ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

CLASS 8

SUBJECT :Arithmetic **STUDY MATERIAL 4**

Sets

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SETS

INTRODUCTION

In everyday life, we come across different collections of objects. For example, a herd of sheep, a cluster of stars, a posse of policemen, etc. In mathematics, we call such collections as 'Sets'. The objects are referred to as the elements of the sets.

Set

A set is a well-defined collection of objects. Let us understand what we mean by a well-defined collection of objects.

We say that a collection of objects is well-defined if there is some reason or rule by which we can say whether a given object of the universe belongs to or does not belong to the collection.

To understand this better, let us look at some examples.

1. Let us consider the collection of odd natural numbers less than 10. In this example, we can definitely say what the collection is. The collection comprises numbers such as 1, 3, 5, 7, and 9.
2. Let us consider the collection of intelligent boys in a class. In this example, we cannot say precisely which boy of the class belong to our collection. Hence, this collection is not well-defined.

Hence, the first collection is a set, whereas the second collection is not a set.

In the first example given above, the set of the first 5 odd natural numbers can be represented as:

$A = \{1, 3, 5, 7, 9\}$.

Elements of a Set

The objects in a set are called its elements or members. We usually denote the sets by capital letters A, B, C or X, Y, Z, etc.

If a is an element of a set A, then we say that a belongs to A and we write as $a \in A$.

If a is not an element of A, then we say that a does not belong to A and we write as $a \notin A$.

Some sets of numbers and their notations:

N = The set of all natural numbers = $\{1, 2, 3, 4, 5, \ldots\}$

W = The set of all whole numbers = $\{0, 1, 2, 3, 4, 5, \ldots\}$

Z = The set of all integers = $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$

Q = The set of all rational numbers = $\left\{x / x = \frac{p}{q}, \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$

Cardinal Number of a Set

The number of elements in a set A is called its cardinal number. It is denoted by $n(A)$. A set which has finite number of elements is a finite set and a set which has infinite number of elements is an infinite set.

For example,

1. The set of vowels in English alphabet is a finite set.
2. The set of all persons living on the earth is a finite set.
3. The set of all natural numbers is an infinite set.

****Proper subsets ,Venn Diagrams and formula on cardinality of Sets can be skipped**

Representation of Sets

We represent sets by the following methods:

1. **Roster method:** In this method, a set is described by listing out all the elements in the set.

   For example,

   (a) Let V be the set of all vowels in English alphabet. Then, we represent V as, $V = \{a, e, i, o, u\}$.

   (b) Let E be the set of all even natural numbers less than 12. Then, we represent the set E as, $E = \{2, 4, 6, 8, 10\}$.

2. **Set-builder method:** In this method, a set is described by using some property common to all its elements.

   For example,

   (a) Let V be the set of all vowels in English alphabet. Then, we represent the set V as, $V = \{x / x$ is a vowel in English alphabet$\}$.

   (b) Let P be the set of all even numbers less than 100.

   Then, we represent the set P as, $P = \{x/x \leq 100$ and x is an even number$\}$.

Some Simple Definitions of Sets

1. **Empty set or null set:** A set with no elements in it is called an empty set (or) void set (or) null set. It is denoted by { } or ϕ. (read as phi)

   For example,

   (a) Set of all natural numbers less than 1.

   (b) Set of all woman Chief Ministers of Andhra Pradesh.

2. **Singleton set:** A set consisting of only one element is called a singleton set.

   For example,

   (a) The set of all even prime numbers is a singleton set as 2 is the only even prime number.

   (b) The set of all positive integers which are factors of every natural number is a singleton set as 1 is the only integer which is a factor of all natural numbers.

3. **Disjoint sets:** Two sets A and B are said to be disjoint, if they have no elements in common.

   For example,

   (a) Sets $A = \{2, 5, 6, 7, 10\}$ and $B = \{1, 3, 4, 8, 9\}$ are disjoint as they have no element in common.

   (b) Sets $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 5, 7\}$ are not disjoint as they have some common elements 2 and 5.

4. **Subset:** Let A and B be two sets. If every element of set A is also an element of set B, then A is said to be a subset of B. We write this symbolically as $A \subseteq B$.

   For example,

   (a) Set $A = \{1, 2, 5, 7\}$ is a subset of set $B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

   (b) Set of all primes is a subset of the set of all natural numbers.

Notes (i) Empty set is a subset of every set.
(ii) Every set is a subset of itself.
(iii) If a set A has n elements, then the number of subsets of A is 2^n.

5. **Proper subset:** If A is a subset of B and $A \neq B$, then A is called a proper subset of B and is denoted by $A \subset B$. If $A \subset B$, then $n(A) < n(B)$.

Notes (i) If a set A has n elements, then the number of non-empty subsets of A is $2^n - 1$.
(ii) If a set A has n elements, then the number of proper subsets of A is $2^n - 1$.

EXAMPLE 10.1

If $N = \{\alpha, \beta, \gamma\}$, then find the number of all possible proper subsets of N.

SOLUTION

If a set has n elements, then the number of proper subsets $= 2^n - 1$.
$= 2^3 - 1 = 8 - 1 = 7$

6. **Universal set:** A set which consists of all the sets under consideration or discussion is called the universal set. It is usually denoted by 'U' or 'μ'.

For example:

Let $A = \{1, 2, 3, 4\}$, $B = \{2, 5, 7\}$ and $C = \{3, 4, 8, 10\}$.

Here, μ can be $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

7. **Complement of a set:** Let μ be the universal set and $A \subset \mu$. Then, the set of all those elements of μ which are not in set A is called the complement of the set A. It is denoted by A' or A^c.

$A' = \{x/x \in \mu \text{ and } x \notin A\}$

For example,

(a) Let $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{2, 3, 5, 7\}$.

Then, $A' = \{1, 4, 6, 8, 9, 10\}$

(b) Let $\mu = N$ (natural numbers set) and E be the set of all even natural numbers.

Then, E' will be the set of all odd natural numbers, i.e., $E' = \{1, 3, 5, 7, 9, 11, \ldots\}$.

Notes (i) A and A' are disjoint sets.
(ii) $\mu' = \phi$ and $\phi' = \mu$

Operations on Sets

1. **Union of sets:** Let A and B be two sets. Then, the union of A and B, denoted by $A \cup B$, is the set of all those elements which are either in A or in B or in both A and B, i.e., $A \cup B = \{x/x \in A \text{ or } x \in B\}$.

For example,

Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8, 9, 10\}$.

Then, $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 9, 10\}$

Notes
1. If $A \subseteq B$, then $A \cup B = B$.
2. $A \cup \mu = \mu$ and $A \cup \phi = A$

2. Intersection of sets: Let A and B be two sets. Then, the intersection of A and B, denoted by $A \cap B$, is the set of all those elements which are common to both A and B.

i.e., $A \cap B = \{x/x \in A \text{ and } x \in B\}$

For example,

Let $A = \{1, 2, 3, 5, 7\}$ and $B = \{2, 4, 7, 10\}$. Then, $A \cap B = \{2, 7\}$.

Notes
1. If A and B are disjoint sets, then $A \cap B = \phi$.
2. If $A \subseteq B$, then $A \cap B = A$.
3. $A \cap \mu = A$ and $A \cap \phi = \phi$

EXAMPLE 10.2

Given that $\mu = \{\text{Whole numbers up to } 36\}$, $A = \{3, 6, 9, ..., 36\}$, and $B = \{4, 8, 12, ..., 36\}$. Find $n\,(A \cap B)'$.

SOLUTION

$\mu = \{0, 1, 2, 3, ..., 36\}$

$A = \{\text{Multiples of 3 from 3 to 36}\}$

$B = \{\text{Multiples of 4 from 4 to 36}\}$

$A \cap B = \{\text{common multiples of 3 and 4}\} = \{12, 24, 36\}$

$\therefore n\,(A \cap B)' = n(\mu) - n(A \cap B) = 37 - 3 = 34$

3. Difference of sets: Let A and B be two sets, then the difference $A - B$ is the set of all those elements which are in A but not in B, i.e., $A - B = \{x/x \in A \text{ and } x \notin B\}$.

For example:

Let $A = \{1, 2, 4, 5, 7, 8, 10\}$ and $B = \{2, 3, 4, 5, 6, 10\}$. Then, $A - B = \{1, 7, 8\}$ and $B - A = \{3, 6\}$.

Notes
1. $A - B \neq B - A$ unless $A = B$.
2. For any set A, $A' = \mu - A$.

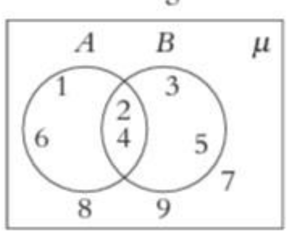
Venn Diagrams

We also represent sets pictorially by means of diagrams called Venn diagrams. In Venn diagrams, the universal set is usually represented by a rectangular region and its subsets by closed regions inside the rectangular region. The elements of the set are written in the closed regions.

For example:

Let $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6\}$, and $B = \{2, 3, 4, 5\}$.

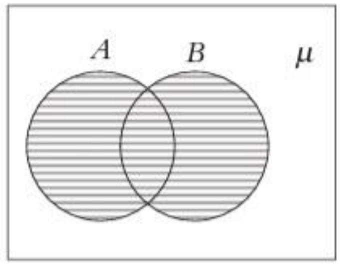
We represent these sets in the form of Venn diagram as follows:



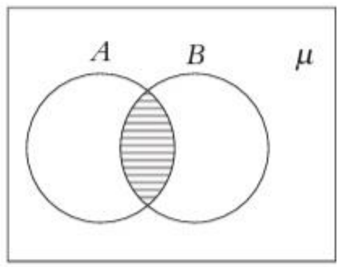
We can also represent the sets in Venn diagrams by shaded regions:

For example,

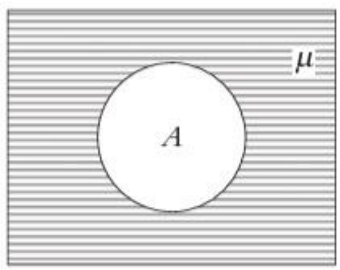
(i) Venn diagram of $A \cup B$, where A and B are two overlapping sets, is



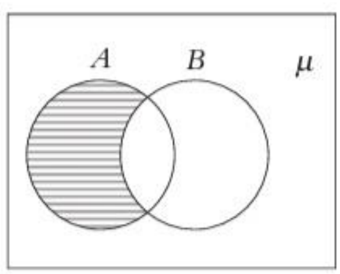
(ii) Let A and B be two overlapping sets. Then, the Venn diagram of $A \cap B$ is



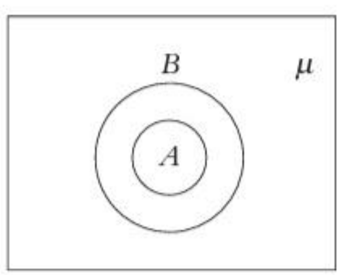
(iii) For a non-empty set A, Venn diagram of A' is



(iv) Let A and B be two overlapping sets. Then, the Venn diagram of $A - B$ is



(v) Let A and B be two sets such that $A \subset B$. We can represent this relation using Venn diagram as follows:



Some Formulae on the Cardinality of Sets

Set $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2, 4, 8, 16\}$

Then, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 16\}$ and $A \cap B = \{2, 4\}$

In terms of the cardinal numbers, $n(A) = 7$, $n(B) = 4$, $n(A \cap B) = 2$, and $n(A \cup B) = 9$.

Hence, $n(A) + n(B) - n(A \cap B) = 7 + 4 - 2 = 9 = n(A \cup B)$.

We have the following formula:

For any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

EXAMPLE 10.3

If $n(A) = 4$, $n(B) = 6$, and $n(A \cup B) = 8$, then find $n(A \cap B)$.

SOLUTION

Given, $n(A) = 4$, $n(B) = 6$, and $n(A \cup B) = 8$.

We know that, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Hence, $8 = 4 + 6 - n(A \cap B) \Rightarrow n(A \cap \mathrm{B}) = 10 - 8 = 2$

EXAMPLE 10.4

If $n(A) = 8$, $n(B) = 6$, and the sets A and B are disjoint, then find $n(A \cup B)$.

SOLUTION

Given, $n(A) = 8$ and $n(B) = 6$.

A and B are disjoint $\Rightarrow A \cap B = \phi \Rightarrow n(A \cap B) = 0$.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) = 8 + 6 - 0 = 14$

EXAMPLE 10.5

There are 40 persons in a group; four of them can speak neither English nor Hindi. The sum of the number of persons who can speak English and that of those who can speak Hindi is 44. Find the number of those who can speak both English and Hindi.

SOLUTION

$n(E \cup H) + n(E' \cap H') = 40$

$n(E' \cap H') = 4 \Rightarrow n(E \cup H) = 36$

$\therefore n(E) + n(H) = 44$

$\Rightarrow n(E \cup H) + n(E \cap H) = 44$

$\therefore n(E \cap H) = 44 - 36 = 8$

Points to Remember

- A set is a well-defined collection of objects.
- Empty set is a subset of every set.
- Every set is a subset of itself and it is called improper subset.
- If a set A has n elements, then the number of subsets of A is 2^n.
- If a set A has n elements, then the number of proper subsets of A is $2^n - 1$.
- If $A \subseteq B$, then $A \cup B = B$.
- If A and B are disjoint sets, then $A \cap B = \phi$.
- If $A \subseteq B$, then $A \cap B = A$.
- $A - B \neq B - A$ unless $A = B$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

QUESTION BANK

CONCEPT APPLICATION

Level 1

Directions for questions 1 to 15: Select the correct answer from the given options.

1. If $\mu = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{2, 5, 8\}$, then find $n(A')$.

   (a) 3 (b) 5
   (c) 4 (d) 6

2. If X = {Non-prime numbers} and Y = {Non-composite numbers}, then $n(X \cap Y)$ = ______.

   (a) 0 (b) 2
   (c) 1 (d) 3

3. If P = {Factors of 6} and Q = {Factors of 12}, then $n(P \cup Q)$ = ______.

   (a) 4 (b) 8
   (c) 10 (d) 6

4. Which of the following is/are true?

   (a) If $M = N$, then $M' = N'$.
   (b) If $M' = N'$, then $M = N$.
   (c) Both (a) and (b)
   (d) Neither (a) nor (b)

5. If $n(A) = 10$, $n(B) = 20$, and $n(A \cup B) = 26$, then $n(A \cap B) =$ ______.

(a) 4 (b) 2

(c) 6 (d) 8

Directions for questions 6 to 9: These questions are based on the following data.

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B = \{2, 4, 6, 8\}$

$C = \{1, 3, 5, 7, 9\}$

6. $(A \cup (B \cup C)) =$ ______

(a) A (b) B

(c) $B \cup C$ (d) Both (a) and (c)

7. $A \cap B =$ ______

(a) B (b) C

(c) $B \cap C$ (d) $A \cap C$

8. $A \cap C =$ ______

(a) B (b) C

(c) $B \cap C$ (d) $A \cap B$

9. $(A \cap B) \cup (A \cap C) =$ ______

(a) A (b) $A \cup B$

(c) $A \cup C$ (d) All of these

Directions for question 10 to 15: Select the correct answer from the given options.

10. Which of the following is a singleton set?

(a) $\{0\}$

(b) $\{\phi\}$

(c) $\{x{:}x$ is an even prime number.$\}$

(d) All of these

11. If $A = \{1, 3, 5, 2, 4, 1, 3, 5, 7, 8, 9, 6, 10\}$, then $n(A) =$ ______.

(a) 13 (b) 8

(c) 10 (d) 9

12. If $A = \{x : x + 5 = 5\}$, then $n(A) =$ ______.

(a) 0 (b) 1

(c) 5 (d) 2

13. If $X = \{1, 2, 3, 4\}$, then which of the following is a correct statement?

(a) $4 \in X$ (b) $4 \subset X$

(c) $4 \notin X$ (d) $4 \not\subset X$

14. $P = \{x{:}x$ is a multiple of 4, $x < 20\}$ and $Q = \{x{:}x$ is a multiple of 6, $x < 30\}$. Find $n(P) + n(Q)$.

The following steps are involved in solving the above problem. Arrange them in sequential order.

(A) $P = \{4, 8, 12, 16\}$ and $Q = \{6, 12, 18, 24\}$

(B) $\Rightarrow n(P) = 4$ and $n(Q) = 4$

(C) $n(P) + n(Q) = 4 + 4 = 8$

(a) CBA (b) ACB

(c) BAC (d) ABC

15. In a group of 50 students, 30 like Basketball, 20 like football and 10 like neither of the games. How many like both the games?

The following steps are involved in solving the above problem. Arrange them in sequential order.

(A) $n(B \cap F) = 50 - 40 = 10.$

(B) $\Rightarrow 40 = 30 + 20 - n(B \cap F)$

$\Rightarrow 40 = 50 - n(B \cap F)$

(C) Let $n(B) = 30$, $n(F) = 20$ and $n(B \cup F) = 50 - 10 = 40$.

(D) We know that $n(B \cup F) = n(B) + n(F) - n(B \cap F)$.

(a) CDBA (b) CBDA

(c) ADBC (d) BCDA

Directions for questions 16 to 19: Match Column A with Column B.

Column A	Column B
16. If $A = \{x{:}x<11, x \in W\}$ and $B = \{x{:}x<11, x \in N\}$	(a) A
17. If $x = \{\}$, then $n(Px))$	(b) B
18. The cardinal number of the set containing the letter of the word 'GOOGLE'	(c) 1
19. If $A \subset B$, then $A \cap B$	(d) 2
	(e) 3
	(f) 4

Directions for questions 20 to 23: State whether the following statements are true or false.

20. If $A = \{T, I, G, E, R\}$ and $B = \{G, I, N, T, E, R\}$, then $A \cup B$ has 6 elements.

21. The cardinal number of the set containing the letters of the word 'GINGERCOOK' is 8.

22. If $A \subset B$, then $A \cup B = A$.

23. If $X = \{1, 2, 3, \{4, 5\}, 6, \{7, 8, 9\}, 10\}$, then $\{4, 5\} \subset X$.

Level 2

Directions for questions 24 to 27: Select the correct answer from the given options.

24. If A = {Positive perfect squares less than 100} and B = {Positive perfect cubes less than 100}, then find $n(A \cap B)$.

(a) 1 (b) 2
(c) 3 (d) 4

25. If μ = {All prime numbers} and O = {All odd prime numbers}, then find $n(O')$.

(a) 1 (b) 2
(c) 3 (d) More than 3

26. μ = {Two digit perfect squares}

X = {Two digit perfect squares for which sum of digits is a perfect square}

Y = {Two digit perfect squares for which sum of digits is a prime number}

Find $(X \cup Y)'$.

(a) {49} (b) {25}
(c) {36} (d) {64}

27. If $\mu = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $X = \{2, 3, 5, 7\}$ and $Y = \{2, 5, 8\}$. Find $n\,(X' \cup Y')$.

(a) 8 (b) 7
(c) 6 (d) 9

Directions for questions 28 to 31: These questions are based on the following data.

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B = \{2, 4, 6, 8\}$

$C = \{1, 3, 5, 7, 9\}$

28. If P = {Factors of 36} and Q = {Factors of 48}, then find $n(P \cap Q)$.

(a) 6 (b) 5
(c) 7 (d) 8

29. $S = abcdef\ldots z$, μ = {Vowels in S} and B = {Vowels in even positions of S}. Find $n(B')$.

(a) 4 (b) 5
(c) 3 (d) 2

30. If N is a natural number, A = {Factors of N} and B = {Multiples of N}, then $n(A \cap B)$ = ______.

(a) 2 (b) 3
(c) 4 (d) 1

31. If μ = {Natural numbers up to 32}, $C = \{2, 5, 8, 11, \ldots, 32\}$ and $D = \{2, 4, 6, 8, \ldots, 32\}$, then find $n(C \cap D')$.

(a) 3 (b) 4
(c) 6 (d) 5

Directions for questions 32 to 65: Select the correct answer from the given options.

32. E = {Natural numbers up to 30}

X = {Multiples of 2 up to 30}

$Y = \{x/x = 4y + 2, y \in E\}$

Find $n(X \cap Y)$.

(a) 2 (b) 5
(c) 7 (d) 9

33. X = {The units digit of the sum of 10 consecutive natural numbers}. Find X.

(a) {5} (b) {2}
(c) {3} (d) {0}

34. If $A = \{1, 2, 3, 4, 8\}$, then which of the following can be concluded?

(a) $8 \in A$ (b) $9 \notin A$
(c) $\{2, 3\} \subset A$ (d) All of these

35. If A and B are two disjoint sets $n(A) + n(B) = 24$, then find $n(A \cup B)$.

(a) 16 (b) 18

(c) 24 (d) Cannot say

36. If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 8, 9\}$, then $(A - B) \cup (B - A) =$ ______.

(a) $\{1, 3, 8, 9\}$ (b) $\{2, 4\}$

(c) A (d) B

37. In the question above, find $(A - B) \cap ((B - A)$.

(a) $\{2, 4\}$ (b) A

(c) B (d) ϕ

38. Which of the following is/are false?

(a) $A - A = \phi$

(b) $A \cup A' = \mu$

(c) Both (a) and (b)

(d) None of these

39. Which of the following is/are true?

(a) $P \cup P' = \mu$

(b) $P \cap P' = \phi$

(c) Both (a) and (b)

(d) None of these

40. If two sets are disjoint, then ______.

(a) they have one element in common

(b) they have 0 as the common element

(c) they have no element in common

(d) they have two elements in common

41. If $Y = \{a, e, \{i, o\}, u\}$, then which of the following is a correct statement?

(a) $\{i, o\} \subset Y$

(b) $\{i, o\} \in Y$

(c) $\{\{i, o\}\} \in Y$

(d) $\{i, o\} \notin Y$

42. If $U = \{x : x$ is an alphabet$\}$ and $C = \{x : x$ is a consonant$\}$, then $C' =$ ______.

(a) $\{a, e, i, o\}$ (b) $\{a, e, i\}$

(c) $\{i, o, u\}$ (d) $\{a, e, i, o, u\}$

43. If $O = \{o\}$, then the number of all possible subsets of O is ______.

(a) 2 (b) 3

(c) 1 (d) 4

44. If $n(A) = 10$, $n(A \cap B) = 5$ and $n(A \cup B) = 35$, then $n(B) =$ ______.

(a) 30 (b) 10

(c) 40 (d) None of these

45. If $X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $Y = \{1, 3, 5, 7, 9\}$, then $X - Y =$ ______.

(a) $\{1, 2, 3, 4, 5\}$

(b) $\{1, 3, 5, 7, 9\}$

(c) $\{0, 2, 4, 6, 8, 10\}$

(d) $\{2, 4, 6, 8, 10\}$

46. Given that A = {Perfect cubes between 10 and 100} and B = {Perfect squares between 10 and 100}. Find $n(A \cap B)$.

(a) 2 (b) 1

(c) 5 (d) 3

ANSWERS

CONCEPT APPLICATION

Level 1

1. (b) 2. (c) 3. (d) 4. (c) 5. (a) 6. (d) 7. (a) 8. (b) 9. (d) 10. (d)
11. (c) 12. (b) 13. (a) 14. (d) 15. (a) 16. (d) 17. (c) 18. (f) 19. (a) 20. True
21. True 22. False 23. False

Level 2

24. (b) 25. (a) 26. (d) 27. (a) 28. (a) 29. (b) 30. (d) 31. (d) 32. (c) 33. (a)
34. (d) 35. (c) 36. (a) 37. (d) 38. (d) 39. (c) 40. (c) 41. (b) 42. (d) 43. (a)
44. (a) 45. (c) 46. (b)

SOLUTIONS

CONCEPT APPLICATION

Level 1

1. $A' = \mu - A$

   $A' = \{1, 3, 4, 6, 7\}$

   $\therefore n(A') = 5$

   Hence, the correct option is (b).

2. 1 is the only number which is neither prime nor composite.

   $\therefore n(X \cap Y) = 1$

   Hence, the correct option is (c).

3. All the factors of 6 are factors of 12.

   $\therefore P$ is a subset of Q. $\Rightarrow P \cup Q = Q$

   $(Q = \{1, 2, 3, 4, 6, 12\}$

   $\therefore n(P \cup Q) = n(Q) = 6.$

   Hence, the correct option is (d).

4. Let U be the universal set.

   $U = M \cup M' = N \cup N'$

   If $M = N$, then $M' = N'$.

   If $M' = N'$, then $M = N$.

   $\therefore$ Options (a) and (b) follow, i.e., Options (c) follows.

   Hence, the correct option is (c).

5. $n(A) + n(B) = n(A \cup B) + n(A \cap B)$

   $10 + 20 = 26 + n(A \cap B)$

   $\therefore n(A \cap B) = 4$

   Hence, the correct option is (a).

6. $B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (1)

   $B \subset A$ and $C \subset A$

   $\therefore A \cup (B \cup C) = A$ (2)

   From Eqs (1) and (2), $A \cup (B \cup C) = A = B \cup C$.

   Hence, the correct option is (d).

7. $B \subset A \Rightarrow A \cap B = B$

   Hence, the correct option is (a).

8. $C \subset A \Rightarrow A \cap C = C$

   Hence, the correct option is (b).

9. $A \cap B = B$; $A \cap C = C$

   $((A \cap B) \cup (A \cap C)) = B \cup C$

   $A = A \cup B = A \cup C = B \cup C$

   Hence, the correct option is (d).

10. $\{0\}$ contains one element, i.e., 0.

    $\{\phi\}$ contains one element, i.e., ϕ.

    $\{x$: x is an even prime number$\} = \{2\}$ contains one element, i.e., 2.

    $\therefore$ All the above are singleton sets.

    Hence, the correct option is (d).

11. $n(A)$ = The number of distinct elements in A.

    $= 10$

    Hence, the correct option is (c).

12. $A = \{x/x + 5 = 5\} \Rightarrow A = \{0\}$

    $(\because x + 5 = 5 \Rightarrow x = 0)$

    $\therefore n(A) = 1$

    Hence, the correct option is (b).

13. $4 \in X$ (since i is an element of X)

    Hence, the correct option is (a).

14. (A), (B), and (C) is the required sequential order.

    Hence, the correct option is (d).

15. (C), (D), (B), and (A) is the required sequential order.

    Hence, the correct option is (a).

16. Option (d): $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and

    $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

    $A - B = \{0, 10\}$ and $n(A - B) = 2$

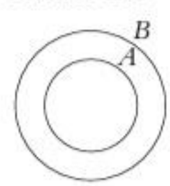
17. Option (c): $n(X) = 0$ ($\because X$ is null set)

    $\therefore n(P(X)) = 2^{n(x)} = 2^0 = 1$

18. Option (f): $A = \{G, O, L, E\}$ is the set containing the letters of the word 'GOOGLE'.

    $\therefore n(A) = 4$

19. Option (a): $\because$ Given $A \subset B$



    $\Rightarrow A \cap B = A$

20. Given $A = \{T, I, G, E, R\}$ and $B = \{G, I, N, T, E, R\}$

$\Rightarrow A \cup B = \{T, I, G, E, R, N\}$

$\Rightarrow n(A \cup B) = 6$

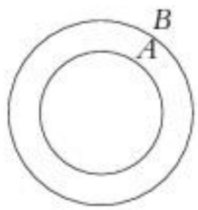
$\therefore$ The given statement is true.

21. $x = \{G, I, N, E, R, C, O, K\}$ is the set containing the letters of the word 'GINGERCOOK'.

$\therefore n(x) = 8$

$\therefore$ The given statement is true.

22. Given $A \subset B$



$\Rightarrow A \cup B = B$

$\therefore$ The given statement is false.

23. Since $\{4, 5\}$ is an element of set X, it is not a subset.

i.e., $\{4, 5\}$ belongs to set X.

$\therefore \{4, 5\}$ is not a subset of X.

$\therefore$ The given statement is false.

Level 2

24. $A = \{1, 4, 9, 16, 25, 36, 49, 64, 81\}$

$B = \{1, 8, 27, 64\}$

$A \cap B = \{1, 64\} \therefore n(A \cap B) = 2$

Hence, the correct option is (b).

25. $O' = \{\text{Set of all even primes}\}$

$\therefore O' = \{2\}$ ($\because$ 2 is the only even prime)

$\therefore n(O') = 1$

Hence, the correct option is (a).

26. $\mu = \{16, 25, 36, 49, 64, 81\}$

$X = \{36, 81\}$

$Y = \{16, 25, 49\}$

$X \cup Y = \{16, 25, 36, 49, 81\}$

$\therefore (X \cup Y)' = \{64\}$

Hence, the correct option is (d).

27. Given, $\mu = \{0, 1, 2, 3, 4 \ldots, 9\}$

$X = \{2, 3, 5, 7\}$ and $Y = \{2, 5, 8\}$

$X' = \{0, 1, 4, 6, 8, 9\}$

$Y' = \{0, 1, 3, 4, 6, 7, 9\}$

$X' \cup Y' = \{0, 1, 3, 4, 6, 7, 8, 9\}$

$n(X' \cup Y') = 8$

Hence, the correct option is (a).

28. $P = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$Q = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

$P \cap Q = \{1, 2, 3, 4, 6, 12\}$

$\therefore n(P \cap Q) = 6$

Hence, the correct option is (a).

29. $E = \{a, e, i\ o, u\}$

The positions of a, e, i, o, and u in S are 1, 5, 9, 15, and 21, respectively.

$\therefore$ Every vowel has an odd position in S.

$\therefore B = \phi$

$\therefore B' = \mu$

$n(B') = n(\mu) = 5$

Hence, the correct option is (b).

30. Every number is a factor and a multiple to itself.

$\therefore n\ (A \cap B) = 1$

Hence, the correct option is (d).

31. $C = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

$D = \{2, 4, 6, 8, \ldots, 32\}$

$D' = \{$All odd natural numbers up to 32$\}$

$C \cap D' = \{5, 11, 17, 23, 29\}$

$\therefore\ n(C \cap D)' = 5$

Hence, the correct option is (d).

32. $E = \{1, 2, 3, \ldots, 30\}$

$X = \{2, 4, 6, \ldots, 30\}$

$Y = \{6, 10, 14, 18, 22, 26, 30\}$

$X \cap Y = Y\ (\because Y \subset X)$

$\therefore\ n(X \cap Y) = 7$

Hence, the correct option is (c).

33. 10 consecutive natural numbers will have their units digits as 0, 1, 2, 3, 4, 6, 7, 8, 9.

$\therefore$ Units digit of their sum = 5

$\therefore\ X = \{5\}$

Hence, the correct option is (a).

34. All the given statements with respect to the set A are true.

Hence, the correct option is (d).

35. $n(A) + n(B) = 24$

A and B are disjoint.

$\therefore\ n(A \cap B) = 0$

$n(A \cup B) + n(A \cap B) = n(A) + n(B)$

$\Rightarrow n(A \cup B) = 24$

Hence, the correct option is (c).

36. $A = \{1, 2, 3, 4\}$

$B = \{2, 4, 8, 9\}$

$A - B = \{1, 3\}$

$B - A = \{8, 9\}$

$\therefore\ (A - B) \cup (B - A) = \{1, 3, 8, 9\}$

Hence, the correct option is (a).

37. $(A - B) \cap (B - A) = \phi$

Hence, the correct option is (d).

38. None of the given statements are false.

Hence, the correct option is (d).

39. Both the given statements are true.

Hence, the correct option is (c).

40. Disjoint sets have no elements in common.

Hence, the correct option is (c).

41. Since $\{i, o\}$ is an element of Y.

$\therefore\ \{i, o\} \in Y$.

Hence, the correct option is (b).

42. $U = \{a, b, c, d, e, f, g, h, i, j, \ldots, z\}$

$C = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

$\therefore\ C' = U - C = \{a, e, i, o, u\}$

Hence, the correct option is (d).

43. If a set has n elements, then number of all possible subsets = 2^n.

Since O contains only one element

$\Rightarrow$ Number of subsets = $2^1 = 2$

Hence, the correct option is (a).

44. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$35 = 10 + n(B) - 5$

$\Rightarrow n(B) = 35 - 10 + 5 = 30$

$\therefore\ n(B) = 30$

Hence, the correct option is (a).

45. $X - Y = \{0, 2, 4, 6, 8, 10\}$

Hence, the correct option is (c).

46. $A = \{27, 64\}$

$B = \{16, 25, 36, 49, 64, 81\}$

$A \cap B = \{64\}$

$\therefore\ n(A \cap B) = 1$

Hence, the correct option is (b).

SELF ASSESSMENT EXERCISE

TEST YOUR CONCEPTS

Very Short Answer Type Questions

Directions for questions 1 to 5: State whether the following statements are true or false.

1. If $A = \{2, 3, 4\}$ and $B = \{3, 5\}$, then $A \cap B$ has only one element.
2. If $A = \{1, 2, 3, 4, 5\}$, then $2 \in A$.
3. If $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, then $A \cup B$ has 5 elements.
4. If $A \subset B$, then $B \cap A = B$.
5. If $A \subseteq B$ and $B \subseteq A$, then $A = B$.

Directions for questions 6 to 10: Fill in the blanks.

6. Two sets having no element in common are called _______ sets.
7. The set of whole numbers is a/an _______ set. (finite/infinite)
8. If $X = \{1, 3, 5, 7, 9\}$, then the cardinal number of X is _______.
9. If $Y = \{2, 4, 6, 8\}$, then the number of proper subsets of X is _______.
10. If $Z = \{a, b, c\}$, then the number of non-empty subsets of Z is _______.

Directions for questions 11 to 14: Which of the following collections is/are sets?

11. All rich people in your city.
12. All intelligent students in your school.
13. All fat boys in your colony.
14. All the boys of your class whose height exceeds 150 cm.

Directions for questions 15 to 28: Select the correct answer from the given options.

15. If a set has 2 elements, then how many proper subsets are there for the given set?
    (a) 4 (b) 3
    (c) 2 (d) 1
16. Which of the following is a null set?
    (a) $\{1\}$
    (b) $\{\phi\}$
    (c) $\{x/x$ is a composite number less than 5.$\}$
    (d) $\{x/x$ is an even prime number more than 2.$\}$
17. If $A = \{a, e, i, o, u, a, e, i\}$, then $n(A) =$ _______.
    (a) 4 (b) 8
    (c) 16 (d) 5
18. If $A = \{x/x + 10 = 10\}$, then $n(A) =$ _______.
    (a) 0 (b) 1
    (c) 4 (d) 2
19. If $A = \{a, b, c\}$ and $X = \{x, y, z\}$, then $A \cap X =$ _______.
    (a) A (b) X
    (c) ϕ (d) μ
20. If $X = \{a, e, i, o, u\}$, then which of the following is a correct statement?
    (a) $e \in X$ (b) $e \subset X$
    (c) $e \notin X$ (d) $e \not\subset X$
21. If $Y = \{\{a, e\}, \{i, o\}, u\}$, then which of the following is a correct statement?
    (a) $\{a, e\} \subset Y$
    (b) $\{a, e\} \in Y$
    (c) $\{\{a, e\}\} \in Y$
    (d) $\{a, e\} \notin Y$
22. If $E = \{x/x$ is a factor of $8\}$, $F = \{x/x$ is a factor of $16\}$, and $G = \{x/x$ is a factor of $13\}$, then which of the following statements is true?
    (a) $E \subset G$ (b) $G \subset E$
    (c) $E \subset F$ (d) $F \subset E$
23. Write the difference of the sets containing the letters of the words 'MATHEMATICS' and 'SOCIALMAN'.
    (a) $\{M, A, T\}$
    (b) $\{L, M, N\}$
    (c) $\{T, H, E\}$
    (d) $\{I, C, S\}$

24. If $n(A) = 20$, $n(B) = 30$ and $n(A \cup B) = 45$, then find $n(A \cap B)$.

(a) 5 (b) 10

(c) 15 (d) 20

25. If $A = \{a, e, i, o, u\}$ and $B = \{a, i, e, c, d\}$, then $n(A - B)$ is ________.

(a) 0 (b) 1

(c) 2 (d) 3

26. If $A = \{1, 2, 3\}$, then $n(P(A))$ is ________.

(a) 3 (b) 8

(c) 4 (d) 6

27. The cardinal number of the set containing letters of the word 'MOONROCK'.

(a) 8 (b) 6

(c) 4 (d) 2

28. If A and B are disjoint, then $n(A \cap B) =$ ________.

(a) 4 (b) 2

(c) 1 (d) 0

ANSWERS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. True
2. True
3. False
4. False
5. True
6. disjoint
7. infinite
8. 5
9. 15
10. 7
11. Not a set
12. Not a set
13. Not a set
14. Set
15. (b)
16. (d)
17. (d)
18. (b)
19. (c)
20. (a)
21. (b)
22. (c)
23. (c)
24. (a)
25. (c)
26. (b)
27. (b)
28. (d)

INDRANIL GHOSH