## ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

# CLASS 8 <br> STUDY MATERIAL 2 <br> Playing with numbers 

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## PLAYING WITH NUMBERS

## Synopsis

- General form of a number: The general form of a number abc is $a b c=a \times 100+b \times 10+c$.


## - Divisibility Rules:

| $\begin{array}{c}\text { Divisibility } \\ \text { factor }\end{array}$ | Conditions | $\begin{array}{c}\text { Example }\end{array}$ |  |
| :---: | :--- | :--- | :--- |
| 2 | $\begin{array}{l}\text { The last digit is } 0 \text { or an } \\ \text { even number. }\end{array}$ | $\begin{array}{l}9340 \\ 3456\end{array}$ | $\begin{array}{l}0 \text { (Last digit } 0 \text { ) } \\ 6 \text { (Last digit is an even number) } \\ \therefore 9340 ~ \& ~ 3456 ~ a r e ~ d i v i s i b l e ~ b y ~\end{array}$ |$]$


| Divisibility factor | Conditions | Example |  |
| :---: | :---: | :---: | :---: |
| 9 | The sum of all the digits of the number is divisible by 9 . | 6048 | $(6+0+4+8)+9=18+9=2$ <br> $\therefore 6048$ is divisible by 9 . |
| 10 | The last digit is 0 . | 9310 | 0 (Last digit is 0 ) <br> $\therefore 9310$ is divisible by 10 . |
| 11 | The difference of the sum of the digits in even places and the sum of the digits in odd places is 0 or is divisible by 11 . | $\begin{aligned} & 1364 \\ & 3729 \\ & \\ & 25176 \end{aligned}$ | $\begin{aligned} & ((3+4)-(1+6))=0 \\ & ((7+9)-(3+2))=11 \end{aligned}$ <br> $\therefore 1364$ and 3729 are divisible by 11 $((5+7)-(2+1+6))=3$ <br> $\therefore 25176$ is not divisible by 11 . |
| 12 | The number is divisible by both 3 and 4 . | $648$ <br> 916 | $(6+4+8=18 \text { and }$ $\text { also } 48+4=12 \text { ) }$ <br> $\therefore 648$ is divisible by 12 . $\begin{aligned} & (9+1+6=16 \text { and } \\ & 16+4=4 \end{aligned}$ <br> $\therefore 916$ is not divisible by 12 as it is not divisible by 3 . |

18 Z is a three digit even number less than 200, and more than 190 that is divisible by 7 . Find $Z$.
(A) 196
(B) 193
(C) 199
(D) 197

19 Identify the even prime number.
(A) 132
(B) 102
(C) 284
(D) 2

20 How many prime numbers are there between 100 and 200?
(A) 25
(B) 19
(C) 21
(D) 20
21. Identify a factor of 261111 .
(A) 8
(B) 3
(C) 4
(D) 2

22
30 is expressed in terms of prime numbers. Which of the expressions is incorrect?
(A) $30=12+18$
(B) $30=11+19$
(C) $30=13+17$
(D) $30=2 \times 3 \times 5$

23 Identify the incorrect statement.
(A) The sum of two odd numbers is always odd.
(B) The product of two odd numbers is always odd.
(C) The sum of two even numbers is always even.
(D) The product of two even numbers is always even.
24) $x^{2}+x+11$ is a prime number. Which of the following is not a value of ' $x$ '?
(A) 1
(B) 3
(C) 4
(D) 2

25 If $a-b$ is a multiple of 7 , which of the following is also a multiple of 7 ?
(A) $\mathrm{b}-\mathrm{a}$
(B) $a b$
(C) $a+b$
(D) $\frac{a+b}{2}$

26 The units digit of a two digit number is 3 times the tens digit. If the digits are reversed, the resulting number is 36 more than the original number. What is the original number?
(A) 26
(B) 36
(C) 93
(D) 39

27 If ' $r$ ' and ' $s$ ' are both odd integers, which of the following must be an even integer?
(A) $\frac{\mathrm{r}+\mathrm{s}}{2}$
(B) 2 rs
(C) $\frac{\mathrm{rs}}{2}$
(D) 3 rs

28 When ' $n$ ' is divided by 5 , the remainder is 4. When 'n' is divided by 4, the remainder is 3 . If $0<n<100$, what is a possible value of ' n '?
(A) 9
(B) 14
(C) 19
(D) 24

29 If ' $n$ ' is an even integer, which of the following must be an odd integer?
(A) $3 n-2$
(B) $n-2$
(C) $3(\mathrm{n}+1)$
(D) $\frac{n}{3}$
30) If $a=4 b+26$, and ' $b$ ' is a positive integer, which of the following does not divide ' a '?
(A) 2
(B) 5
(C) 4
(D) 6
31) If $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d are consecutive multiples of 5 and $\mathrm{a}<\mathrm{b}<\mathrm{c}<\mathrm{d}$, what is the value of $(\mathrm{a}-\mathrm{c})(\mathrm{d}-\mathrm{b})$ ?
(A) 100
(B) -100
(C) 25
(D) -25
(32) The remainder when ' $x$ ' is divided by 5 equals the remainder when ' $x$ ' is divided by 4 . Which of the following could not be a value of ' $x$ '?
(A) 20
(B) 24
(C) 21
(D) 22
33. If ' $q$ ' is an integer between 50 and 70 which can be expressed as $7 \mathrm{j}+3$, for some integer $j$ what is a possible value of ' $q$ '?
(A) 59
(B) 53
(C) 64
(D) 68
(34) What is the remainder when the sum of three consecutive even integers is divided by 6 ?
(A) 4
(B) 1
(C) 0
(D) 3
(35) How many numbers between 1 and 200 are multiples of 5 and are divisible by 3 ?
(A) 100
(B) 50
(C) 95
(D) 93

36 Find the value of $P, Q, R$ and $S$ respectively. PQRS
$\times 9$
SRQP
(A) $1,0,8,9$
(B) $8,1,0,9$
(C) $1,0,9,8$
(D) $1,9,0,8$

37 Find the value of $P, Q, R$ and $S$ respectively. R S

Q R S

+ Q PR S

$$
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$$

(A) $6,1,7,3$
(B) 1,7,3,6
(C) $7,1,6,3$
(D) 1,6,3, 7

38 Find $\mathrm{D}+\mathrm{E}+\mathrm{F}$ from the following.

$$
D E F \times D=10 D F
$$

(A) 12
(B) 4
(C) 3
(D) 5

## Previous Contest Questions

1 X $X$ is the sum of the digits of a 3 digit number subtracted from the number. Which of the following is true?
(A) X is divisible by 5 .
(B) $X$ is not divisible by 6 .
(C) X is divisible by 9 .
(D) $X$ is not divisible by 9 .

2 If $X+X+X=Y X$, find the respective values of $X$ and $Y$.
(A) 6,1
(B) 5,1
(C) 1,5
(D) 1,6
3. Which of the following is the usual form of $100 \times 8+10 \times 5+9$ ?
(A) 985
(B) 895
(C) 809
(D) 859
4. Which of the following integers has the most divisors?
(A) 88
(B) 91
(C) 99
(D) 101
5. Which of the following numbers is exactly divisible by 99 ?
(A) 114345
(B) 135792
(C) 3572404
(D) 913464

6 Which is the expanded form of 407?
(A) $4 \times 100+7 \times 10$
(B) $4 \times 100+0 \times 10+7 \times 1$
(C) $4 \times 10+7$
(D) $4 \times 10+0 \times 10+7 \times 1$

7 In the product $B A \times B 3=57 A$, what are the respective positional values of $B$ and A?
(A) 6,7
(B) 5,2
(C) 7,4
(D) 2,5

8 In which of the following pairs of numbers is it true that their sum is 11 times their product?
(A) $1,1 / 11$
(B) $1,1 / 10$
(C) $1,1 / 12$
(D) 1,10
9. $X$ is a prime number greater than 50 but less than 70 . What is the greatest possible value of $X$ ?
(A) 53
(B) 61
(C) 67
(D) 59
10. M is a composite number between 70 and 90 . Which of the following is the greatest possible value of $M$ ?
(A) 70
(B) 88
(C) 72
(D) 90
(11) Which of the following is true?
(A) 7 is the least composite number.
(B) 53 is the only prime number between 50 and 60 .
(C) 97 is the only prime number between 90 and 100.
(D) 1 is the least prime number.

12 Which of the given statements is false?
(A) 47 is the largest prime between the numbers 1 and 50.
(B) 89 is the only prime between 85 and 95.
(C) 99 is the largest composite number less than 100.
(D) There are 10 prime numbers between 1 and 20 .

## SOLUTIONS

## [完 Multiple Choice Questions

1. (C) The last digit is 5 , the number is divisible by 5 .

Since $311636-10=311626+7$ $=44568$, the number is divisible by 7 also.
2. (D) A number divisible by 3 and 4 is divisible by 12 .
3. (A) Adding the digits at odd places of the given number $653^{\circ} 47$, we get 13. Now if the number is divisible by 11, the sum of digits at even places should also be 13 , so that $13-13=0$ is divisible by 11 .

We have $7+{ }^{*}+5=13$.
$\therefore *=13-12=1$ is the required value.
4. (D) 1001 is divisible by 13 , as 1001 $=13 \times 77$.
5. (A)
6. (B)
7. (A)
8. (A)
9. (B) The prime number after 32 is 37 which is got by adding 5 to 32 .
10. (C) $153=3 \times 51$. So, 153 is not prime.

IIFPrevious Contest Questions

1. (C) Consider an example 452
$4+5+2=11$ is the sum of the digits.
$\mathrm{X}=452-11=441$
$4+4+1=9$ is divisible by 9 .
So, option (C) is correct.
2. (B) $\mathrm{X}+\mathrm{X}+\mathrm{X}=\mathrm{YX}$

Consider 5, 1 for the respective values of X and Y .
Then $5+5+5=15$ is correct.
3. (D) $100 \times 8+10 \times 5+9=859$
4. (A) 57. (D) 6. (B) 7. (D) 8. (B)
9. (C) The prime numbers that lie between 50 and 70 are 53,59,61 \& 67 .
Therefore, the greatest possible value of X is 67 .
10. (B) The composite numbers between 70 and 90 are 72, 74, $7576,77,78$, $80,81,82,84,85,86,87$ and 88 . Thus, $\mathrm{M}=88$.
11. (C) The only prime number between 90 and 100 is 97 .
11. (B) $147=7 \times 21$ is a composite number.
12. (D) The prime numbers between 30 and 60 are $31,37,41,43,47,53$ and 59. The greatest of these is 59.
13. (D) The statement in option (D) is true according to the test of divisibility by 8 .
14. (C) According to the problem, a fivedigit number between 10000 and 10010 divisible by 5 is 10005 .
15. (B) $672 \div 8=84$
$\therefore 505672$ is a multiple of 8 .
16. (B) 17. (D) 18. (A) 19. (D)
20. (C) The prime numbers between 100 and 200 are $101,103,107,109$, $113,127,131,137,139,149,151$, $157,163,167,173,179,181,191$, 193, 197 and 199 which are 21 in number.
21. (B)
22. (A)
23. (A)
24. (B) For $x=3, x^{2}+x+11=9+3+11$ $=33$ is not a prime number.
25. (A) 26. (A) 27. (B) 28. (C)
29. (C) Consider $\mathrm{n}=10$ (Even)
$\Rightarrow 3(\mathrm{n}+1)=3(11)=33 \rightarrow$ Odd
30. (C) $\mathrm{a}=4 \mathrm{~b}+26$

Consider $\mathrm{b}=1$. Then $\mathrm{a}=4(1)+26$ $=30$, which is divisible by 2,5 and 6 , but not 4 .
Hence option (C) is the answer.
31. (B) 32. (B) 33. (A) 34. (C) 35. (D)
36. (A) Consider $\mathrm{P}=1$ and $\mathrm{S}=9$.

Then 1QR9

$$
\begin{array}{r}
\times 9 \\
\hline 9 \mathrm{RQ1} \\
\hline
\end{array}
$$

$\mathrm{Q}=0$ (else there will be regrouping in the thousands place) and $\mathrm{R}=8$.

1QR9
So, $\frac{\frac{\times 9}{9 \mathrm{R} \mathrm{Q} \mathrm{1}}}{\text { (A) }}$ is correct.

## SELF ASSESSMENT EXERCISE

## MATHEMATICAL REASONING

1. The number $\left(10^{n}-1\right)$ is divisible by 11 for
$\qquad$ _-
(A) $n \in N$
(B) Odd values of $n$
(C) Even values of $n$
(D) $n$ is the multiple of 11
2. The values of $A$ and $B$ in the given addition respectively are $\qquad$ -.
(A) 4,7
(B) 7,4
(C) 5,6

23 A
(D) 6,5
3. The greatest value that must be given to $x$ so that the number $7713 \times 8$ is divisible by 4 is $\qquad$ _.
(A) 1
(B) 6
(C) 8
(D) 7
4. If $1 A$, where $A$ and $B$ are single digit $\begin{array}{r}\times \quad A \\ \hline B 6 \\ \hline\end{array}$
numbers, such that $B-A=3$, then the values of $A$ and $B$ respectively are $\qquad$ .
(A) 4, 5
(B) 9,6
(C) 5,4
(D) 6,9
5. Suppose that the division $\mathrm{N} \div 5$ leaves a remainder of 4 and the division $\mathrm{N} \div 2$ leaves a remainder of 1 . What must be the ones digit of N ?
(A) 7
(B) 3
(C) 9
(D) 4
6. Given that the number 148101a095 is divisible by 11 , where $a$ is single digit number, what are the possible values of $a$ ?
(A) 4
(B) 1
(C) 7
(D) 9
7. The largest natural number by which the product of three consecutive even natural numbers is always divisible, is $\qquad$ _.
(A) 16
(B) 24
(C) 48
(D) 96
8. If in a number, difference between the sum of digits at its odd places and that of digits at the even places is given 0 , then the number is divisible by $\qquad$ -.
(A) 7
(B) 9
(C) 5
(D) 11
9. A 5-digit number $x y 235$ is divisible by 3 such that $x+y<5$, where $x$ and $y$ are single digits, then possible values of $(x, y)$ are $\qquad$ _.
(A) $(1,1)$ or $(4,0)$
(B) $(1,1)$ or $(2,0)$
(C) $(1,1)$ or $(0,2)$
(D) $(2,0)$ or $(0,2)$
10. If $1 A B$ and there is no carry on addition, $\begin{array}{r}+C C A \\ +697 \\ \hline\end{array}$
then the value of $B$ is $\qquad$ .
(A) 5
(B) 4
(C) 3
(D) 2
11. If $N$ divided by 5 leaves a remainder of 3 , then one's digit of $N$ must be $\qquad$ .
(A) Either 3 or 6
(B) Either 3 or 8
(C) Either 8 or 1
(D) Either 8 or 6
12. Given that the number $67 y 19$ is divisible by 9 , where $y$ is a single digit, what is the least possible value of $y$ ?
(A) 3
(B) 9
(C) 7
(D) 4
13. A 3 -digit number ' $c b a$ ' is divisible by 3 if (A) $a+2 b+c$ is divisible by 3
(B) $2 a+b+c$ is divisible by 3
(C) $a+b+2 c$ is divisible by 3
(D) $a+b+c$ is divisible by 3
14. If $A B$, then the value of $B$ is $\qquad$ $\begin{array}{r}\times A 3 \\ \hline 57 B \\ \hline\end{array}$
(A) 5
(B) 2
(C) 0
(D) 4
15. In a division, the divisor is 12 times the quotient and 5 times the remainder. If the remainder is 48 , then dividend is $\qquad$ .
(A) 240
(B) 576
(C) 4800
(D) 4848

## ACHIEVERS SECTION (HOTS)

16. Which of the following statements is INCORRECT?
(A) All even natural numbers which are divisible by 3 are also divisible by 6 .
(B) If a natural number is divisible by 21, then it is divisible by both 3 and 7 .
(C) If $A B \times 4=192$, then $A+B=10$
(D) A number of the form $14 N+2$ leaves the remainder 2 when divided by 7 .
17. Fill in the blanks.
(i) If sum of 3-digit numbers $x y z, y z x$ and $z x y$ is divided by $(x+y+z)$, then quotient is $\qquad$ -
(ii) The difference between 2-digit numbers $a b$ and $b a$, (where $a>b$ ) is divided by 3 . The quotient is $\qquad$ _.
(iii) Sum of a 2-digit number and the number obtained by reversing its digits is always divisible by $\qquad$ R -

|  | $\mathbf{P}$ | $\mathbf{Q}$ | $\mathbf{R}$ |
| :--- | :--- | :--- | :--- |
| (A) | 111 | $3(a+b)$ | 11 |
| (B) | 99 | $(a+b)$ | 7 |
| (C) | 111 | $3(a-b)$ | 11 |
| (D) 99 | $(a-b)$ | 3 |  |

18. Match the following.

## Column-I

(P) If $213 \times 27$ is divisible by 9 ,
then $x=$
(Q) If $2415 x$ is
divisible by 6 ,
then $x=$
(R) If $23245 x$ is divisible by
4 and 3 ,
then $x=$
(S) If $7251 \times 93$ is divisible by 11 ,
then $x=$
(A) (P) $\rightarrow$ (iii); (Q) $\rightarrow$ (ii); (R) $\rightarrow$ (iv); (S) $\rightarrow$ (i)
(B) (P) $\rightarrow$ (ii); (Q) $\rightarrow$ (iv); (R) $\rightarrow$ (i); (S) $\rightarrow$ (iii)
(C) (P) $\rightarrow$ (iii); (Q) $\rightarrow$ (iv); (R) $\rightarrow$ (i); (S) $\rightarrow$ (ii)
(D) (P) $\rightarrow$ (ii); (Q) $\rightarrow$ (iii); (R) $\rightarrow$ (i); (S) $\rightarrow$ (iv)
19. How many 5-digit numbers of the form $A A B A A$ is divisible by 33 ?
(A) 1
(B) 3
(C) 0
(D) infinite
20. Find the value of $A, B$ and $C$ respectively.
(i)
(ii)

## (i) <br> $$
\begin{array}{r} A 83 \\ \times \quad \mathrm{C} 9 \\ \hline A 04 A \\ +15 \mathrm{BBO} \\ \hline \text { CCA0A } \\ \hline \end{array}
$$

(ii) $\qquad$
(A) $2,6,7$

9, 5, 2
(B) $6,7,2$

4, 3, 1
(C) 7, 5, 2

9, 2, 5
(D) 7, 6, 2

4, 1, 3


ANSWERS: 1-C, 2-A, 3-C, 4-D, 5-C, 6-A, 7-C, 8-D, 9-B, 10-C, 11-B, 12-D, 13-D, 14-A, 15-D, 16-C, 17-C, 18-C, 19-B, 20-D

