ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-14

SUBJECT – MATHEMATICS

1st - Term

Chapter: Sequence & Series **Class: XI**

Topic: Problems on GP **Date: 08.07.2020**

Illustration a, b, c are three consecutive terms of an AP and x, y, z as three consecutive terms of a GP, then prove that $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$.

Solution: Let r is the common ratio of a GP, then

$$y = x \times r, z = x \times r^2$$
$$x^{b-c} \times y^{c-a} \times z^{a-b} = x^{b-c}(x \times r)^{c-a}(x \times r^2)^{a-b}$$
$$= x^{b-c+c-a+a-b} \times r^{c-a+2a-2b} \qquad [2b = a + c, a, b, c \text{ are in AP}]$$
$$= x^0 \times r^{c+a-2b} = x^0 \times r^{2b-2b} = 1$$

Illustration The fourth, seventh and last terms of a GP are 10, 80 and 2560. Find a, r and n.

Solution: The last term of a GP is

$$T_n = ar^{n-1}$$

So $$T_4 = 10 = a r^3 \qquad (1)$$

$$T_7 = 80 = a\,r^6 \quad (2)$$
$$T_n = 2560 = a\,r^{n-1} \quad (3)$$

Divide Eq. (2) by Eq. (1). We get

$$\frac{80}{10} = \frac{ar^6}{ar^3} \Rightarrow r^3 = 8 \Rightarrow r = 2$$

Putting the value of r in Eq. (1), we get

$$a = \frac{10}{8}$$

Putting the value of a and r in Eq. (3), we get

$$2560 = \frac{10}{8} \times 2^{n-1} \Rightarrow n = 12$$

Illustration Find the sum of the given series 2, 6, 18, … up to 7 terms.

Solution: Given $a = 2, r = 3$. The sum of the given series is

$$S_7 = \frac{a(r^7 - 1)}{(r-1)} = \frac{2(3^7 - 1)}{(3-7)}$$
$$= \frac{2(3^7 - 1)}{-4} = -\frac{1}{2}(3^7 - 1)$$

Illustration Find the sum of the series $(a^2 - b^2), (a-b), \left(\frac{a-b}{a+b}\right) \ldots$ upto n.

Solution: The first term is $A = a^2 - b^2$, and the common ratio is $r = \frac{1}{a+b}$. The sum of the series is given by

$$S_n = \frac{(a^2 - b^2)\left[\left(\frac{1}{a+b}\right)^n - 1\right]}{\left[\frac{1}{a+b} - 1\right]}$$
$$= \frac{(a-b)(a+b)[(a+b)^{-n} - 1]}{\frac{(1-a-b)}{(a+b)}}$$

$$= \frac{(a-b)(a+b)^2[(a+b)^{-n}-1]}{(1-a-b)}$$

$$= \frac{(a-b)[(a+b)^n-1]}{(a+b)^{n-2}[(a+b)-1]}$$

Illustration Find the sum of 10 terms of the GP

1, 1/2, 1, 1/4, 1/8, …

Solution: Given $a = 1$, $r = 1/4$. So, $S_{10} = \frac{1[1-(1/2)^{10}]}{1-1/2} = \frac{2[2^{10}-1]}{2^{10}}$

$= 2^{-9}[2^{10}-1] = \frac{1023}{512}$.

Illustration Find the sum up to 7 terms of the sequence

$$\left(\frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}\right), \left(\frac{1}{5^4}+\frac{2}{5^5}+\frac{3}{5^6}\right), \left(\frac{1}{5^7}+\frac{2}{5^8}+\frac{3}{5^9}\right), \cdots$$

Solution: The given sequence can be written as

$$\left(\frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}\right), \frac{1}{5^3}\left(\frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}\right), \frac{1}{5^3}\left(\frac{1}{5}+\frac{2}{5^2}+\frac{3}{6^3}\right)$$

This is a GP with

$$a = \frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3} = \frac{38}{125}$$

$$r = \frac{1}{5^3}$$

Hence the sum upto 7 terms is

$$S_7 = a\left(\frac{1-r^n}{1-r}\right) = \frac{38}{125}\left(\frac{1-(1/5^3)^n}{1-(1/5^3)}\right) = \frac{19}{62}\left(1-\frac{1}{5^{21}}\right)$$

Illustration Find the sum of the following infinite series:

(A) $1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\frac{1}{3^4}+\cdots\infty$ **(B)** $8+4\sqrt{2}+4+\cdots\infty$

Solution:

(A) $a=1$ and $r=-\frac{1}{3}$

$$S_\infty=\frac{1}{1+1/3}=3/4$$

(B) $a=8$ and $r=\frac{1}{\sqrt{2}}$

$$S_\infty=\frac{8}{1-\frac{1}{\sqrt{2}}}=\frac{8\sqrt{2}}{\sqrt{2}-1}=\frac{8\sqrt{2}(\sqrt{2}+1)}{1}=16+8\sqrt{2}$$

Illustration The sum of first two terms of an infinite GP is 5 and each term is three times the sum of the succeeding terms. Find the GP.

Solution: Let the GP be $a, ar, ar^2, \ldots, \infty$. Given

$$a+ar=5 \tag{1}$$

As given in question $a_n=3[a_{n+1}+a_{n+2}+\cdots+\infty]$. So

$$ar^{n-1}=3\,[ar^n+ar^{n+1}+\cdots+\infty]$$

$$=3ar^n[1+r+r^2+\cdots+\infty]$$

$$\Rightarrow 1=3r\,\frac{[1]}{1-r}$$

$$\Rightarrow 1-r=3r$$

$$\Rightarrow 1=4r$$

$$\Rightarrow r=\frac{1}{4}$$

Now, putting in Eq. (1)

$$a+a/4=5\Rightarrow a=4\Rightarrow r=\frac{1}{4}, a=4$$

Illustration How many terms of the series $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ amount to $39 + 13\sqrt{3}$?

Solution: Here the first term (b) is $\sqrt{3}$ and the common ratio (r) is $\sqrt{3}$. So

$$S_n = \frac{b(1-r^n)}{1-r}$$

$$\Rightarrow\ 39+13\sqrt{3} = \frac{\sqrt{3}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}}$$

$$\Rightarrow\ \frac{(39+13\sqrt{3})(1-\sqrt{3})}{\sqrt{3}} = 1-(\sqrt{3})^n$$

$$\Rightarrow\ (\sqrt{3})^n - 1 = 26$$

$$\Rightarrow n = 6$$

Illustration Find three numbers in GP whose sum is 65 and whose product is 3375.

Solution: Let the numbers be a/r, a, ar. Then

$$\frac{a}{r} + a + ar = 65 \qquad (1)$$

and

$$\frac{a}{r} \times a \times ar = 3375 \qquad (2)$$

$$\Rightarrow a^3 = 3375$$

$$\Rightarrow a = 15$$

Now from Eq. (1)

$$a\left[\frac{1}{r} + 1 + r\right] = 65$$

$$\Rightarrow\ 15\left[\frac{1}{r} + 1 + r\right] = 65$$

$$\Rightarrow 3\,[1 + r + r^2] = 13r$$

$$\Rightarrow 3 + 3r + 3r^2 - 13r = 0$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow r = 3, \frac{1}{3}$$

Illustration The product of three numbers in GP is 216. If 2, 8, 6 are added then the result converts to AP. Find the numbers.

Solution: Let the numbers $a/r, a, ar$. Then

$$\frac{a}{r} \times a \times ar = 216 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

So the numbers become

$$\frac{6}{r}, 6, 6r$$

According to the given condition the numbers become

$$\frac{6}{r} + 2, 6 + 8, 6r + 6$$

Since they are in AP, we have

$$\Rightarrow 14 = \frac{\frac{6}{r} + 2 + 6r + 6}{2}$$

$$\Rightarrow 14 = \frac{6\left(\frac{1}{r} + r\right) + 8}{2}$$

$$\Rightarrow 14 = 3\left(\frac{1}{r} + r\right) + 4$$

$$\Rightarrow 14 = \frac{3(1 + r^2) + 4r}{r}$$

$$\Rightarrow 14r = 3 + 3r^2 + 4r$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow r = 3, 1/3$$

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