck** our answer by substituting the value of x in the equation.

3x – 10 = 14

3(8) – 10 = 14

24-10 = 14

14 = 14

Here, LHS = RHS, so our solution is correct.

2. Transposing Method

In this method, we need to transpose or transfer the constants or variables from one side to another until we get the solution. When we transpose the terms the sign will get changed.

Example

Find the solution for 2z +10 = 4.

Solution

Step 1: We transpose 10 from LHS to RHS so that all the constants come in the same side.

2z = 4 -10 (sign will get changed)

2z = -6

Step 2: Now divide both the sides by 2.

2z/2 = - 6/2

z = - 3

Here z = -3 is the solution of the equation.

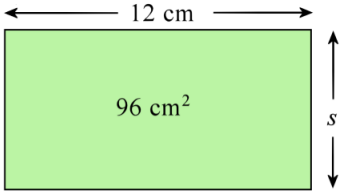
Some Applications of Linear Equation

We can use the concept of linear equations in our daily routine also. There are some situations where we need to use the variable to find the solution. Like,

- What number should be added to 23 to get 75?
- If the sum of two numbers is 100 and one of the no. is 63 then what will be the other number?

Example

What is the height of the rectangle whose perimeter is 96 cm^2 and the length is 12 cm?



Solution

Let the height of the rectangle be ‘s’.

Area of rectangle = Length × Breadth

$96 = S \times 12$

Now, this is a linear equation with variable s.

We need to divide both sides by 12 to find the solution.

$96/12 = 12s/12$

$s = 8$

Hence the height of the rectangle is 8 cm.

Solving Equations having the Variable on both Sides

As the equation can have the variable on both the sides also so we should know how to solve such problems.

In this type of problems, we need to bring all the constants on one side and all the terms having variables on the other side. Then they can be solved easily.

Example

Find the solution of $2x-3 = 6 - x$.

Solution

Step 1: Bring all the terms including variable x on LHS and the constants on the RHS.

$2x + x = 6 + 3$ (sign will change while changing the position of the terms)

Step 2: Solve the equation

$3x = 9$

Step 3: Divide both the sides by 3 to get the solution.

$3x/3 = 9/3$

$x = 3$

Hence the solution of the equation is $x = 3$.

Some More Applications

Example

Renu's age is four times that of her younger brother. Five years back her age was 9 times her brother's age. Find their present ages.

Solution

Let the Renu's brother age = x

Renu's age = 4x (as her age is 4 times that of her younger brother)

Five years back her age was = $9(x - 5)$ which is equal to $4x - 5$

$9(x - 5) = 4x - 5$

$9x - 45 = 4x - 5$

$9x - 4x = -5 + 45$ (by transferring the variable and constants on different sides)

$5x = 40$

x = 40/5 = 8

Renu's brother age = x = 8 years

Renu's age = 4x = 4(8) = 32 years.

Reducing Equations to Simpler Form

When linear equations are in fractions then we can reduce them to a simpler form by-

- Taking the LCM of the denominator
- Multiply the LCM on both the sides, so that the number will reduce without the denominator and we can solve them by the above methods.

Example

Solve the linear equation

$$\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+\frac{1}{4}+1$$

Solution

As the equation is in complex form, we have to reduce it into a simpler form.

Step 1: Take the L.C.M. of the denominators, 2, 3, 4, and 5, which is 60.

Step 2: Multiply both the sides by 60,

$$\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+\frac{1}{4}+1$$

30x −12 = 20x + 15 + 60

Step 3. Bring all the variables on the LHS and all the constants on the RHS

30x − 20x = 15 + 12 + 60

10x = 87

Step 4: Dividing both the sides by 10

x = 8.7

Equations Reducible to the Linear Form

Sometimes there are some equations which are not linear equations but can be reduced to the linear form and then can be solved by the above methods.

Example

$$\text{Solve } \frac{x+1}{2x+3}=\frac{3}{8}$$

Solution

This is not a linear equation but can be reduced to linear form

Step 1: Multiply both the sides by (2x + 3).

$$\frac{x+1}{2x+3} \times (2x+3) = \frac{3}{8} \times (2x+3)$$

$$x+1 = \frac{3(2x+3)}{8}$$

Now, this is a linear equation.

Step 2: Multiply both the sides by 8.

8(x + 1) = 3(2x + 3)

8x + 8 = 6x + 9

8x - 6x = 9 - 8

2x = 1

x = 1/2

Hence the solution for the equation is x = 1/2.

Solved Numerical Problems

1. The sum of three consecutive multiples of 4 is 444. Find these multiples.

Solution:

If x is a multiple of 4, the next multiple is x + 4, next to this is x + 8.

Their sum = 444

According to the question,

x + (x + 4) + (x + 8) = 444

⇒ x + x + 4 + x + 8 = 444

⇒ x + x + x + 4 + 8 = 444

⇒ 3x + 12 = 444

⇒ 3x = 444 - 12

⇒ x = 432/3

⇒ x = 144

Therefore, x + 4 = 144 + 4 = 148

Therefore, x + 8 - 144 + 8 – 152

Therefore, the three consecutive multiples of 4 are 144, 148, 152.

2. The denominator of a rational number is greater than its numerator by 3. If the numerator is increased by 7 and the denominator is decreased by 1, the new number becomes 3/2. Find the original number.

Solution:

Let the numerator of a rational number = x

Then the denominator of a rational number = x + 3

When numerator is increased by 7, then new numerator = x + 7

When denominator is decreased by 1, then new denominator = x + 3 - 1

The new number formed = 3/2

According to the question,

$(x + 7)/(x + 3 - 1) = 3/2$

$\Rightarrow (x + 7)/(x + 2) = 3/2$

$\Rightarrow 2(x + 7) = 3(x + 2)$

$\Rightarrow 2x + 14 = 3x + 6$

$\Rightarrow 3x - 2x = 14 - 6$

$\Rightarrow x = 8$

The original number i.e., x/(x + 3) = 8/(8 + 3) = 8/11

3. The sum of the digits of a two digit number is 7. If the number formed by reversing the digits is less than the original number by 27, find the original number.

Solution:

Let the units digit of the original number be x.

Then the tens digit of the original number be 7 - x

Then the number formed = $10(7 - x) + x \times 1$

$= 70 - 10x + x = 70 - 9x$

On reversing the digits, the number formed

$= 10 \times x + (7 - x) \times 1$

$= 10x + 7 - x = 9x + 7$

According to the question,

New number = original number - 27

$\Rightarrow 9x + 7 = 70 - 9x - 27$

$\Rightarrow 9x + 7 = 43 - 9x$

$\Rightarrow 9x + 9x = 43 - 7$

$\Rightarrow 18x = 36$

$\Rightarrow x = 36/18$

$\Rightarrow x = 2$

Therefore, $7 - x$

$= 7 - 2$

$= 5$

The original number is 52

4. A motorboat goes downstream in river and covers a distance between two coastal towns in 5 hours. It covers this distance upstream in 6 hours. If the speed of the stream is 3 km/hr, find the speed of the boat in still water.

Solution:

Let the speed of the boat in still water = x km/hr.

Speed of the boat downstream = (x + 3) km/hr.

Time taken to cover the distance = 5 hrs

Therefore, distance covered in 5 hrs = $(x + 3) \times 5$ **(D = Speed × Time)**

Speed of the boat upstream = (x - 3) km/hr

Time taken to cover the distance = 6 hrs.

Therefore, distance covered in 6 hrs = 6(x - 3)

Therefore, the distance between two coastal towns is fixed, i.e., same.

According to the question,

$5(x + 3) = 6(x - 3)$

$\Rightarrow 5x + 15 = 6x - 18$

$\Rightarrow 5x - 6x = -18 - 15$

$\Rightarrow -x = -33$

⇒ x = 33

Required speed of the boat is 33 km/hr.

5. Divide 28 into two parts in such a way that 6/5 of one part is equal to 2/3 of the other.

Solution:

Let one part be x.

Then other part = 28 - x

It is given 6/5 of one part = 2/3 of the other.

⇒ 6/5x = 2/3(28 - x)

⇒ 3x/5 = 1/3(28 - x)

⇒ 9x = 5(28 - x)

⇒ 9x = 140 - 5x

⇒ 9x + 5x = 140

⇒ 14x = 140

⇒ x = 140/14

⇒ x = 10

Then the two parts are 10 and 28 - 10 = 18.

6. A total of $10000 is distributed among 150 persons as gift. A gift is either of $50 or $100. Find the number of gifts of each type.

Solution:

Total number of gifts = 150

Let the number of $50 is x

Then the number of gifts of $100 is (150 - x)

Amount spent on x gifts of $50 = $ 50x

Amount spent on (150 - x) gifts of $100 = $100(150 - x)

Total amount spent for prizes = $10000

According to the question,

$50x + 100 (150 - x) = 10000$

$\Rightarrow 50x + 15000 - 100x = 10000$

$\Rightarrow -50x = 10000 - 15000$

$\Rightarrow -50x = -5000$

$\Rightarrow x = 5000/50$

$\Rightarrow x = 100$

$\Rightarrow 150 - x = 150 - 100 = 50$

Therefore, gifts of $50 are 100 and gifts of $100 are 50.

Previous Years Solution

2019

1st Term

(iii) Solve: : $\frac{x}{3} + 2x = 14$
Ans: x+6x/3=14
Or 7x=42
Or x= 42/7=6

(ii) Solve : 8y – 3 – 5y = 24
Ans: 8y – 3 – 5y = 24
Or 8y-5y-3=24
Or 3y-3=24
Or 3y=24+3=27
Or y=9

OR

Solve: 14 + 2n – 6 + 8n = 4n – 21 + n + 34
Ans: 14 + 2n – 6 + 8n = 4n – 21 + n + 34
Or 14-6+2n+8n=4n+n-21+34
Or 8+10n=5n+13
Or 10n-5n=13-8
Or 5n=5
Or n=1

(iii)One third of a number is 2 more than one fourth of the number; find the numbers.
Ans: Let the number be x. Then
One third of the number=x/3, one fourth of the number=x/4
The problem states that x/3-x/4=2
4x-3x=24
x=24
The required number is 24.

OR

If a number is multiplied by 5 and 8 is subtracted from the product, the result is 12. Find the number
Ans: Let the required number be x . Then 5x is 5 times the number. If 8 is subtracted from 5x, the result is 5x-8. The problem states that this is equal to 12.
5x-8=12
5x=12+8
5x=20
x=4
The required number is 4.

(iv) Solve: 5(p+4) -8(4-p) = 25 -3(7-p) +144
Ans: 5(p+4) -8(4-p) = 25 -3(7-p) +144
Or 5p+20-32+8p=25-21+3p+144
Or 5p+8p+20-32=25-21+144+3p
Or 13p-12=148+3p
Or 13p-3p=148+12
or 10p=160
or p=16

OR

Solve: 1.32y + 0.02y = 1.19+y.
Ans: 1.32y + 0.02y = 1.19+y.

Or 132y/100+2y/100-y=119/100
Or 132y+2y-100y=119
Or 34y=119
Y=119/34=7/2
(v) Three years ago Raja was 5 years older than Rina was then. If he is now twice as old as she is, find their present ages.
Ans: Let Rina's present age is x. 3 years ago her age was x-3.
So Raja's present age is 2x and 3 years ago his age was 2x-3.
It is given that Raja's age 3 years ago = 5 more than Rina's age 3 years ago.
Or 2x-3=x-3+5
Or 2x-x=-3+5+3
Or x=5
So Rina is now 5 years old and Rja is 10 years old.

2018

1st Term

i) After solving $\frac{2x}{3}=8$,we get, x=12

ii)if 8t=32 then,t=4. True
iii)If 3x+2=14, then x=3. False

(iii) $\frac{a}{3} - 7 = 4$
$\Rightarrow \frac{a}{3} = 4 + 7$
$\Rightarrow \frac{a}{3} = 11$
$\Rightarrow a = 11 \times 3 = 33$

(ii) $8y - 3 - 5y = 24$
$\Rightarrow 3y = 24 + 3$
$\Rightarrow 3y = 27$
$\Rightarrow y = \frac{27}{3} = 9.$

Or

$14 + 2n - 6 + 8n = 4n - 21 + n + 34$
$\Rightarrow 2n + 8n - 4n - n = 34 - 21 - 14 + 6$
$\Rightarrow 10n - 5n = 40 - 35$
$\Rightarrow 5n = 5$
$\Rightarrow n = \frac{5}{5} = 1.$

(iii) Let the smaller number be x
∴ the greater number be x + 3
BTP
$x + (x + 3) = 11$
$\Rightarrow x + x + 3 = 11$
$\Rightarrow 2x + 3 = 11$
$\Rightarrow 2x = 11 - 3$
$\Rightarrow 2x = 8$
$\Rightarrow x = \frac{8}{2} = 4$
So, the numbers are 4 and 4 + 3 = 7.

Or

Let Sayan's age be x years; then Bubai's age be (x + 7) years.
BTP
$x + (x+7) = 33$
$\Rightarrow x + x + 7 = 33$
$\Rightarrow 2x + 7 = 33$
$\Rightarrow 2x = 33 - 7$
$\Rightarrow 2x = 26$
$\Rightarrow x = \frac{26}{2} = 13$
∴ Sayan's age is 13 years and Bubai's age is (13 + 7) = 20 years.

(iv) $\frac{x}{2} + \frac{x}{3} = 10$
$\Rightarrow \frac{3x+2x}{6} = 10$
$\Rightarrow 3x + 2x = 60$
$\Rightarrow 5x = 60$
$\Rightarrow x = \frac{60}{5}$
$\Rightarrow x = 12.$

(v) Let the number be x, then BTP
$\frac{x}{3} = \frac{x}{4} + 2$
$\Rightarrow \frac{x}{3} - \frac{x}{4} = 2$
$\Rightarrow \frac{4x-3x}{12} = 2$
$\Rightarrow x = 24$
The number is 24.

Or

Let the number be x, the BTP
$x + 14 = 35$
$\Rightarrow x = 35 - 14$
$\Rightarrow x = 21.$
The number is 21.

2nd Term

iii) Solving $2+\frac{2x}{3}=8$, we get x=8. <u>False</u>

3rd Term

ii)Find the value of x,If $\frac{2x}{3}=8$.

Ans x=12

v) If $-\frac{4}{5}y = -12$, then find the value of y.

Ans. y= 15

ii)solve: $\frac{x+3}{2} - \frac{3x+1}{4} = \frac{2(x-2)}{3} - 2$

Ans. $\frac{2(x+3)-(3x+1)}{4} = \frac{2(x-2)-6}{3}$

Or-x+5/4= 2x-10/3

Or, -3x+15=8x-40 or, -11x = -55 or, x=5

iii)The sum of two numbers is 64 and the second number is 16 less than the first. Find the numbers.

Ans, x+x-16 =64 or, 2x= 64+16 or, x=40 therefore x-16= 40-16=24

Exercise Problems

1. Solve each of the following equations and represent the solution graphically.

(a) 7x + 2 = -19

(b) 3x - 1/3 = 5

(c) 5x - 4 = 21

(d) -7x = 21

(e) 18 - 7x = -3

(f) 3(x + 4) = 21

(g) $2x/3 - 3x/5 = 8$

(h) $3x - 9 = 5x - 3$

(i) $3(x - 3) = 4(2x + 1)$

2. Solve each of the following equations and check your solution by substituting in the equation.

(a) $3x/2 - 10 = 1/2$

(b) $x/2 - x/3 = 8$

(c) $6x - 9 - 2(1 - x) = x + 9$

(d) $2(x - 2) - 5(x - 5) = 4(x - 8) - 2(x - 2)$

(e) $(2 - y)/(y + 7) = 3/5$

(f) $(3x + 2)/(2x - 3) = - 3/2$

(g) $(x - 8)/3 = (x - 3)/5$

(h) $0.25(4y - 3) = 0.5y - 9$

3. Convert the following statements into equations.

(a) 5 added to a number is 9.

(b) 3 subtracted from a number is equal to 12.

(c) 5 times a number decreased by 2 is 4.

(d) 2 times the sum of the number x and 7 is 13.

4. A number is 12 more than the other. Find the numbers if their sum is 48.

5. Twice the number decreased by 22 is 48. Find the number.

6. Seven times the number is 36 less than 10 times the number. Find the number.

7. 4/5 of a number is more than 3/4 of the number by 5 . Find the number.

8. The sum of two consecutive even numbers is 38. Find the numbers.

9. The sum of three consecutive odd numbers is 51. Find the numbers.

10. Rene is 6 years older than her younger sister. After 1 0 years, the sum of their ages will be 50 years. Find their present ages.

11. The length of a rectangle is 10 m more than its breadth. If the perimeter of rectangle is 80 m, find the dimensions of the rectangle.

12. A 300 m long wire is used to fence a rectangular plot whose length is twice its width. Find the length and breadth of the plot.

13. The denominator of a fraction is greater than the numerator by 8. If the numerator is increased by 17 and denominator is decreased by 1, the number obtained is 3/2, find the fraction.

14. A sum of $2700 is to be given in the form of 63 prizes. If the prize is of either $100 or $25, find the number of prizes of each type.

15. In a class of 42 students, the number of boys is 2/5 of the girls. Find the number of boys and girls in the class.

16. Among the two supplementary angles, the measure of the larger angle is 36° more than the measure of smaller. Find their measures.

17. My mother is 12 years more than twice my age. After 8 years, my mother's age will be 20 years less than three times my age. Find my age and my mother's age.

18. In an isosceles triangle, the base angles are equal and the vertex angle is 80°. Find the measure of the base angles.

19. Adman's father is 49 years old. He is 5 years older than four times Adman's age. What is Adman's age?

20. The cost of a pencil is 25 cents more than the cost of a eraser. If the cost of 8 pencils and 10 erasers is $12.80, find the cost of each.

21. Divide 36 into two parts in such a way that 1/5 of one part is equal to 1/7 of the other.

22. The length of the rectangle exceeds its breadth by 3 cm. If the length and breadth are each increased by 2 cm, then the area of new rectangle will be 70 sq. cm more than that of the given rectangle. Find the length and breadth of the given rectangle.