ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-8

SUBJECT – MATHEMATICS

Pre-test

Chapter: Relations & Functions **Class: XII**

Topic: Mapping or Function **Date: 09.06.2020**

UNIT-I : RELATIONS AND FUNCTIONS (WBCHSE SYLLABUS).

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1. Relations and Functions :

Types of relations : reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. Inverse Trigonometric Functions :

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

Total Marks : 08

-: MAPPING OR FUNCTION :-

Topic Covered:-

1. Recapitulation (Some definitions)
2. Kinds of Function
3. Composition of Function
4. Inverse of a function.

1. Recapitulation (Some definitions) :-

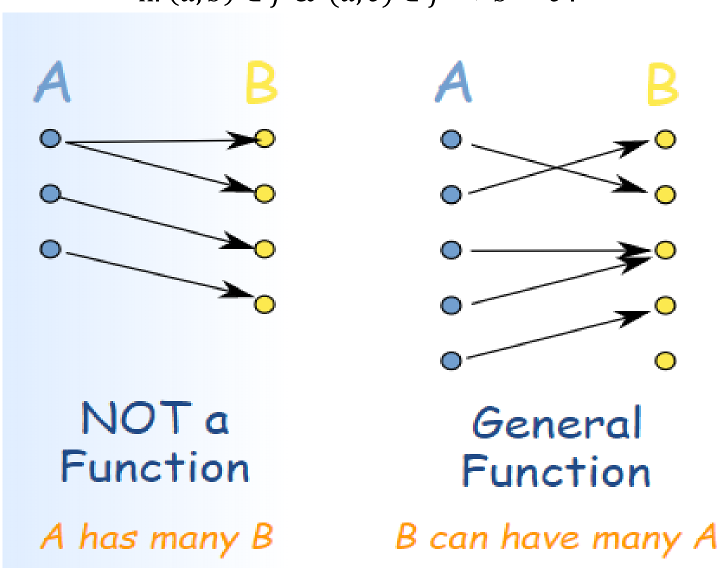
A) Definition of Function :- A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

In other words, a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element.

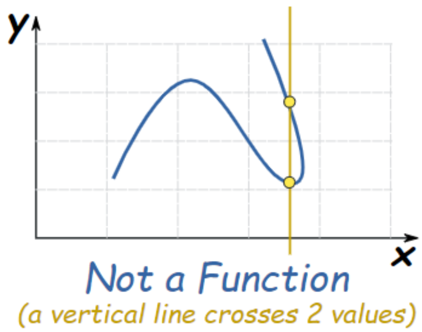
i.e. $f: A \to B$ will be a function , if

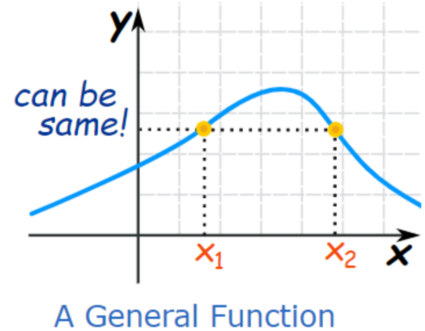
i. For each $a \in A$, $\exists b \in B$ $such\ that$ $(a, b) \in f$

ii. $(a, b) \in f$ & $(a, c) \in f$ $\Rightarrow b = c$.



- If B is subset of the set of all real variables i.e. $\mathbb{R}$, then the function is called real valued function.
- If both A & B are subsets of the set of all real variables i.e. $\mathbb{R}$, then the function is called real function.
- Graphical representation,



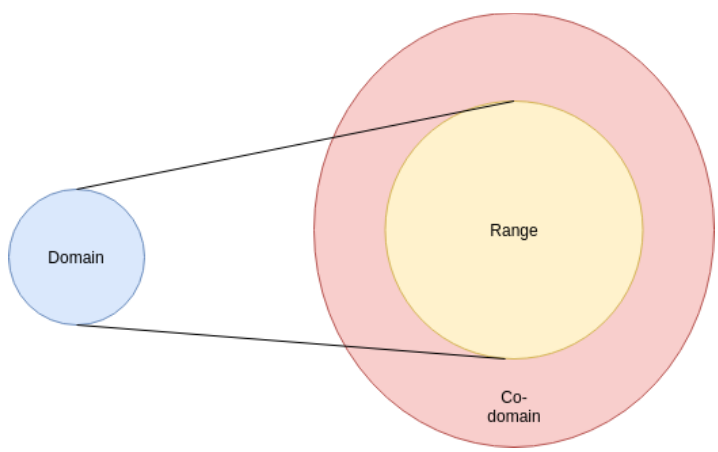


B) Domain, co-domain & range of a function :- Let . $f: A \to B$ be a function.

i. Then the set A is known as the domain of f.

ii. Then the set B is known as the co-domain of f.

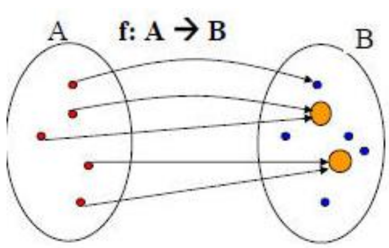
iii. $f(A) = \{ f(x) : x \in A\}$ = Range of . Clearly, $f(A) \subseteq B$.



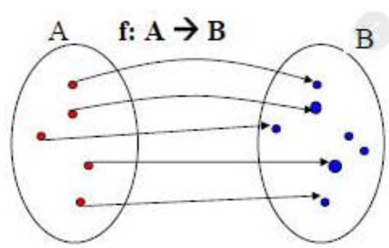
2. Kinds of Function :-

A) **<u>One-one (or injective) function :-</u>** A function $f: A \to B$ is defined to be one-one (or injective), if the images of distinct elements of A under f are distinct.

i.e. $f: A \to B$ is injective $\Leftrightarrow$ $f(a) = f(b) \Rightarrow a = b$; $\forall a, b \in A \Leftrightarrow a \neq b \Rightarrow f(a) \neq f(b); \forall a, b \in A$

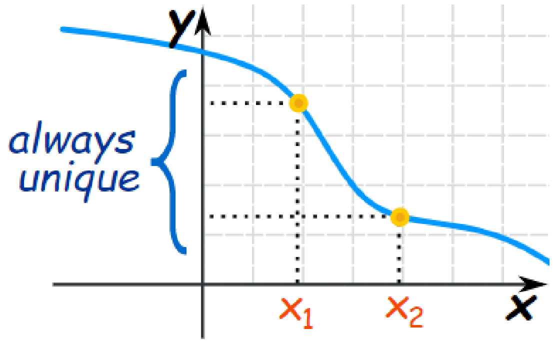


Not injective function



Injective function

➢ Graphical representation,

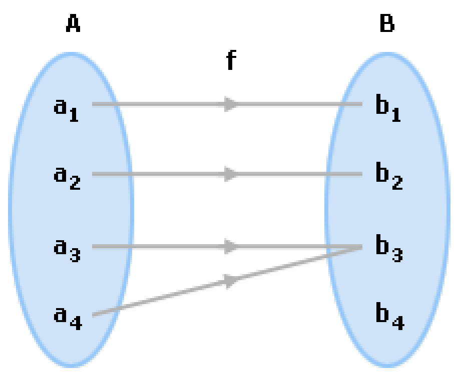


"Injective" (one-to-one)

❖ Example :- Let, $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = 3x^3 - 2, \forall x \in \mathbb{R}$.

B) **<u>Many-one function :-</u>** A function $f: A \to B$ is defined to be many-one, if two or more elements of A have the same image in B under f.

i.e. $f(a) = f(b);\ when\ a \neq b\ \ for\ some\ a, b \in A$.

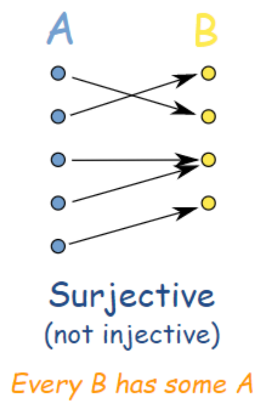


Many-one function

❖ Example :- Let, $f: \mathbb{Z} \to \mathbb{Z}$, defined by $f(x) = |x|, \forall x \in \mathbb{Z}$.

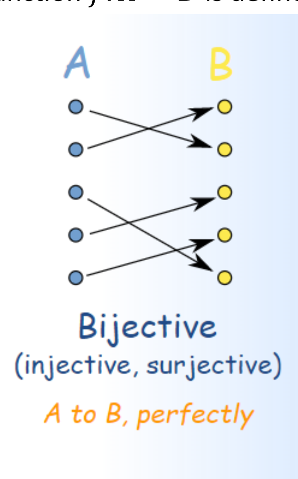
C) **<u>Into function :-</u>** A function $f: A \to B$ is defined to be Into function if the range set is a proper subset of B . i.e. $\{ f(x) : x \in A\} \subset B$.

D) **On-to (or surjection) function :-** A function $f: A \to B$ is defined to be on-to (or surjective), if every element in B , has at least one preimage in A under f.
In other words, the range set $\{ f(x) : x \in A\} = B$.
i.e. $\forall y \in B$, $\exists\, x \in A$ $such\ that$ $f(x) = y$.



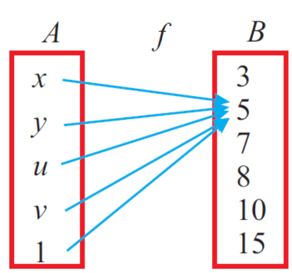
❖ Example :- Let, $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x^3 + 2$, $\forall x \in \mathbb{R}$.

E) **Bijective function :-** A function $f: A \to B$ is defined to be bijective if it is One-one & Onto.



❖ Example :- Let, $A = \{x \in \mathbb{R} : -1 \le x \le 1\} = B$. $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x|x|, \forall x$.

F) **Constant function :-** A function $f: A \to B$ is defined to be constant if the range set is singleton set.
i.e. all elements in A have the same image in B under F.



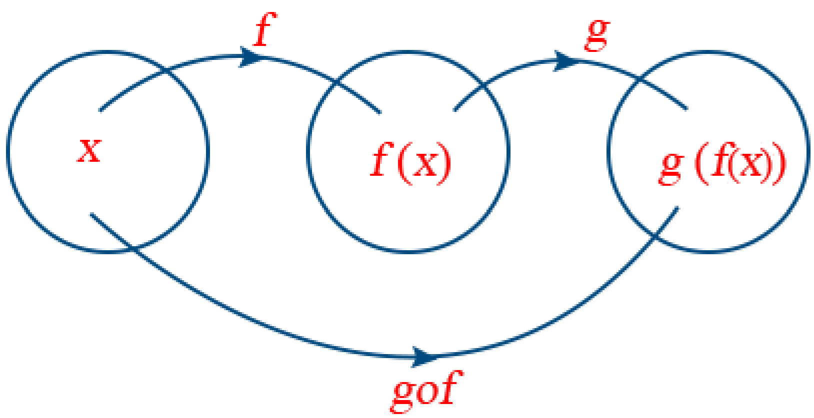
❖ Example :- Let, $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = 5$, $\forall x \in \mathbb{R}$.

G) **Identity function :-** A function $f: A \to A$ is defined to be Identity function if each element of A is mapped on itself. It is denoted by I_A.
i.e. $I_A: A \to A$ is given by $I_A(x) = x, \forall x \in A$.

- ✓ Note that identity map is a bijective mapping.

3. Composition of Function :-

Let, $f: A \to B$ *and* $g: B \to C$ be two functions. Then the function $g \circ f: A \longrightarrow C$ defined by $(g \circ f)(x) = g(f(x)), \forall x \in A$ is called the composition of f *and* g.



- ❖ Example :- Let, $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ where $f(x) = \sin x$ *and* $g(x) = x^2$. Then, $(g \circ f)(x) = g(f(x)) = g(\sin x) = (\sin x)^2$.

➢ Properties of composition of functions :-

a) The composition of functions is associative.
b) The composition of functions, in general, is not commutative.
c) The composition of two bijective functions is bijective.
d) The composition of any function with the identity function is the function itself.
e) Let, $f: A \to B$ *and* $g: B \to A$ be two functions such that $g \circ f = I_A$. Then f is an injection and g is a surjection.
f) Let, $f: A \to B$ *and* $g: B \to A$ be two functions such that $f \circ g = I_B$. Then g is an injection and f is a surjection.

4. Inverse of a function :-

Let , $f: A \to B$ be a bijective function.. Then a function $g: B \to A$ which associates each element $y \in B$ to a unique element $x \in A$ such that $f(x) = y$ is called the inverse of f.
The inverse of f is generally denoted by f^{-1}.

- Properties of inverse of function :-

a) The inverse of bijective mapping is unique.
b) The inverse of bijective mapping is bijective.
c) $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
d) Let, $f: A \to B$ and $g: B \to A$ be two functions such that both $g \circ f$ and $f \circ g$ are identity mappings on A and B respectively. Then $g^{-1} = f$.

Prepared by -

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