ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

CLASS 8

SUBJECT : Arithmetic

Study Material 6

Squares and Cube roots-2

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SQUARES AND SQUARE ROOTS

Synopsis

- **Square:** The square of a number is the product obtained when a number is multiplied by itself.
- **Perfect Square:** A perfect squares are the shares of whole numbers. Perfect squares are formed by multiplying a whole number by itself.
- **Properties of Squares**

(i) A number ending in 2, 3, 7 or 8 is never a perfect square. All square numbers end in 0, 1, 4, 5, 6 or 9.

(ii) A number ending in an odd number of zeroes is never a perfect square.

(iii) Square numbers have only even number of zeros at the end.

(iv) Squares of even numbers are even.

(v) Squares of odd numbers are odd.

(vi) For every natural number n, $(n + 1)^2 - n^2 = (n + 1) + n$.

e.g., $9^2 - 8^2 = 9 + 8 = 17$

(vii) A triplet (a, b, c) of three natural numbers 'a', 'b' and 'c' is called a Pythagorean triplet if $a^2 + b^2 = c^2$.

(viii) For any natural number $m > 1$, we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$. So, 2m, $(m^2 - 1)$ and $(m^2 + 1)$ form a Pythagorean triplet.

(ix) The square of a natural number 'n' is equal to the sum of the first 'n' odd numbers.

(x) If a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

(xi) There are no natural numbers 'm' and 'n' for which $m^2 = 2n^2$.

(xii) There are 2n non-perfect square numbers between the squares of the numbers n and (n + 1).

- **Square root:** Square root is the inverse operation of square.

(i) The square root of a number 'x' is a number which when multiplied by itself gives 'x' as the product. We denote the square root of 'x' by $\sqrt{x}$.

(ii) There are two integral square roots of a perfect square number. The positive square root of a number is denoted by the symbol $\sqrt{\ }$.

(iii) If x and y are positive numbers, work out the square root of the numerator and denominator separately.

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

(vi) Square root of a number can be found using the following methods.

(a) Repeated subtraction (b) Prime factorisation and (c) Division

◆ **Determining the square roots of positive numbers without using a calculator**

(i) The square root of a fraction is determined by finding the square root of the numerator and denominator separately.

(ii) Some fractions must be reduced to fractions with perfect squares as their numerators and denominators before their square roots can be calculated.

(iii) To find the square root of a mixed number, first change the mixed number into an improper fraction.

(iv) The square root certain decimals are obtained by first changing the decimals into fractions with perfect squares as their numerators and denominators.

◆ **Estimating the number of digits in the square root of a given number:** Place bars over every two digits from the right. The number of bars obtained is the number of digits in the square root of the number.

e.g., $\sqrt{\overline{9}} = 3$; $\sqrt{\overline{25}} = 5$; $\sqrt{\overline{1}\,\overline{00}} = 10$; $\sqrt{\overline{1}\,\overline{69}} = 13$; $\sqrt{\overline{1}\,\overline{44}\,\overline{00}} = 120$

◆ Estimate the square root of a number by determining the range of the square root of that number.

$\sqrt{193}$

$169 < 193 < 196$ ¬ Determine the range between two known perfect squares.

$\sqrt{169} < \sqrt{193} < \sqrt{196}$ ¬ Square root the range.

$13 < \sqrt{193} < 14$ ¬ Estimated answer.

(i) The square root of a number with one bar has one digit.

(ii) The square root of a number with two bars has two digits. The square root of a number with three bars has three digits.

◆ To compute the square or square root of a mixed number, first convert it into an improper fraction.

FINDING SQUARE ROOTS BY VEDIC METHODS

Square root of any number means to get a number which is multiplied by itself gives the given number. In the conventional method of finding the square root, the divisor goes on becoming larger at each step. This increases the calculation time as well as the complexity of the problem. Here, we shall try to learn some speedy Vedic Methods of finding the square roots of perfect square numbers. Before proceeding for finding square roots, let us have a look into the known facts of squares and square roots.

The basic rules for extracting square roots are :

(*i*) The given number is arranged in two-digit groups from right to left; and a single digit (if any) left over at the left and is counted as a group by itself.

(*ii*) The number of digits in the square root will be the same as the number of two-digit groups in the given number including a single digit group (if any). Thus, 36 will count as one group, 169 as two groups and 1225 as two groups.

(*iii*) If the number contains n digits then the square root will contain $\frac{n}{2}$ (when n is even) and $\frac{n+1}{2}$ (when n is odd) digits. Thus, one or two digit number will have the square root of one digit, three and four digit number will have the square root of two digits, 5 and 6 digit number will have the square root of 3 digits and so on.

(*iv*) The squares of first nine natural numbers are :

$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, 9^2 = 81.$

This means :

(*a*) unit digit of the perfect square number is 1, 4, 5, 6, 9 or 0.

(*b*) a perfect square number cannot end in 2, 3, 7 or 8.

(*c*) the relation between the unit digit of a perfect square number and the unit digit of its square root is as follows :

Unit digit of the number	1	4	5	6	9	0
Unit digit of square root	1 or 9	2 or 8	5	4 or 6	3 or 7	0

(*d*) If there are odd number of zeros at the end (on right side) of a number, then it will not be a perfect square.

VILOKANAM (OBSERVATION) METHOD

Square root of one or two digit number is well known from the table given in the beginning. Now we shall learn the method of finding square root of 3 or 4 digit perfect square number by *Vilokanam* method.

Look at the unit digit of the given number and decide about the unit digit of the square root from the following data :

Unit digit of the number	0	1	4	5	6	9
Unit digit of square root	0	1 or 9	2 or 8	5	4 or 6	3 or 7

Now ignore the last two digits (unit digit and ten's digit) of the given number and find out the greatest number whose square is less than or equal to the remaining part of the given number. Then adjust the above obtained unit digits on its right side and get two numbers. Find out the unique number with its unit digit 5 which lies between these two

numbers and obtain the square of this unique number. If the given number is less than this square number then the smaller number among above obtained two numbers is the square root of the given number; otherwise another one is the required square root of the given number. Let us learn it with the help of some examples.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the square root of 841.*

Solution. The given number is 841.

Its unit digit is 1, therefore, the unit digit of the square root will be 1 or 9.

Ignoring the last two digits (unit digit and ten's digit) we get 8.

The greatest number whose square is less than or equal to 8 is 2.

Adjusting above obtained two unit digits 1 or 9 to the right of 2, we get two numbers 21 and 29.

The unique number with unit digit 5 which lies between 21 and 29 is 25.

$(25)^2 = 625$

(By *Ekādhikena sūtra* : $(25)^2 = (2 \times 3)\ 25 = 625$).

Since $841 > 625$, therefore, the required square root is 29.

Hence, $\sqrt{841} = 29$.

Example 2. *Find the square root of 4356.*

Solution. The given number is 4356.

Its unit digit is 6, therefore, the unit digit of the square root will be 4 or 6.

Ignoring the last two digits (unit digit and ten's digit), we get 43.

The greatest number whose square is less than or equal to 43 is 6.

On adjusting above obtained two unit digits 4 or 6 to the right of 6, we get two numbers 64 and 66.

The unique number with unit digit 5 which lies between 64 and 66 is 65.

$(65)^2 = 4225$.

(By *Ekādhikena sūtra* : $(65)^2 = (6 \times 7)\ 25 = 4225$)

Since $4356 > 4225$, therefore, the required square root is 66.

Hence, $\sqrt{4356} = 66$.

Example 3. *Find the square root of 8649.*

Solution. The given number is 8649.

Its unit digit is 9, therefore, the unit digit of the square root is 3 or 7.

Ignoring the last two digits (unit digit and ten's digit), we get 86.

The greatest number whose square is less than or equal to 86 is 9.

On adjusting above obtained two unit digits 3 or 7 to the right of 9, we get two numbers 93 or 97.

The unique number with unit digit 5 which lies between 93 and 97 is 95.

$(95)^2 = 9025$

(By *Ekādhikena sūtra* : $(95)^2 = (9 \times 10)\ 25 = 9025$)

Since $8649 < 9025$, therefore, the required square root is 93.

Hence, $\sqrt{8649} = 93$.

CUBES AND CUBE ROOTS

Synopsis

- **Cubes**
  - (i) The cube of a number is the product of the number multiplied by itself twice.
  - (ii) Write the cube of a number using the cube symbol or notation.
  - (iii) 8^3 is read as 'eight cubed' or 'the cube of eight', or 'eight to the power of three.
- **Estimating the cubes of numbers**

  Estimate the cube of a number by determining the range in which its value lies.

  e.g., Estimate the cube of 10.6 by determining the range in which its value lies.

  Solution

  $10 < 10.6 < 11$ ¬ Determine the range

  $103 < (10.6)^3 < 113$ ¬ Cube the range

  $1\,000 < (10.6)^3 < 1331$ ¬ Estimated answer

  $\therefore$ $(10.6)^3$ is between 1000 and 1331.
- **Perfect cube**
  - (i) A natural number is said to be a perfect cube if it is the cube of some natural number.
  - (ii) Cubes of all even natural numbers are even.
  - (iii) Cubes of all odd natural numbers are odd.
  - (iv) Cubes of negative integers are negative.
- **Cube roots of numbers**
  - (i) The cube root of a number is a number which, when multiplied by itself twice, equals the given number.
  - (ii) The symbol used for cube root is $\sqrt[3]{\ }$.
  - (iii) The cube root of a number 'x' is that number whose cube gives 'x'. It is denoted as $\sqrt[3]{x}$.
  - (iv) For any positive integer 'x', $\sqrt[3]{-x} = -\sqrt[3]{x}$.
  - (v) For any two integers 'a' and 'b',

    (a) $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$ (b) $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$
  - (iv) Cube root of a number can be found by prime fatorisation.
- **Determining the cube roots**
  - (i) To find the cube roots of fractions, reduce the fractions to numerators and denominators that are cubes of integers. Then, find the cube roots of those integers.
  - (ii) The find the cube roots of decimals, convert the decimals to fractions so that the numerators and denominators are cubes of integers. Then, solve the cube roots of those integers.

CUBE ROOT OF A PERFECT CUBE

Table 1. Table of cubes.

1	2	3	4	5	6	7	8	9	10
1	8	27	64	125	216	343	512	729	1000

An inspection of this table reveals that each cube ends in a different digit. The digit corresponds to the cube root in all cases except 2 and 3 and 7 and 8. In these four cases the final digit of the cube is the difference between the cube root and 10.

To see how this information is used by the lightning calculator, let us suppose that a spectator calls out the cube 250047. The last number is a 7 which tells the performer immediately that the last number of the cube root must be 3. The first number of the cube root is determined as follows. Discard the last three figures of the cube (regardless of the size of the number) and consider the remaining figures—in this example they are 250. In the above table, 250 lies between the cubes of 6 and 7. The *lower* of the two figures—in this case 6—will be the first figure of the cube root. The correct answer, therefore, is 63.

One more example will make this clear. If the number called out is 19,683, the last digit, 3, indicates that the last digit of the cube root is 7. Discarding the final three digits leaves 19, which falls between the cubes of 2 and 3. Two is the lower number, therefore we arrive at a final cube root of 27.

QUESTION BANK -SQUARES AND SQUARE ROOTS

Multiple Choice Questions

A B C D

1 What is the least perfect square which is divisible by 2, 4 and 6 ?

(A) 36 (B) 64
(C) 16 (D) 18

2 What is the number of digits in the square root of 390625 ?

(A) 4 (B) 6
(C) 5 (D) 3

3 What is the least perfect square exactly divisible by each of the numbers 6, 9, 15 and 20 ?

(A) 3600 (B) 900
(C) 400 (D) 225

4 The sides of a rectangular field are 80 m and 18 m respectively. What is the length of its diagonal ?

(A) 84 m (B) 98 m
(C) 82 m (D) 86 m

5 What is the least number which must be subtracted from 2509 to make it a perfect square ?

(A) 6 (B) 9
(C) 12 (D) 14

6 What is the square root of $42\frac{583}{1369}$?

(A) $6\frac{19}{37}$ (B) $4\frac{2}{11}$
(C) $7\frac{2}{121}$ (D) $6\frac{12}{37}$

7 If $\sqrt{1+\frac{27}{169}} = 1+\frac{x}{13}$, what is the value of '$x$' ?

(A) 1 (B) 14
(C) 10 (D) 12

8 What is the value of $(501)^2 - (500)^2$?

(A) 1 (B) 101
(C) 1001 (D) 100

9 Which of the following is a Pythagorean triplet?

(A) (6, 8, 10) (B) (3, 4, 7)
(C) (5, 12, 18) (D) (2, 3, 6)

10 What is the value of
$1 + 3 + 5 + 7 + 9 + \ldots\ldots + 25$?

(A) 196 (B) 625
(C) 225 (D) 169

11 What is the smallest number by which 396 must be multiplied so that the product is a perfect square ?

(A) 5 (B) 11
(C) 3 (D) 2

12 Find the square root of

$$\frac{0.081}{0.0064} \times \frac{0.484}{6.25} \times \frac{2.5}{12.1}.$$

(A) 0.45 (B) 0.75
(C) 0.95 (D) 0.99

13 Find the approximate value of

$$\frac{1+\sqrt{0.01}}{1-\sqrt{0.1}}.$$

(A) 0.6 (B) 1.1
(C) 1.6 (D) 1.7

14 If $x * y = \sqrt{x^2 + y^2}$, find the value of $(1 * 2\sqrt{2})(1 * -2\sqrt{2})$.

(A) −7 (B) 0 (C) 2 (D) 9

15 Evaluate $\sqrt{41-\sqrt{21+\sqrt{19-\sqrt{9}}}}$.

(A) 3 (B) 6 (C) 5 (D) 6.4

16 Which of the following is a four digit perfect square number whose first two digits and last two digits taken separately are also perfect squares ?

(A) 1681 (B) 1636
(C) 3664 (D) 6481

17 Find the number, which when subtracted from its square results in 12.

(A) 6 (B) 5
(C) 4 (D) 8

18 Which is the smallest natural number which when added to the difference of squares of 17 and 13 gives a perfect square ?

(A) 1 (B) 5
(C) 11 (D) 24

19 If $\sqrt{\frac{16}{49}} = \frac{x}{49}$, find the value of '$x$'.

(A) 4 (B) 7
(C) 16 (D) 28

20 The area of a square field is $80\frac{244}{729}$ sq. m. What is the length of each side of the field?

(A) 8.96 m (B) 10.26 m
(C) 13.54 m (D) 9.86 m

21 If $\sqrt{2^n} = 64$, what is the value of 'n' ?

(A) 2 (B) 4
(C) 6 (D) 12

22 Evaluate $\sqrt{\frac{36.1}{102.4}}$.

(A) $\frac{29}{32}$ (B) $\frac{19}{72}$
(C) $\frac{19}{32}$ (D) $\frac{29}{62}$

23 Given $\frac{x}{\sqrt{2.25}} = 550$, find the value of '$x$'.

(A) 825 (B) 82.5
(C) 3666.66 (D) 2

24 If $\sqrt{24} = 4.899$, find the value of $\sqrt{\frac{8}{3}}$.

(A) 0.544 (B) 2.666
(C) 1.633 (D) 1.333

25 Evaluate $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}}$.

(A) $\frac{1}{2\sqrt{3}}$ (B) $-\frac{1}{2\sqrt{3}}$
(C) 1 (D) $\frac{5\sqrt{3}}{6}$

26 Find the value of the expression $\sqrt{0.0009} \div \sqrt{0.01}$.

(A) 3 (B) 0.3
(C) $\frac{1}{3}$ (D) 0.9

27 If $\sqrt{2401} = \sqrt{7^x}$, find the value of 'x'.

(A) 3 (B) 4
(C) 5 (D) 6

28 Find the value of $\sqrt{0.064}$.

(A) 0.8 (B) 0.08
(C) 0.008 (D) 0.253

29 Find the value of $\sqrt{\frac{0.16}{0.4}}$.

(A) 0.2 (B) 0.02
(C) 0.63 (D) $\frac{2\sqrt{5}}{5}$

30 Which of the following is the smallest number of 4 digits, which is not a perfect square ?

(A) 1024 (B) 1089
(C) 1000 (D) 1156

31 A gardener wants to plant 17956 trees and arranges them in such a way that there are as many rows as there are trees in a row. What is the number of trees in a row ?

(A) 144 (B) 136
(C) 154 (D) 134

32 A group of students decided to collect as many rupees from each member of the group as is the number of members. If the total collection amounts to ₹ 2209, what is the number of members in the group ?

(A) 37 (B) 47
(C) 107 (D) 43

33 A general wishes to draw up his 36562 soldiers in the form of a solid square. After arranging them, he found that some of them are left over. How many are left?

(A) 36 (B) 65
(C) 81 (D) 97

Previous Contest Questions

1 Find the value $\sqrt{\frac{0.289}{0.00121}}$.

(A) $\frac{1.7}{11}$ (B) $\frac{17}{11}$

(C) $\frac{170}{11}$ (D) $\frac{17}{110}$

2 If $\sqrt{75.24+x}=8.71$, find the value of 'x'.

(A) 0.6241 (B) 6.241
(C) 62.41 (D) 624.1

3 If $\sqrt{3}=1.732$, find the approximate value of $\frac{1}{\sqrt{3}}$.

(A) 0.617 (B) 0.313
(C) 0.577 (D) 0.173

4 If $\sqrt{0.04\times0.4\times a}=0.4\times0.04\times\sqrt{b}$, what is the value of $\frac{a}{b}$?

(A) 0.016 (B) 1.60
(C) 0.16 (D) 16

5 Find the value of $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$.

(A) 0 (B) 1
(C) 2 (D) $2^{31/32}$

6 Find the largest number of 5 digits, which is perfect square.

(A) 99999 (B) 99764
(C) 99976 (D) 99856

7 Which smallest number must be added to 269 to make it a perfect square ?

(A) 31 (B) 16
(C) 20 (D) 7

QUESTION BANK – CUBES AND CUBE ROOTS

Multiple Choice Questions

A B C D

1. If 72K is a perfect cube, find the value of K.
(A) 1 (B) 2 (C) 3 (D) 4

2. If 'a', 'b', 'c' and 'd' are consecutive natural numbers and $a^3 = b^3 + c^3 + d^3$, what is the least value of 'a' ?
(A) 6 (B) 9 (C) 3 (D) 12

3. Find the number which is not a perfect cube among the following.
(A) 1331 (B) 216 (C) 243 (D) 512

4. What is the smallest number by which 2560 must be multiplied so that the product is a perfect cube ?
(A) 5 (B) 25 (C) 10 (D) 15

5. Find the smallest number by which 8788 must be divided so that the quotient is a perfect cube.
(A) 4 (B) 12 (C) 16 (D) 32

6. What is the cube root of 1.331 ?
(A) 0.11 (B) 0.011
(C) 11 (D) 1.1

7. Find the value of $\sqrt[3]{343} \times \sqrt[3]{-64}$.
(A) 28 (B) –28 (C) 18 (D) –18

8. The length of each side of a cubical box is 2.4 m. What is its volume ?
(A) 13.824 cu m (B) 13.824 cu cm
(C) 13.824 m^2 (D) 13.824 cm^2

9. The cube of a number is 8 times the cube of another number. If the sum of the cubes of numbers is 243, what is difference of the numbers ?
(A) 3 (B) 4 (C) 6 (D) –6

10. The difference of the cube and the square of a natural number is 48. Find the number.
(A) 6 (B) 5 (C) 4 (D) 8

11. Which of the following numbers is a perfect cube ?
(A) 108 (B) 343 (C) 243 (D) 5324

12. Which of the following is the cube of an even natural number ?
(A) 729 (B) 3375
(C) 1331 (D) 13824

13. Find the cube root of 0.001728.
(A) 0.12 (B) 1.2
(C) 12 (D) 2.6

14. Evaluate $\sqrt[3]{-2744} \div \sqrt[3]{0.008}$.
(A) 70 (B) – 70
(C) 14 (D) – 14

15. Identify the digit in the units place of $\sqrt[3]{21952}$.
(A) 8 (B) 4 (C) 6 (D) 7

16. The quotient of the cube root of a number and 25 is 5. Identify the number.
(A) 5^3 (B) 25^3
(C) 125^2 (D) 125^3

17. The product 864 × n is a perfect cube. What is the smallest possible value of 'n'?
(A) 2 (B) 1 (C) 4 (D) 3

18. Find the units place in the cube of 126.
(A) 4 (B) 6 (C) 8 (D) 1

19. Find the value of $\sqrt[3]{0.000216}$.
(A) 0.006 (B) 0.6
(C) 0.36 (D) 0.06

20. Find the value of $\sqrt{\sqrt[3]{0.000064}}$.
(A) 0.2 (B) 0.02
(C) 0.002 (D) 0.04

21. If $P = \sqrt[3]{\sqrt[3]{a^9}}$, what is the value of P ?
(A) $(a^3)^3$ (B) a^3 (C) a^9 (D) a

22. By which number should 5184 be divided to make it a perfect cube ?
(A) 2 (B) 4 (C) 3 (D) 6

23 The cube root of a number when divided by 7 results in 49. Identify the number.

(A) 343^3 (B) 33^3 (C) 43^3 (D) 343^7

24 Evaluate $\sqrt[3]{\frac{343\times125}{0.064}}$.

(A) 875 (B) 8.75 (C) 807.5 (D) 87.5

25 A metallic cuboid with dimensions 16 cm × 8 cm × 4 cm was melted and recast into a cube. What is the length of the edge of the cube ?

(A) 8 cm (B) 2 cm (C) 4 cm (D) 16 cm

26 What is the volume of a cube of edge 15 cm ?

(A) 15625 cm^3 (B) 3.375 cm^3
(C) 3375 cm^3 (D) 225 cm^3

27 The volume of liquid in a measuring jar is 200 cm^3. A cube of side 7 cm is immersed in the liquid. What is the reading on the measuring jar ?

(A) 800 cm^3 (B) 543 cm^3
(C) 643 cm^3 (D) 843 cm^3

28 The volume of material used to make a cube is 4913 cm^3. What is the length of the edge of the cube ?

(A) 16 cm (B) 27 cm
(C) 18 cm (D) 17 cm

29 A tank holds 27000 litres of water. How many litres of water would a similar pond hold, if its dimensions were double the first one ?

(A) 17286 litres (B) 216000 litres
(C) 15625 litres (D) 432000 litres

30 A cubic wooden block has an edge of 0.21 m. What is its volume ?

(A) 9261 cm^3 (B) 926.1 m^3
(C) 92.61 cm^3 (D) 92.61 m^3

31 A cube is made of 24389 cm^3 of wood. What is the measure of its edge ?

(A) 39 cm (B) 29 cm
(C) 2.9 cm (D) 19 cm

32 A water tank holds 79507 cm^3 of water. What is the measure of its edge ?

(A) 23 cm (B) 33 cm
(C) 43 cm (D) 38 cm

33 A wooden cuboid is 24 cm by 30 cm by 36 cm. 8 cubes of equal sides are cut off from the 8 corners of the cuboid. The volume of the remaining block is 20088 cm^3. What is the length of the edge of each cube ?

(A) 6 cm (B) 9 cm (C) 1 cm (D) 7 cm

Previous Contest Questions

1 Find the value of $\sqrt[3]{0.125}+3$.

(A) 8 (B) 3.5 (C) 2 (D) 0.35

2 If $x=\sqrt[3]{-3\frac{3}{8}}$, find the value of '$x$'.

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) $-\frac{3}{4}$ (D) $-\frac{3}{2}$

3 Find the value of $\sqrt[3]{\frac{-192}{81}}$.

(A) $-\frac{5}{3}$ (B) $\frac{-4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{13}{9}$

4 What is the value of $\sqrt[3]{216}$?

(A) Less than 6.
(B) Greater than 6.
(C) Equal to 6.
(D) Equal to 9.

5 Given that $512 = 8^3$ and $3.375 = 1.5^3$, find the value of $\sqrt[3]{512}\times\sqrt[3]{3.375}$.

(A) 12 (B) 9.5 (C) 8 (D) 1.5

6 Given that $\sqrt[3]{x}=-6$, find the value of 'x'.

(A) 216 (B) 18 (C) –18 (D) –216

7 The length of each edge of a cube is 9 m. Find the volume of the cube.

(A) 27 m^3 (B) 81 m^3
(C) 108 m^3 (D) 729 m^3

SOLUTIONS-SQUARES AND SQUARE ROOTS

Multiple Choice Questions

1. (A) 2. (D) 3. (B) 4. (C) 5. (B) 6. (A)

7. (A) $\sqrt{1+\frac{27}{169}} = \sqrt{\frac{196}{169}} = \frac{14}{13}$

$= 1\frac{1}{13} = 1+\frac{1}{13} \Rightarrow x = 1$

8. (C) We have, $(n+1)^2 - n^2 = (n+1) + n$.

So, $(501)^2 - (500)^2 = 501 + 500$
$= 1001$

9. (A) General form of Pythagorean triplet is $(2m, m^2 - 1, m^2 + 1)$.

Let $m = 3$.

Then $2m = 6$, $m^2 - 1 = 3^2 - 1 = 8$ and $m^2 + 1 = 3^2 + 1 = 10$.

$\therefore$ (6, 8, 10) is a Pythagorean triplet.

10. (D) The given expression is the sum of 13 odd numbers.

So, the required value is $13^2 = 169$.

11. (B) 12. (A) 13. (C) 14. (D) 15. (B)

16. (A) By checking the given options only (A) has a number that satisfies the given conditions as $1681 = 41^2$.

17. (C) Let 'x' be the required number. Then
$x^2 - x = 12$

$\Rightarrow x(x-1) = 12$

By inspection, we have

$4(4-1) = 12$

$\Rightarrow 4 \times 3 = 12$

$\Rightarrow x = 4$

18. (A) $17^2 - 13^2 = 289 - 169 = 120$

$120 + 1 = 121 = 11^2$

$\therefore$ The required number is 1.

19. (D) $\sqrt{\frac{16}{49}} = \frac{4}{7} = \frac{x}{49}$

$\Rightarrow x = 28$

20. (A) Length of each side of the square field

$= \sqrt{80\frac{244}{729}} = \sqrt{\frac{58564}{729}}$

$= \frac{242}{27} = 8.96$ m

21. (D) $\sqrt{2^n} = 64 = 2^6 \Rightarrow 2^{\frac{n}{2}} = 2^6$

So, $\frac{n}{2} = 6$ or $n = 12$.

22. (C) $\sqrt{\frac{36.1}{102.4}} = \sqrt{\frac{361}{1024}} = \frac{19}{32}$

23. (A) Given $\frac{x}{\sqrt{2.25}} = 550$.

Then, $\frac{x}{1.5} = 550$.

$\therefore x = 550 \times 1.5 = \left(\frac{550 \times 15}{10}\right) = 825$

24. (C) $\sqrt{\frac{8}{3}} = \frac{\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{24}}{3} = \frac{4.899}{3} = 1.633$

25. (A) $\frac{\sqrt{4}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{4}} = \frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2}$

$= \frac{4-3}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$

26. (B) Given expression

$= \frac{\sqrt{0.0009}}{\sqrt{0.01}} = \frac{\sqrt{0.0009}}{\sqrt{0.0100}}$

$= \sqrt{\frac{9}{100}} = \frac{\sqrt{9}}{\sqrt{100}} = \frac{3}{10} = 0.3$

27. (B) $\sqrt{2401} = \sqrt{7^x} \Rightarrow 7^x = 2401 \Rightarrow x = 4$

28. (D) 29. (C) 30. (C) 31. (D) 32. (B)

33. (C)

Previous Contest Questions

1. (C) $\sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{0.28900}{0.00121}} = \sqrt{\frac{28900}{121}}$

$= \frac{\sqrt{28900}}{\sqrt{121}} = \frac{170}{11}$

2. (A) $75.24 + x = 8.71 \times 8.71$

$\Rightarrow x = 0.6241$

3. (C) $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$= \frac{1.732}{3} = 0.577$

4. (A) 5. (D) 6. (D) 7. (C)

SOLUTIONS- CUBES AND CUBE ROOTS

Multiple Choice Questions

1. (C) $72 = 2\times2\times2\times3\times3$

If K = 3,

$72\ K = 72\times3 = 216 = 6^3$ is a perfect cube.

2. (A) $a = 6$ as $6^3 = 3^3 + 4^3 + 5^3$

3. (C) $1331 = 11^3$, $216 = 6^3$ and $512 = 8^3$.

But, 243 is not a perfect cube.

4. (B) $2560 = 2\times2\times2\times4\times4\times4\times5$

Clearly, on multiplication by $5\times5 = 25$, it becomes a perfect cube.

5. (A) $8788 = 4\times13\times13\times13$

On division by 4 it becomes a perfect cube.

6. (D) $\sqrt[3]{1.331} = \sqrt[3]{(1.1)^3} = 1.1$

7. (B) $\sqrt[3]{343}\times\sqrt[3]{-64} = \sqrt[3]{343\times-64}$

$= -\sqrt[3]{7\times7\times7\times4\times4\times4}$

$= -7\times4 = -28$

8. (A) 9. (A) 10. (C) 11. (B) 12. (D)

13. (A) $\sqrt[3]{0.001728} = \sqrt[3]{(0.12)^3} = 0.12$

14. (B) $\sqrt[3]{-2744}\div\sqrt[3]{0.008} = (-14)\div0.2$

$= -70$

15. (A) $21952 = 2^3\times2^3\times7^3$

$\therefore\ \sqrt[3]{21952} = \sqrt[3]{2^3\times2^3\times7^3}$

$= 2\times2\times7 = 28$

$\therefore$ The required units digit is 8.

16. (D) According to the problem if 'x' is the number. then $\sqrt[3]{x}+25$ is 5

$\Rightarrow \sqrt[3]{x} = 125 \Rightarrow x = 125^3$

17. (A) $864\times2 = 1728 = 12^3$ which is a perfect cube. Hence, the smallest possible value of 'n' is 2.

18. (B) 126 ends in 6. So its cube also ends in 6.

19. (D) 20. (A) 21. (D) 22. (C) 23. (A)

24. (D) $\sqrt[3]{\frac{343\times125}{0.064}} = \sqrt[3]{\frac{7^3\times5^3}{(0.4)^3}}$

$= \frac{7\times5}{0.4} = \frac{7\times5\times\cancel{10}^{5}}{\cancel{4}_{2}} = \frac{175}{2} = 87.5$

25. (A) Volume of the metallic cubold

$= 16\times8\times4\ cm^3 = 512\ cm^3$

$\therefore$ The edge of the cube $= \sqrt[3]{512}$

$= 8$ cm.

26. (C) Edge of the cube = 15 cm

$\Rightarrow$ Its volume $= 15^3\ cm^3$

$= 3375\ cm^3$

27. (B) Volume of the liquid in the jar $= 200\ cm^3$.

Side of the immersed cube = 7 cm

$\Rightarrow$ Its volume $= 7^3\ cm^3 = 343\ cm^3$

Therefore, the reading on the measuring jar is $200 + 343\ cm^3$

$= 543\ cm^3$

28. (D) Volume of the material used = volume of the cube $= 4913\ cm^3$

$\therefore$ The length of its edge

$= \sqrt[3]{4913} = 17$ cm

29. (B) 30. (A) 31. (B) 32. (C) 33. (B)

Previous Contest Questions

1. (B) $\sqrt[3]{0.125}+3 = 0.5+3 = 3.5$

2. (D) $\sqrt[3]{-3\frac{3}{8}} = \sqrt[3]{\frac{-27}{8}} = \sqrt[3]{\frac{-3^3}{2^3}} = \frac{-3}{2}$

3. (B) $\sqrt[3]{\frac{-192}{81}}$

$= \sqrt[3]{\frac{(-2)\times(-2)\times(-2)\times2\times2\times2\times\cancel{3}}{3\times3\times3\times\cancel{3}}}$

$= \frac{-2\times2}{3} = \frac{-4}{3}$

4. (C) $\sqrt[3]{216} = 2\times3 = 6$

5. (A) $\sqrt[3]{512}\times\sqrt[3]{3.375} = 8\times1.5 = 12$

6. (D) $\sqrt[3]{x} = -6$

$\Rightarrow x = (-6)^3 = (-6)\times(-6)\times(-6)$

$= -216$

7. (D) Volume of cube $= 9\times9\times9\ m^3$

$= 729\ m^3$